INDUSTRIAL ELECTRICITY

PART I

ELECTRICAL ENGINEERING TEXTS

A series of textbooks outlined by a committee of well-known electrical engineers of which Harry E. Clifford, Gordon McKay Professor of Electrical Engineering, Harvard University, is Chairman and Consulting Editor.

Laws—
ELECTRICAL MEASUREMENTS

Lawrence—
PRINCIPLES OF ALTERNATING-CURRENT MACHINERY

Lawrence—
PRINCIPLES OF ALTERNATING CURRENTS

Langsdorf—
PRINCIPLES OF DIRECT-CURRENT MACHINES

Dawes—
COURSE IN ELECTRICAL ENGINEERING
Vol. I.—Direct Currents
Vol. II.—Alternating Currents

Dawes—
INDUSTRIAL ELECTRICITY—PART I

Dawes—
INDUSTRIAL ELECTRICITY—PART II

ELECTRICAL ENGINEERING TEXTS

INDUSTRIAL ELECTRICITY

PART I

BY

CHESTER L. DAWES, S. B.

Assistant Professor of Electrical Engineering, The Harvard Engineering School;
Member, American Institute of Electrical Engineers, Etc.

FIRST EDITION
SECOND IMPRESSION

McGRAW-HILL BOOK COMPANY, INC.
NEW YORK: 370 SEVENTH AVENUE
LONDON: 6 & 8 BOUVERIE ST., E. C. 4
1924

PRINTED IN THE UNITED STATES OF AMERICA

THE MAPLE PRESS COMPANY, YORK, PA.

PREFACE

The rapid increase in the industrial applications of electricity has resulted in the establishment of elementary electrical engineering courses in many technical high schools and in other schools not of collegiate grade. Because of the resulting demand for textbooks suitable for such courses, the author was requested to prepare such books. Since a considerable portion of the author's two volumes "A Course in Electrical Engineering" (Vol. I, "Direct Currents" and Vol. II, "Alternating Currents") is devoted to fundamental considerations, the Consulting Editor of these Electrical Engineering Texts and the author both agreed that some of this material could be utilized advantageously in the preparation of the present volumes. As the chapters in the author's Volume I, "Direct Currents" are arranged in the order which would naturally be followed by the majority of teachers giving courses in magnetism and direct currents, the arrangement and titles of chapters in the present volume conform very closely to those given in the previous work.

Throughout the text, the attempt has been made to explain in simple language the principles underlying electrical engineering and electrical-engineering apparatus and to give a bird's-eye view of electrical engineering and its problems to the student who is beginning to study the subject, either by himself or in courses of the grade of those given in the technical high schools.

With the possible exception of the incandescent lamp, nearly every important industrial application of electricity involves an electric circuit interlinked with a magnetic circuit. A knowledge of the underlying principles of these two circuits is necessary for an understanding of these industrial applications, and the first five chapters in this volume are arranged to give the student a good grounding in the elementary principles of magnetism and the electric circuit. A large number of illustrative problems and their solutions are given in order to show concrete applications of these principles.

Owing to the increasing industrial importance of batteries, due in large measure to their wide use for automobiles and radio apparatus, their underlying principles, their care and applications are given in some detail in Chapter VI. Industrial installations of electrical apparatus require a knowledge of conductor resistance, insulation resistance, volts, amperes, power, energy, etc. Chapter VII discusses various commercial methods of making electrical measurements and also considers the simple principles underlying electrical-measuring instruments. The author is convinced that ability to make electrical measurements accurately and intelligently is important to every person engaged in electrical work.

Sufficient theory and descriptive matter are given in the chapters on the magnetic circuit and on electrostatics to enable a student, pursuing an elementary course, to understand the phenomena which he may be called upon to analyze in industrial apparatus. The last four chapters are devoted to the construction and operation of electrical machinery. The ordinary operating characteristics are given and briefly analyzed with reference to the underlying principles of the electric and magnetic circuits. Considerable attention is also given to the relation of these characteristics to the industrial applications of direct-current machinery. Since the electrical equipment of automobiles represents a very common industrial application of electricity, a typical ignition system is described in Chapter VIII and a typical lighting and starting system is described in Chapter XIII. Direct-current power distribution is given in Part II, with alternating-current power transmission and distribution, since the two are so closely related.

The author is greatly indebted to Prof. H. E. Clifford, the Consulting Editor of these Electrical Engineering Texts, for his careful review of the manuscript and his many criticisms and suggestions.

C. L. D.

Harvard University, Cambridge, Mass.
August, 1924.

CONTENTS

CHAPTER V

CHAPTER VI

CHAPTER VII

CHAPTER VIII

CHAPTER IX

CHAPTER X

CHAPTER XIII

APPENDIX A

APPENDIX B

APPENDIX C

APPENDIX D

APPENDIX E

APPENDIX F

INDUSTRIAL ELECTRICITY

PART I

CHAPTER I

MAGNETS AND MAGNETISM

1. Magnets and magnetism are involved in the operation of practically all electrical apparatus. Therefore, an understanding of their underlying principles is essential to a clear conception of the operation of all such apparatus.

2. Magnetic Materials.—Iron (or steel) is far superior to all other metals and substances as a magnetic material and is practically the only metal used for magnetic purposes. Cobalt and nickel (and some of their alloys) possess magnetic properties, which are far inferior to those of iron. Liquid oxygen is also attracted to the poles of magnets.

3. Natural Magnets.—Magnetic phenomena were first noted by the ancients. Certain stones, notably at Magnesia, Asia Minor, were found to have the property of attracting bits of iron, hence the name *magnets* was given to these magic stones. The fact that such stones had the property of pointing north and south, if suspended freely, was not discovered until the tenth or twelfth century. The practical use of such a stone in navigation gave it the name of *Lodestone* or leading stone. Natural magnets are composed of an iron ore known in metallurgy as magnetite, having the chemical composition Fe_3O_4.

4. Artificial Magnets.—If a piece of hardened steel be rubbed with lodestone it will be found to have acquired a very appreciable amount of magnetism, which it will retain indefinitely.

Such a steel magnet is called an *artificial magnet.* Artificial magnets commonly derive their initial excitation from an electric current, as will be shown later. If a piece of soft steel or soft iron be similarly treated, it retains but a very small portion of the magnetism initially imparted to it.

These properties make it desirable to use hardened steel when a permanent magnet is desired and to use soft iron or steel when it is essential that the magnetism respond closely to changes of magnetizing force. It is found that even hardened steel ages or loses some of its magnetism with time. Where a high degree of permanency is desired, as in electrical instruments, or even in magnetos, the magnets are aged artificially.

5. Magnetic Field.—It is found that magnetism manifests itself as if it existed in lines called *lines of magnetism* or *lines*

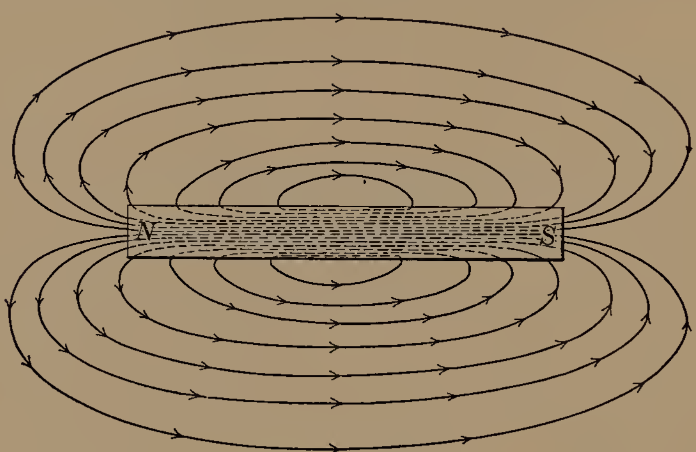


Fig. 1.—Magnetic field about a bar magnet.

of induction. These magnetic lines taken as a whole are called the *magnetic flux.* The region or space in which these lines exist is called the *magnetic field.* Further, if the lines of induction of a field due to a magnet be determined experimentally, it is found that they seem to emanate from one region of the magnet and enter some other region, as shown in the bar magnet, Fig. 1. These regions are called the *poles* of the magnet. The two poles are distinguished by the position which they seek if suspended freely. The one which points north is called the *north-seeking pole,* or *north pole* for short, and the other the *south-seeking pole,* or *south pole.* In practice it is assumed that

the lines of induction leave the magnet at the north pole and reenter the magnet at the south pole. Within the magnet the lines of induction continue from the south to the north pole so that each line of induction forms a closed loop, as shown in Fig. 1. The plane midway between the poles is the *neutral zone* or *equator* of the magnet. The entire path through which the lines of induction pass is called the *magnetic circuit.*

6. Effect of Breaking a Bar Magnet.—Neither a north pole nor a south pole can exist alone. For every north pole there

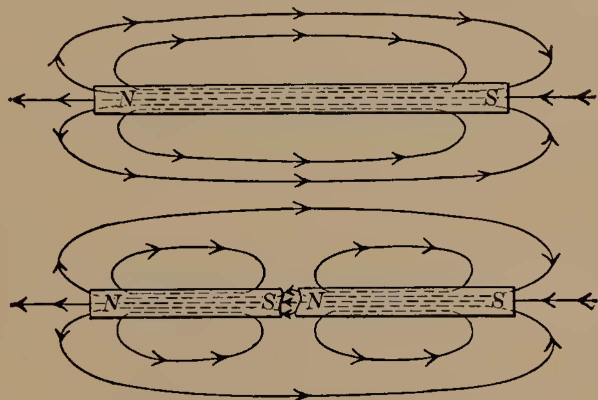


FIG. 2.—Effect of breaking a bar magnet.

exists an equal south pole. If an ordinary bar magnet be broken at the middle, or at various points, each fragment will constitute a bar magnet having its north and its south pole lying in the same respective directions as those of the original magnet. This phenomenon is easily explained by noting that the lines of induction still continue to pass from one fragment to the next adjacent one, and in so doing constitute north and south poles, as shown in Fig. 2. Experimentally, this phenomenon may be illustrated by magnetizing a highly-tempered steel knitting needle and breaking it at various points.

7. Weber's Theory.—An explanation of the appearance of north and south poles on breaking a magnet, and of other phenomena associated with the magnetization of iron, is offered by Weber's theory, which has been expanded by Ewing. Each molecule of a magnet is assumed to be a very small magnet, as shown in Fig. 3 (*a*). Under ordinary conditions the small magnets are arranged in a haphazard way, as shown at (*a*), so that

the various north and south poles all neutralize one another, and no external effect is produced. Upon the application of a magnetizing force, however, the small magnets tend to arrange themselves in such a manner that their axes are parallel and their north poles are all pointing in the same general direction as the magnetizing force. This is shown in Fig. 3 (*b*). Figure 3 (*c*) shows a section of bar magnet to which a magnetizing force has

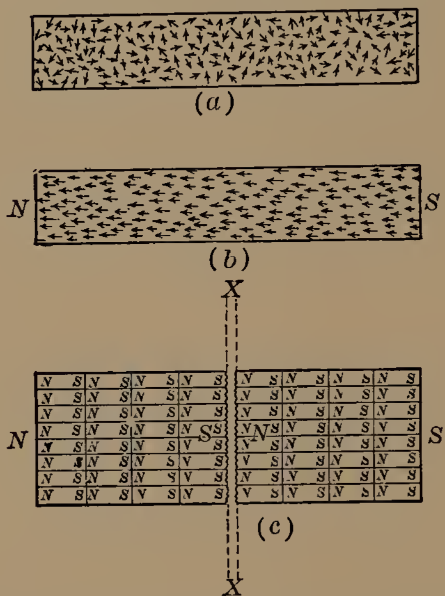


FIG. 3.—Weber's theory of molecular magnets.

been applied. At one end of the magnet the molecular *N*-poles combine to form the north pole of the magnet. Likewise, at the other end of the magnet the molecular *S*-poles combine to form the south pole of the magnet. Within the magnet, however, each *N*-pole is neutralized by the adjacent *S*-pole. If the magnet be broken at *XX*, the molecular *N*- and *S*-poles no longer neutralize one another along the sections thus produced, so that a new *N*-pole is formed on the left-hand section and a new *S*-pole on the right-hand section.

This theory is further substantiated by grinding a permanent magnet into very small particles. Each of the small particles possesses the properties of the bar magnet, each having its own north and its own south pole. Further, the theory offers a rational explanation of saturation, hysteresis, etc., occurring

in iron subjected to a magnetizing force. These effects will be considered later.

8. Magnetic Force.—When a freely-suspended north pole is brought in the vicinity of another north pole, it is repelled, whereas, if a south pole is brought in the vicinity of a north pole, it is attracted to the north pole. South poles are also found to repel one another. From this it may be stated that *like poles repel one another and unlike poles attract one another.*

9. Pole Strength.—The force of attraction (or repulsion) between two given poles is found to be inversely as the square of the distance between the poles, provided the dimensions of the poles are small compared with the distance between them. *A unit magnetic pole is one of such strength that if placed at a distance of one centimeter in free space from a similar pole of equal strength will repel it with a force of one dyne.*

Pole strength is measured by the number of unit poles which, if placed side by side, would be equivalent to the pole in question.

The force f, existing between poles in air, may be formulated as follows:

$$f = \frac{m\,m'}{r^2} \text{ dynes} \tag{1}$$

where m and m' are the respective pole strengths (in terms of a unit pole) of two magnetic poles, placed a distance r cm. apart, as shown in Fig. 4. This force may be attraction or repulsion, according as the poles are unlike or like.

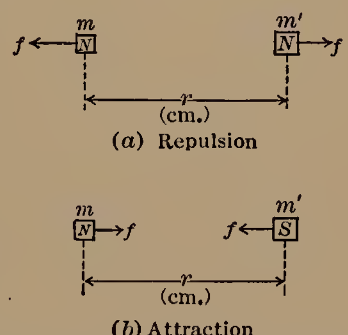


Fig. 4.—Repulsion and attraction between magnetic poles.

Example.—Two north poles, one having a strength of 500 units and the other a strength of 150 units, are placed 4.0 in. (10.16 cm.) apart in air. What is the force in grams acting between these poles, and in what direction does it act?

$$4.0 \text{ in.} = 4.0 \times 2.54 = 10.16 \text{ cm.}$$

$$f = \frac{500 \times 150}{(10.16)^2} = \frac{75{,}000}{103.2} = 727 \text{ dynes}$$

$$\frac{727}{981} = 0.742 \text{ gram.}$$ Poles repel each other. *Ans.*

10. Lines of Force.—Thus far the magnetic field has been studied only with respect to the lines of magnetism or induction. If a single north pole be placed in such a field, two effects will be observed:

1. This pole will be urged along the lines of induction.
2. The force urging this pole will be greatest where the lines of induction are most dense, and, moreover, the force at a point will be proportional to the number of lines per unit area at the point taken perpendicular to the lines in the field in which the pole finds itself.

From these statements it can be seen that *lines of force*, similar to lines of induction, can be drawn to represent the force at the various points in the magnetic field. In much of the literature of the subject, lines of induction and lines of force are used indiscriminately. The fallacy of so doing is immediately apparent upon considering a solid bar magnet. The lines of induction pass completely through the solid metal of the magnet, whereas the lines of force terminate at the poles. To be sure, a magnetic force does exist within the magnet, but this force can be determined only by making a cavity of special form in the magnet, and the force acting under these conditions is quite distinct from that indicated by the number of lines of induction passing through the bar. In air, however, the lines of force and lines of induction coincide.

11. Field Intensity.—It has been stated that the force acting on a magnetic pole placed in a magnetic field is proportional to the number of lines of force per square centimeter at that point. *Unit field intensity is defined as the field strength which will act on a unit pole with a force of one dyne.* Field intensity is usually given in *dynes per unit pole* and is usually represented by the symbol H. It follows that one *line of force* perpendicular to and passing through a square centimeter represents unit field intensity.. It is evident that if a pole of m units be placed in a field of intensity H, as shown in Fig. 5, the force acting on this pole will be

$$f = m \times H \text{ dynes} \tag{2}$$

A pole placed in such a field must be of such small magnitude that it will have no appreciable disturbing effect upon the magnetic field.

Example.—A total flux of 150,000 lines exists in air between two parallel pole faces, each 10 cm. square. The field is uniformly distributed. With what force (grams) will a pole, having a strength of 200 units, be acted upon if placed in this field?

Flux density $= \frac{150,000}{10 \times 10} = 1,500$ lines per square centimeter, or 1,500 gausses. Being in air, this value of flux density also equals the field intensity, H.

$$f = m \times H = 200 \times 1,500 = 300,000 \text{ dynes}$$

$$\frac{300,000}{981} = 306 \text{ grams. } \textit{Ans.}$$

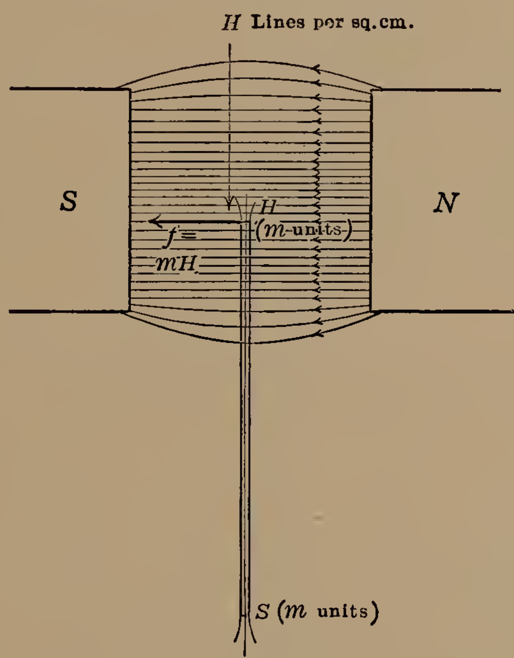


FIG. 5.—Force acting on magnetic pole in a magnetic field.

12. Flux Density.—Flux density is the number of lines of induction per unit area taken perpendicular to the induction. In free space, flux density and field intensity are numerically the same, but within magnetic material the two are entirely different. The two should not be confused. The unit of flux density (one line per square centimeter) is often called the *gauss*, but the expression "lines per square centimeter" and "lines per square inch" are often used in practical work when speaking of flux density.

13. The Compass Needle.—The compass consists of a hardened steel needle or small bar, permanently magnetized and

accurately balanced upon a sharp pivot. The north-seeking end or north pole points north, and the south-seeking end points south. The north pole of the needle is usually colored blue, or given some distinguishing mark. With the exception of a few used for lecture purposes, the needle is enclosed in an air-tight case for mechanical protection. Mariners' compasses are mounted carefully upon gimbals, so that they always hang level. Unless the compass is compensated, large errors may be introduced into mariners' compasses by the magnetic effects of the steel hull and the masses of iron in the machinery of the ship. This compensation is accomplished by placing heavy iron balls near the compass, which neutralize the magnetic effect of the ship itself.

By means of the compass, the polarity of a magnet is readily determined. The south pole of the compass points to the *north* pole of the magnet, as shown in Fig. 6. Likewise, the *north*

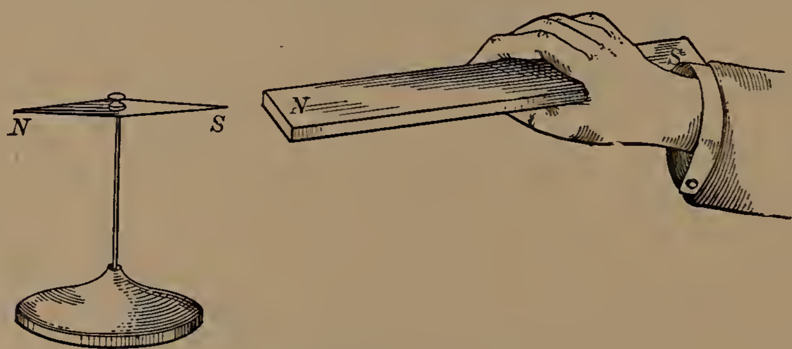


FIG. 6.—Compass needle and bar magnet.

pole of the compass points to the *south* pole of the magnet. This action of the compass needle follows immediately from the law that like poles repel and unlike poles attract each other. The compass is, therefore, very useful in practical work, for it enables one to determine the polarity of the various poles of motors and generators, thus showing whether or not the exciting coils are connected correctly.

Further, the compass needle always tends to set itself in the direction of the magnetic field in which it finds itself, the north end of the needle pointing in the direction of the lines of force or magnetic lines. This is illustrated in Fig. 7. By placing a

small compass at various points in the neighborhood of a magnet, and drawing an arrow at each point, the arrow pointing in the same direction as the needle, the field around the magnet may be mapped, as shown in Fig. 7. In mapping a field in this manner, it must be remembered that the earth's field may exert considerable influence on the compass needle in addition to the effect of the field being studied.

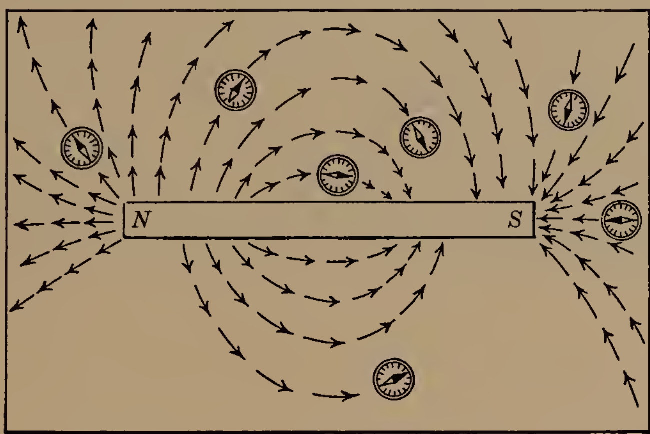


FIG. 7.—Exploring the field about a bar magnet with a compass.

14. Magnetic Figures.—If a card be placed over a magnet and iron filings be sprinkled over the card, a magnetic figure is obtained. The filings at each point set themselves in the direction of the line of force at that point, and the resultant figure shows in close detail the character of the magnetic field. Figure 8 shows the magnetic field due to two bar magnets placed side by side, with unlike poles adjacent. On the other hand, Fig. 9 shows the field due to these same bar magnets when like poles are adjacent. It will be noted in Fig. 8 that the lines of force seem like elastic bands stretched from one pole to the other, acting to pull the unlike poles together. In Fig. 9, the lines of force from the two like poles appear to repel one another, indicating a state of repulsion between the poles.

15. Magnetic Induction.—If a magnet is brought near a piece of soft, non-magnetized iron, the piece of iron becomes magnetized by *induction.* If the north pole of the magnet is brought near the iron, a south pole is induced in that part of the

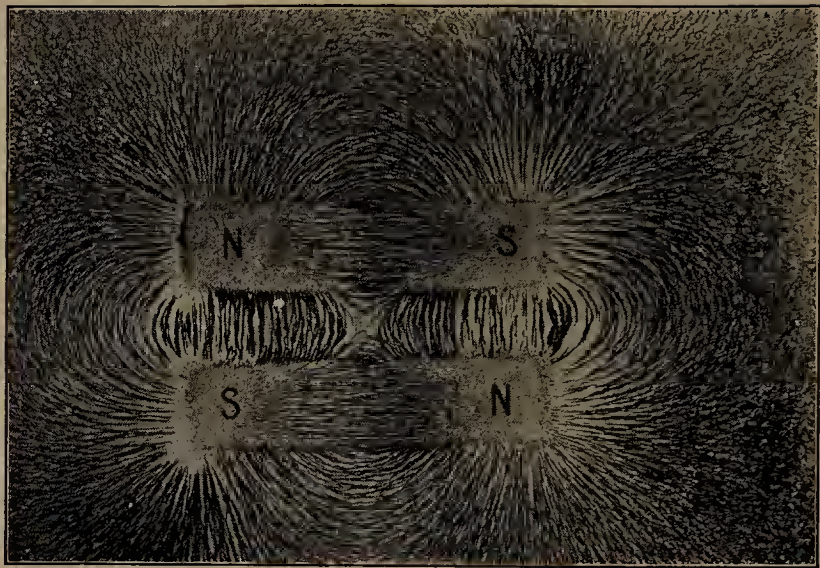


FIG. 8.—Magnetic figure, unlike poles adjacent.

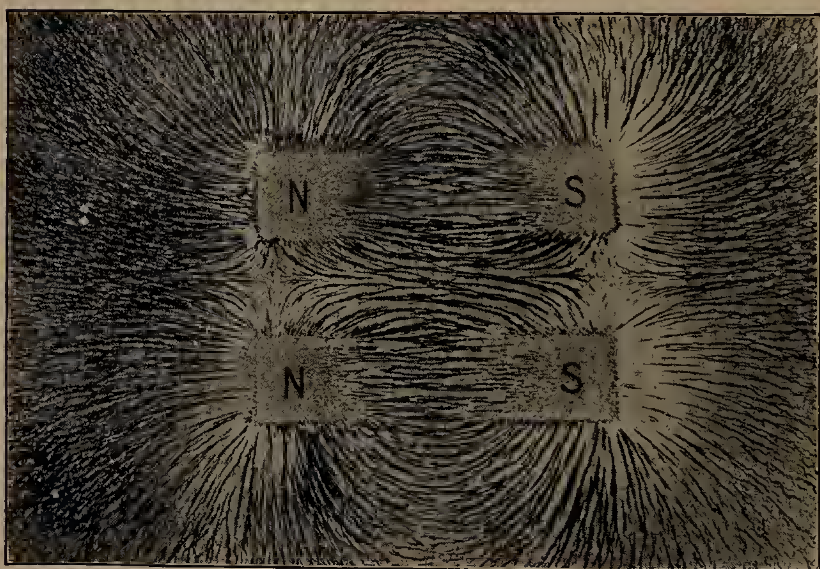


FIG. 9.—Magnetic figure, like poles adjacent.

iron nearest the inducing magnet and, if the south pole of the magnet is brought near the iron, a north pole is similarly induced. This is illustrated in Fig. 10. The reason that poles are

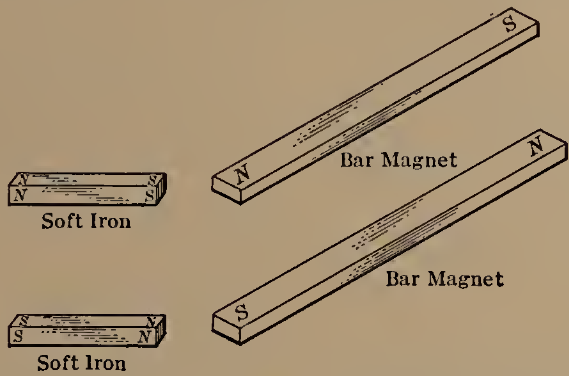


FIG. 10.—Poles produced by magnetic induction.

induced in soft iron is indicated in Fig. 11, where a bar of soft iron is brought in proximity to a north pole. A considerable number of the lines of induction leaving this north or inducing pole enter the adjacent end of the soft-iron bar readily and leave the soft-iron bar at its further end, because the magnetic con-

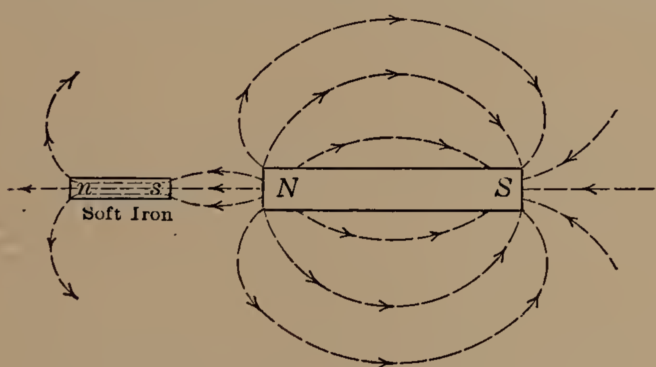


FIG. 11.—Relation of induced poles and magnetic lines.

ductivity of the iron is considerably greater than that of the air. This makes the end of the soft-iron bar nearest the inducing pole a south pole and its further end a north pole. From the foregoing, the ability of magnets to attract soft iron is readily understood. An opposite pole to that of the magnet is induced in the adjacent portions of the iron, and these two poles, being of unlike polarity, attract each other.

It is sometimes noticed that if a comparatively weak north pole be brought into the vicinity of a strong north pole, attraction between the two results, rather than the repulsion which might be expected. This is no violation of the laws governing the attraction and repulsion of magnetic poles, but comes from the fact that the strong north pole induces a south pole, which overpowers the existing weak north pole and results in attraction. It is easy to reverse the polarity of a compass needle by holding one end too close to a strong magnetic pole of the same polarity.

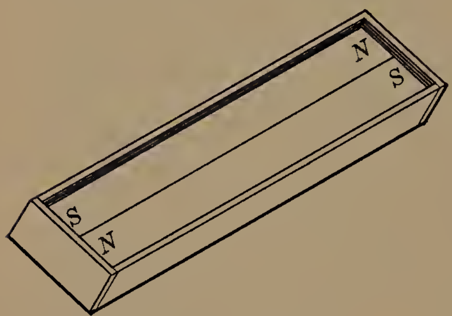


Fig. 12.—Proper method of "keeping" bar magnets.

For a similar reason, when two bar magnets are put away in a box, the adjacent ends should be of opposite polarity, as shown in Fig. 12. They will retain their magnetism better under these conditions. When a horseshoe magnet is not in use, a "keeper" of soft iron should be placed across the poles.

16. Law of the Magnetic Field.—*The magnetic field always tends so to conform itself that the maximum amount of flux is attained.* This offers further explanation of the attraction of iron to poles of magnets. The iron is drawn toward the magnet so that the magnetic lines may utilize it as a part of their return path, since iron conducts these lines much better than the air. This is illustrated in the horseshoe magnet of Fig. 14. The armature is drawn toward the poles of the magnet, and the return path through the air is materially shortened, so that the number of magnetic lines is materially increased. The maximum flux exists when the armature is in contact with the poles.

17. Other Forms of Magnets.—The simple bar magnet frequently is not suitable for practical work. For the same amount of material, other forms are more powerful and more compact. Figure 13 (*a*) shows a closed ring magnet. All the magnetic flux is contained within the ring and little or no external effect is noted. This type is not very useful. If the ring be cut as shown in Fig. 13 (*b*), however, a north and a south pole are

obtained. The flux in the gap formed by the cut can be utilized. For example, the moving coil of a galvanometer, or the vibrator of an oscillograph may be suspended therein.

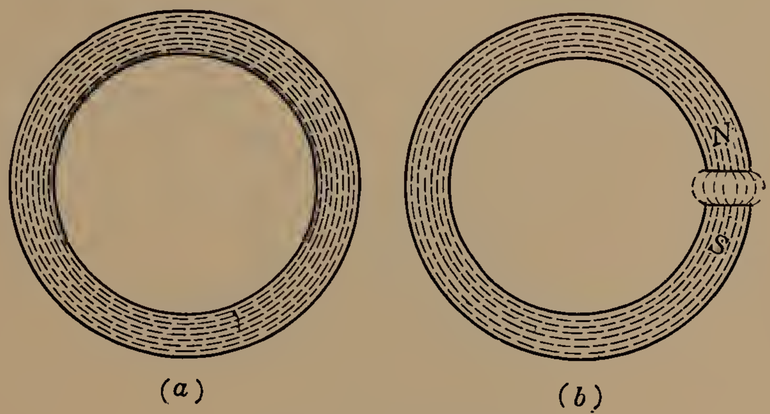


FIG. 13.—Ring magnets.

The horseshoe magnet, shown in Fig. 14, is very useful, for two reasons. Since the two poles are near each other, a comparatively strong field exists between them. Further, if the function of the magnet is to exert a pull upon an armature, each pole is equally effective. Figure 114, Chap. VII, page 121, shows a horseshoe magnet such as is used in Weston direct-current instruments.

18. Laminated Magnets.—It is found that thin steel magnets are stronger in proportion to their weight than thick

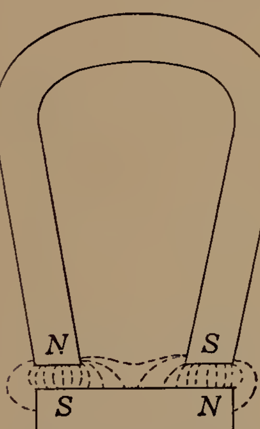


FIG. 14.—Horseshoe magnet attracting a soft-iron armature.

FIG. 15.—Compound or laminated bar magnet.

ones. For a given amount of material, a magnet made up of several laminations, as shown in Figs. 15 and 16, is more powerful than one made of a solid piece of metal. This is

probably due in a large measure to the fact that during heat treatment the material within the thicker magnets is not properly tempered. Figure 16 shows the form of horseshoe magnet generally used for telephone and ignition magnetos.

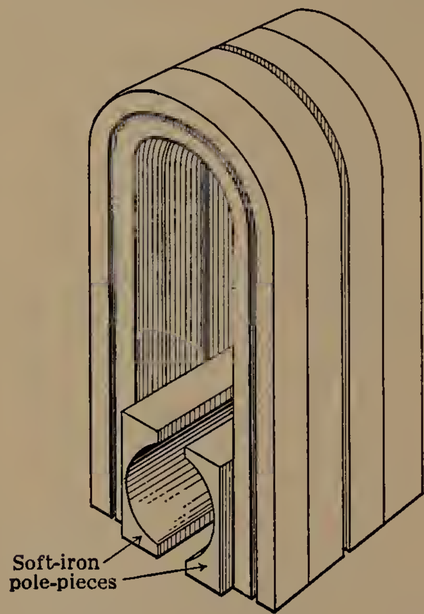


FIG. 16.—Compound horseshoe magnet used in magnetos.

19. Magnet Screens.—There is no known insulator for magnetic flux. No appreciable change in the flux or in the pull

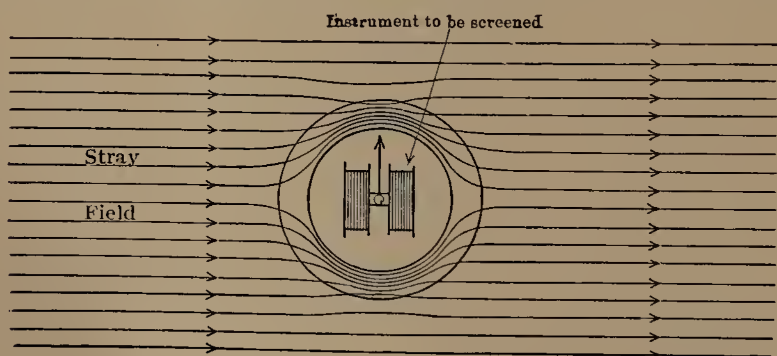


FIG. 17.—Magnetic screen.

of a magnet is noticed if glass, paper, wood, copper, or other such material be placed in the magnetic field. However, it is often desirable to shield galvanometers and electrical measuring

instruments from the earth's field and from stray fields due to generators, conductors carrying currents, etc. This is done by surrounding the instrument with an iron shell, as shown in Fig. 17. This shell by-passes practically the entire flux and thus prevents it from affecting the sensitive portions of the instrument. The smaller the openings in the shell, the more effective the screening becomes.

20. Magnetizing.—A magnet may be magnetized by merely rubbing it with another magnet. The resulting polarity at any point is opposite to that of the last pole which came in con-

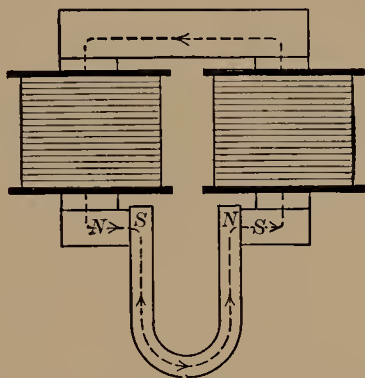


Fig. 18.—Magnetizing a horseshoe magnet with an electromagnet.

tact with this point. Stronger magnets may be obtained by placing them between the poles of a very powerful electromagnet. Figure 18 shows this method of magnetizing a horseshoe magnet. An armature or "keeper" should be placed across the poles of the horseshoe magnet before removing it from the electromagnet. The most common method of magnetizing permanent magnets, however, is to insert the magnets in a suitable exciting coil and cause a heavy current to flow in the coil.

21. The Earth's Magnetism.—The earth behaves as if it were a huge bar magnet, the poles of which are not far from the geographical poles. The north magnetic pole (corresponding to the south pole of a magnet) is situated in Boothia Felix, about 1,000 miles from the geographical north pole. The south magnetic pole has never been located, but experiment points to the existence of two south poles. Due to the non-coincidence of the geographical and magnetic poles and to the presence of magnetic materials in the earth, the compass points to the true north in

only a few places on the earth's surface. The deviation from the true north is called the *declination,* and magnetic maps are provided showing the declination at various parts of the earth. At New York it is about 9° west.

A freely suspended and balanced needle does not take up a position parallel to the earth's surface, when under the influence of the earth's magnetism alone, but assumes a position making some angle with the horizontal. This angle is called the *dip* of the needle. At New York it is about 70° north.

The field intensity (total, not horizontal) of the earth's field at New York is about 0.61 c.g.s. units, although this value changes slightly with the time.

CHAPTER II

ELECTROMAGNETISM

22. Magnetic Field Surrounding a Conductor.—It had long been suspected that some relation existed between electricity and magnetism, but it remained for Oersted in 1819 to show that this relation not only existed but that it was a definite relation.

If a compass be brought into the neighborhood of a single conductor carrying an electric current, the needle deflects, indicating the presence of a magnetic field. It is further observed that the needle always tends to set itself at right angles to the current. When it is held above the conductor, the needle points in a direction opposite to that which it assumes when held beneath the conductor. Further investigation shows that the magnetic flux exists in circles about the conductor (if there is no other magnetic field in the vicinity), as shown in Figs. 19, 20

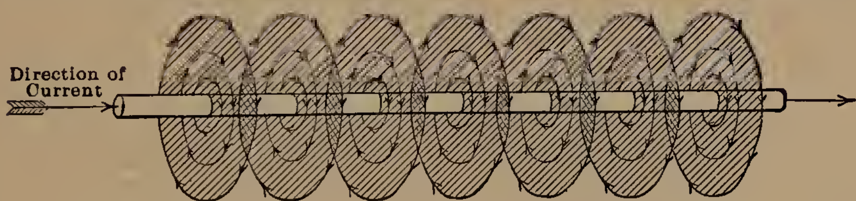


FIG. 19.—Magnetic field about a straight conductor.

and 21. These circles have their common center on the axis of the conductor and their planes are perpendicular to the conductor. If the current in the conductor be reversed, the direction in which the compass needle is deflected will also reverse, showing that the direction of this magnetic field depends on the direction of the current. This relation is shown in Fig. 19. The fact that the magnetic flux exists in circles perpendicular to the conductor explains the reversal in direction of the compass needle when moved from a point above the conductor to a point beneath it, since the direction of the field above the conductor must be

opposite to that beneath the conductor. This is illustrated in Figs. 20 and 21.[1]

The experiment shown in Fig. 22 is illustrative of this concentric relation of the magnetic flux to the conductor. A conductor

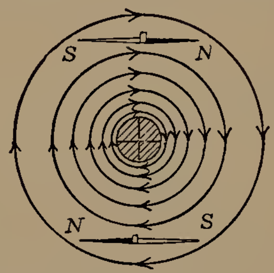


FIG. 20.—Lines of flux surrounding a cylindrical conductor—current inwards.

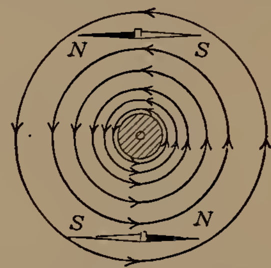


FIG. 21.—Lines of flux surrounding a cylindrical conductor—current outwards.

carrying a current is brought vertically down through a horizontal sheet of cardboard. Iron filings sprinkled on the cardboard form concentric circles. (A current of about 100 amp. is necessary to obtain distinct figures.) If four or more compasses are arranged as shown in Fig. 22, they will indicate, by the direction in which the needles point, that the magnetic lines are circles having the axis of the wire as a center.

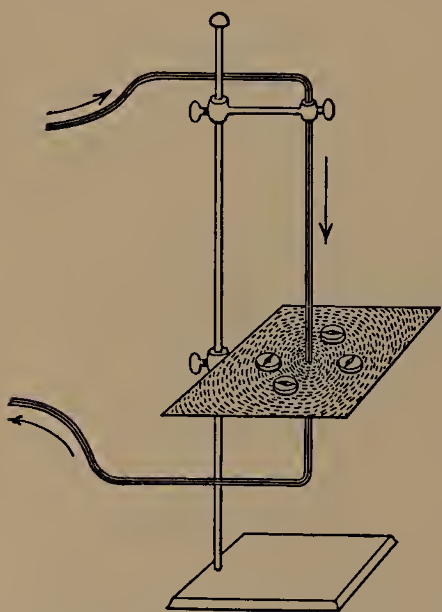


FIG. 22.—Investigation of the magnetic field surrounding a conductor.

23. Relation of Magnetic Field to Current.—A definite relation exists between the direction of the current in a conductor and the direction of the magnetic field surrounding the conductor. There are two simple rules by which this relation may be remembered.

Hand Rule.—Grasp the con-

[1] A circle having a cross inside (⊕) indicates that the current is flowing into the paper, the cross representing the feathered end of an arrow. A circle having a dot at the center (⊙) indicates that the current is flowing out of the paper, the dot representing the approaching tip of an arrow.

ductor in the right hand with the thumb pointing in the direction of the current. The fingers will point in the direction of the lines of flux (Fig. 23).

Corkscrew Rule.—The direction of the current and that of the resulting magnetic field are related to each other as the forward travel of a corkscrew and the direction in which it is rotated.

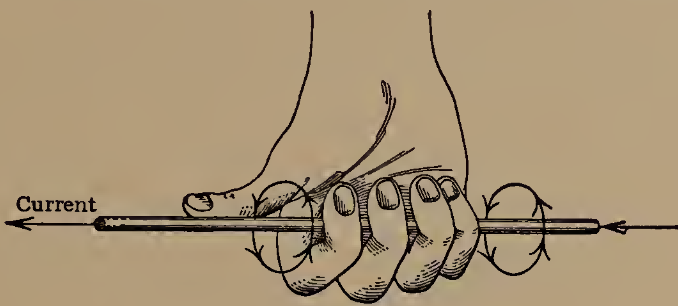


FIG. 23.—Hand rule.

This last rule is probably the most common and the most easily remembered. It must not be inferred from this rule, however, that the magnetic field exists in spirals about the conductor. It exists actually in planes perpendicular to the conductor.

24. Magnetic Field of Two Parallel Conductors.—When each of two parallel conductors carries an electric current, flowing

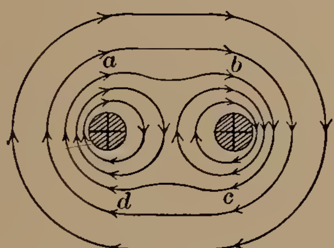


FIG. 24.—Magnetic field about two parallel conductors—current in same direction.

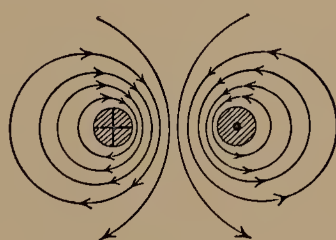


FIG. 25.—Magnetic field about two parallel conductors—current in opposite directions.

in the same direction, there is a tendency for the two conductors to be drawn together. The reason for this is obvious. In Fig. 24, the lines of force encircle each conductor in the same direction (corkscrew rule), and the resultant field is an envelope of lines tending to pull the conductors together. Further reason for this attraction is given by the rule of Par. 16 stating that the magnetic field tends so to conform itself that the number

of magnetic lines is a maximum. The pulling together of the conductors reduces the length of path *abcd* through which the lines must pass. In any given plane, perpendicular to the conductors, the field *due to each conductor separately* is still circular in form, but the *resultant magnetic field* is no longer circular, as is shown in Fig. 24.

In Fig. 25 is shown the field which results when two parallel conductors carry current in opposite directions. In any given plane, perpendicular to the conductors, the magnetic lines are circles, but these circles are not concentric either with one another

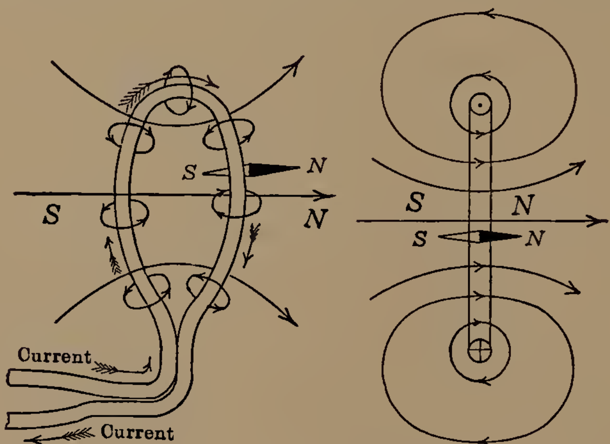


Fig. 26.—Magnetic field produced by a single-turn carrying current.

or with the conductor. The lines are crowded between the conductors, and therefore tend to push the conductors farther apart. Again, when the conductors separate, the area through which the flux passes is increased, so that the electric circuit in this case tends so to conform itself that the magnetic flux is a maximum.

From the foregoing, the following rules may be formulated. *Conductors carrying current in the same direction attract one another; conductors carrying current in opposite directions repel one another.*

All electric circuits tend to take such a position as will make their currents parallel and flowing in the same direction.

This effect is especially pronounced in modern, large-capacity power systems. Bus-bars have been wrenched from their clamps; transformer coils have been pulled out of place and transformers

wrecked by the forces produced by the enormous currents arising under short-circuit conditions.

25. Magnetic Field of a Single Turn.—If a wire carrying a current be bent into a loop, a field similar to that shown in Fig. 26 results. The direction of the field is determined by the corkscrew rule. The electric circuit has a north pole and a south pole, which possess all the properties of similar poles of a short permanent magnet. A compass needle placed in this field assumes the direction shown, the north pole pointing in the direction of the magnetic field.

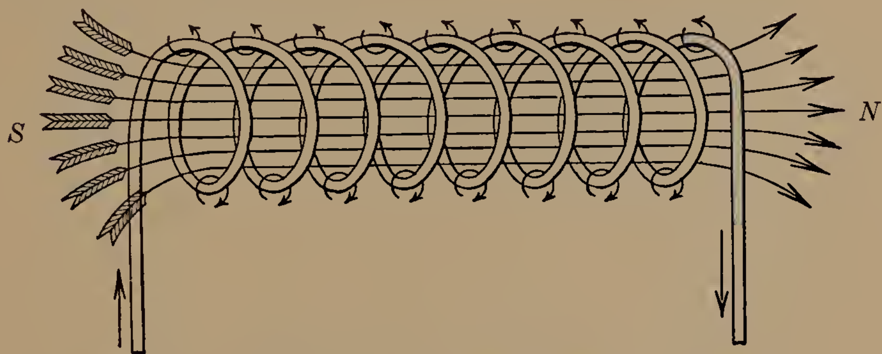


FIG. 27.—Magnetic field produced by a helix or solenoid.

26. The Solenoid.—An electric conductor wound in the form of a helix and carrying current is called a *solenoid*. A simple solenoid and the magnetic field produced within it are shown in Fig. 27. The solenoid may be considered as consisting of a large number of single turns, such as the turn in Fig. 26, placed together.

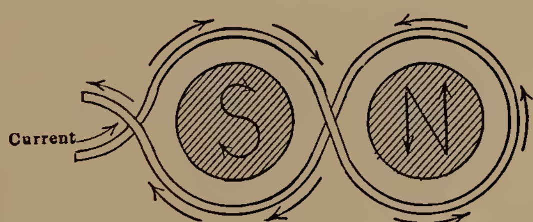


FIG. 28.—Relation of magnetic poles to direction of exciting current.

The solenoid winding may consist of several layers, as shown in Fig. 29. The solenoid, like the bar magnet, has a north and a south pole as shown in Fig. 27.

The relation of the direction of the flux within the solenoid to the direction in which the current flows in the helix may be

determined by the hand rule or by the corkscrew rule of Par. 23. Another simple method is shown in Fig. 28, where the arrows at the ends of the *N* and the *S* show the direction of current in the coil. For example, when looking down upon a north pole, the current direction in the coil will be counter-clockwise, as shown by the *N*; when looking down upon a south pole, the direction of the exciting current will be clockwise, as shown by the *S*.

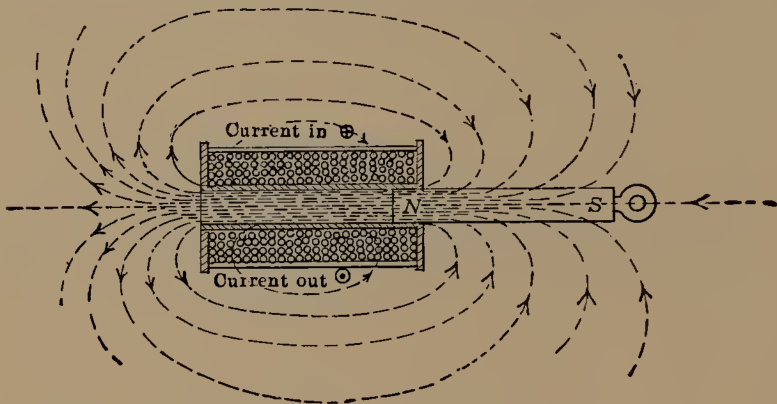


FIG. 29.—Simple solenoid and plunger.

27. The Plunger Solenoid. Industrial Applications of Solenoids.—In practice, the solenoid is used for tripping circuit breakers, for operating contactors in automatic motor-starters, for operating voltage-regulating devices, for arc-lamp feeds, for operating valves, and for numerous other purposes. In practically all cases a soft-iron (or steel) plunger or armature is

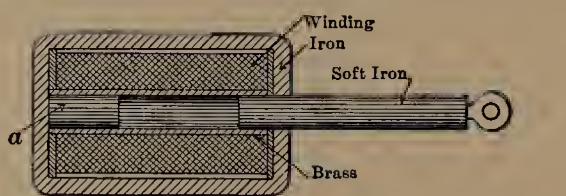


FIG. 30.—"Iron-clad" solenoid and plunger with stop.

necessary to obtain the tractive effort required of the solenoid. The operation of a solenoid and plunger is indicated in Fig. 29. The flux due to the solenoid produces magnetic poles on the plunger. The pole nearer the solenoid will be of such sign that it will be urged along the lines of force (see Par. 10), and in such a direction as to be drawn within the solenoid.

A position of equilibrium is reached when the center of the plunger reaches the center of the solenoid (Fig. 29). Figure 30 shows an "iron-clad" solenoid commonly used for tractive work. The iron-clad feature increases the range of uniform pull and produces a very decided increase of pull as the plunger approaches the end of the stroke. When a stop "*a*" is used, the solenoid becomes a *plunger electromagnet.* This changes the characteristics of the solenoid, in that the maximum pull now occurs when the end of the plunger is near the stop.

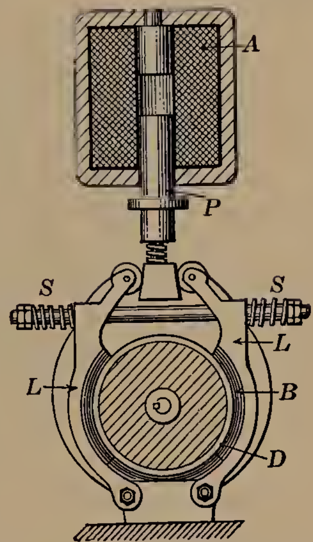


FIG. 31.—Plunger electromagnet operating a crane brake.

An important practical application of the solenoid occurs in the braking of elevators and cranes. When the power is removed from the lifting motor or when the power is interrupted due to a broken wire or other accident, the brake must be applied immediately. One method of accomplishing this is shown in Fig. 31. When the power, for any reason, is interrupted, the plunger *P* of the solenoid *A* drops, due partly to gravity and partly to the action of the springs *S*. The springs *S* immediately force the levers *L* against the brake bands *B*, pressing these against the brake drum *D*, thus effecting the braking action. When the power is applied to the lifting motor, the plunger *P* is pulled up, thus releasing the brake. A plunger electromagnet is most suitable for this purpose because the stroke is short and the pull must be positive.

28. The Telegraph Relay.—A common use of the solenoid is illustrated by the relay or the sounder used in telegraphy. To increase the pull upon the armature, two solenoids are used, each being placed on one of the legs of a horseshoe or U-shaped magnet. When the coils *C* (Fig. 32) are excited, the iron armature *A* is attracted because of the tendency of the magnetic lines to make their path of minimum length. As a rule, the armature *A* is not allowed to close the magnetic circuit completely, for under these conditions the magnetic lines still exist after the excitation

is removed, preventing rapid release of the armature. The stop S' prevents the armature making contact with the cores FF and thus completely closing the magnetic circuit. The two sets of contacts D close any secondary circuit that the relay may be operating. The spring T draws the armature back against a stop S when the excitation is removed.

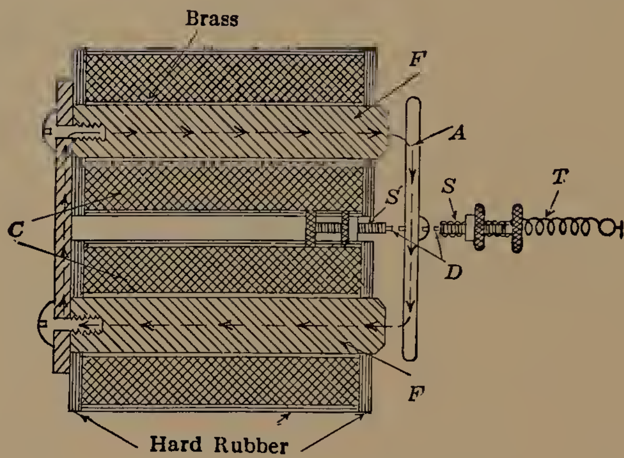


Fig. 32.—Telegraph relay.

29. Electric Bells, Buzzers.—Electric bells and buzzers also utilize the principle of the electromagnet in their operation. A

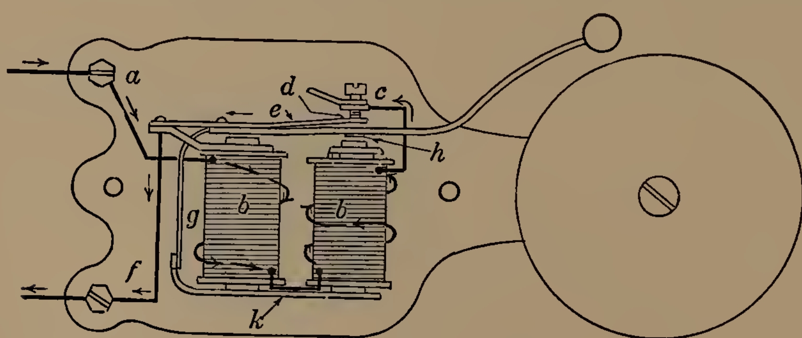


Fig. 33.—Electric bell.

diagram of a typical electric bell is given in Fig. 33. The path of the current is indicated by the arrows, the upper binding post a being assumed positive. Current leaving the binding post a passes through the two solenoids b, b, which are fastened to the iron yoke k. The solenoids b, b are connected in series and their magnetic fields act in opposite directions. The current next

passes to the stud c, then through the contact point d (usually of tungsten or platinum) to the contact spring e, which is fastened to the iron clapper-arm, and so to the binding post f. The spring g holds the clapper away from the gong and holds the spring e against contact d. The current causes the solenoids b, b to attract the clapper-arm. The clapper strikes the gong and the contact at d is broken, interrupting the current. The solenoids thus being deenergized, the spring g is able to restore the clapper-arm to its initial position, again closing the contact at d. The cycle is then repeated. A more or less rapid striking of the gong results. The small copper stop h, which prevents the clapper-arm from coming in contact with the core of the solenoid and sticking, should be noted.

30. The Lifting Magnet.—Lifting magnets are used commercially to handle iron and steel in various forms. A considerable saving of time and labor is effected by their use, because chains and slings for holding the load are unnecessary. They are useful for handling steel billets in rolling mills, although the

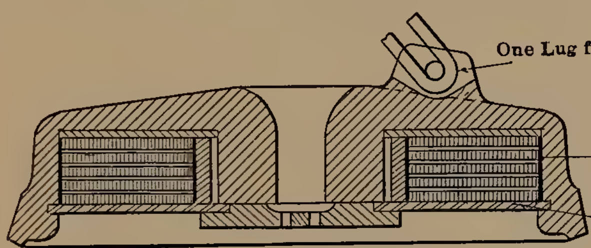


FIG. 34.—Typical lifting magnet.

billets cannot be picked up when red hot, as they lose their magnetic properties at high temperatures. Lifting magnets effect a great saving of labor when small pieces of iron, such as scrap iron, are handled, for they will pick up large quantities at every lift. Without such a magnet it would be necessary to handle each individual piece by hand. Figure 34 shows in cross-section a typical Cutler-Hammer lifting magnet.

Figure 35 shows a lifting magnet in actual operation.

It should be understood that the magnet itself does little or no work in the actual lifting, but, like a hook and chain, merely serves as a holding device. The actual work is performed by the engine or motor which operates the steel ropes or chains attached to the magnet.

31. Magnetic Separator.—Another important application of magnetic principles is found in the magnetic separator shown in Fig. 36. It is especially designed to remove steel and iron

FIG. 35.—Cutler-Hammer 36-in. magnet, handling heavy steel castings.

from coal, rock, ore, etc., but it may be used for separating steel shot from molding sand, iron chips from machine-shop turnings, etc. The material is fed on an endless belt running at a speed of about 100 ft. per minute. The belt passes over a magnetized

pulley. The non-magnetic material immediately drops off into a hopper, but the magnetic material is held by the pulley until

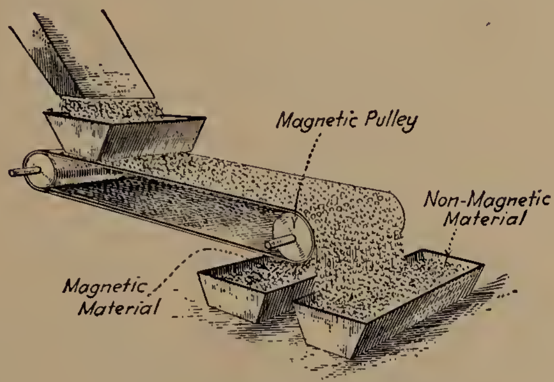


FIG. 36.—Magnetic separator

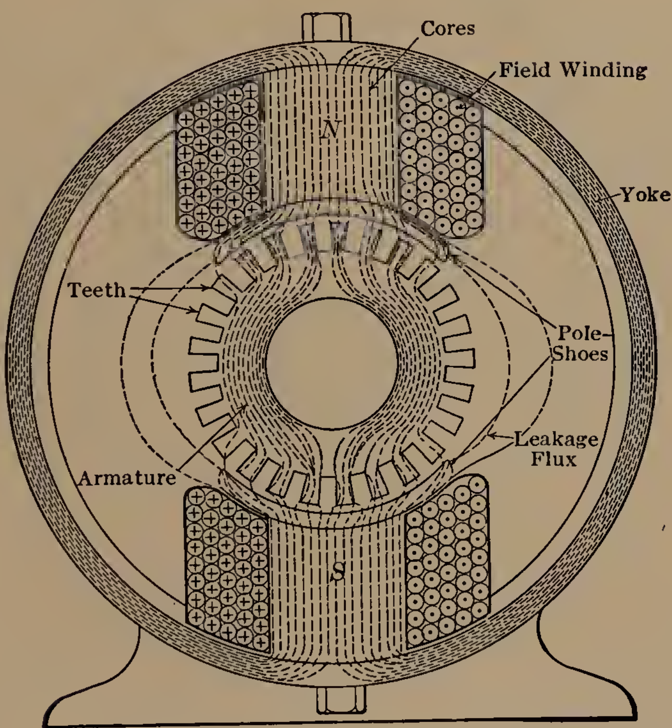


FIG. 37.—Magnetic circuit and field windings of a modern bipolar generator.

the belt leaves the pulley, when the material drops into a second hopper. The pulley is magnetized by concentric exciting coils, to which current is carried by means of slip-rings.

32. The Magnetic Circuits of Dynamos.—One of the most important uses of electromagnets is in the magnetic circuits of generators and motors.

The magnetic circuit of a bipolar generator of modern design is shown in Fig. 37. Some of the flux, called *leakage flux*, passes between the field-cores and shoes without entering the armature. In this type of machine the long air-path existing between the

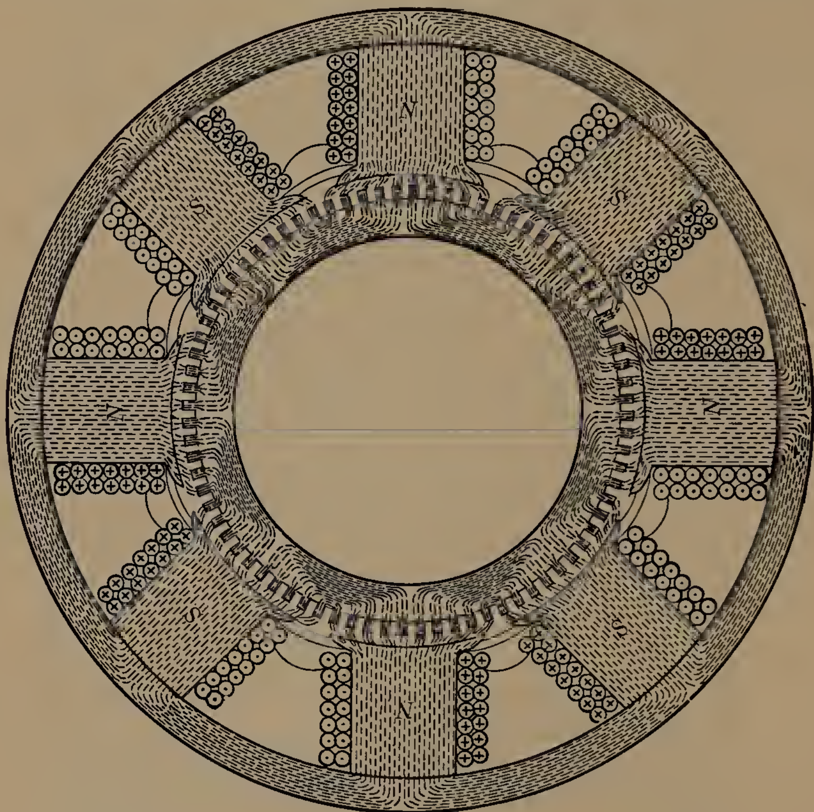


Fig. 38.—Magnetic circuits of a multipolar generator.

pole-shoes reduces this leakage flux to a minimum. It is to be noted that the flux in the cores divides as it passes into the armature and into the yoke. Ordinarily the yoke need be only one-half the cross-section of the field-cores. Direct-current machines of the bipolar type are made usually in small units.

Figure 38 shows the more complex magnetic circuits of a multipolar generator having eight poles, the poles being alternately north and south. The flux passing through the field-cores

divides, both on reaching the yoke and on reaching the armature iron, and the cross-section of the yoke and the armature iron need be only approximately one-half that of the cores. In both Figs. 37 and 38 the magnetic leakage is materially reduced by placing the exciting ampere-turns on the pole-cores as near the armature as possible.

It should be understood that, of itself, magnetic leakage does not directly lower the efficiency of a machine, since to maintain a constant magnetic field *does not require an expenditure of energy* (see page 172). However, both the yokes and the cores must have sufficiently large cross-sections to carry the leakage flux in addition to the useful armature flux. This increases both the amount of iron and the amount of field copper.

CHAPTER III

RESISTANCE

33. Electrical Resistance.—The current flowing in an electric circuit depends not only on the electromotive force impressed on the circuit, but on the circuit properties as well. For example, if a copper wire be connected across the terminals of a battery, a current will flow through this wire. This is shown in Fig. 39 (*a*), where the ammeter connected in the circuit indicates considerable

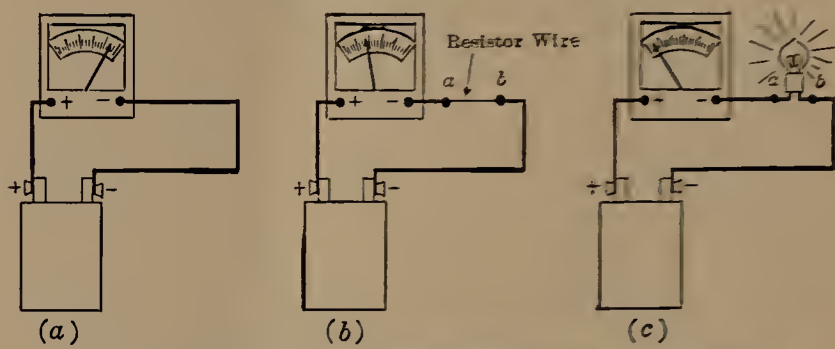


FIG. 39.—Effect of introducing resistance in an electric circuit.

current. If this copper wire is opened and a short length of resistor wire of smaller cross-section than the copper is introduced between two points, as *ab* (Fig. 39 (*b*)) two effects are noticed. The deflection of the ammeter pointer decreases, indicating that less current flows in the circuit. A perceptible amount of heat is developed in the resistor wire *ab*. If the resistor wire is removed and a small incandescent lamp substituted, the deflection of the ammeter pointer decreases further and the heat developed in the lamp filament is sufficient to bring it to incandescence.

It is clear, therefore, that the resistor wire and the incandescent lamp tend to reduce the flow of current and at the same time cause heat to be developed within themselves. This property of

tending to prevent the flow of current and at the same time causing heat dissipation is called *resistance.*

Resistance in the electric circuit may be likened in its effect to friction in mechanics. For example, if a street car is running at a uniform speed on a straight, level track, friction tends to reduce the speed of the car. Likewise, resistance tends to reduce the flow of electric current. The power which is used in maintaining the speed of the car is converted by friction into heat. Likewise, the power expended in maintaining the current through resistance is converted into heat. Friction tends to impede the flow of water in a pipe or in a flume, some of the energy of the water being expended in overcoming this friction. The loss of energy is represented by a loss of head. This energy loss is largely absorbed by the water, and careful measurements would show a slight increase in its temperature. It will be shown in the next chapter that when an electric current flows through a resistance there is a loss of voltage or electric pressure, and also a loss of energy, both these losses being directly proportional to the amount of resistance.

All[1] substances have resistance, but the resistance of some substances is so great that it practically prevents the flow of current This leads to the classification of substances into conductors and insulators. No substance is a perfect conductor and no substance is a perfect insulator. Silver, one of the best conductors, has appreciable resistance, and glass or porcelain, among the best insulators known, allow small currents to flow, and therefore are not perfect insulators. The best conductors are the metals, silver coming first and copper second. Carbon and ordinary water also may be classed as conductors. Acid, alkaline, and salt solutions are fair conductors. Distilled or pure water, however, is a good insulator. Oils, glass, porcelain, silk, paper, cotton, ebonite, fiber, paraffin, rubber, etc. may be considered as non-

[1] Professor Kamerlingh-Onnes of Leyden, in 1914, was able to produce a circuit in which an electric current showed no diminution in strength 5 hr. after the electromotive force had been removed. The current was induced magnetically in a short-circuited coil of lead wire at −270° C. in the presence of liquid helium and the inducing source then removed. Liquid helium has the lowest temperature known, being in the neighborhood of absolute zero (−273° C.). This experiment indicates that the resistance of the lead was practically zero at this extremely low temperature.

conductors or good insulators. Wood, either dry or impregnated with oil, is a good insulator, but wood containing moisture is a partial conductor.

34. Unit of Resistance.—The ohm is the practical unit of resistance. The unit as adopted in the United States is specifically defined by an act of Congress as follows:

The *International Ohm* is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area, and of a length of 106.300 centimeters.

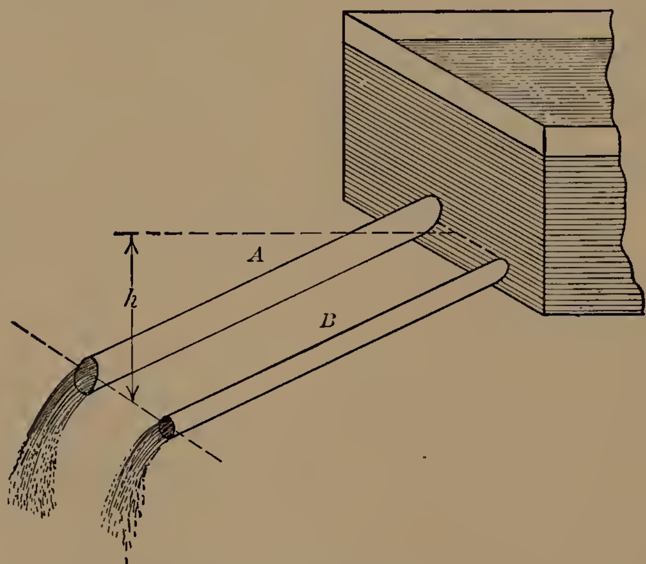


FIG. 40.—Water discharge through two pipes of equal lengths but having unequal cross-sections.

One volt impressed across a resistance of *one* ohm causes a current of *one* ampere to flow. Also, *one* ampere flowing through *one* ohm for *one* second dissipates as heat *one joule.*

The resistance of insulating substances is ordinarily of the magnitude of millions of ohms, so that it is awkward to express this resistance in terms of a unit as small as the ohm. The *megohm,* equal to 1,000,000 (10^6) ohms, is the unit ordinarily used under these conditions. (The prefix "mega" means million.)

On the other hand, the resistance of bus-bars and short pieces of metals may be so low that the ohm is too large a unit for convenience. Under these conditions the *microhm* is used as the

unit, and is equal to $\frac{1}{1,000,000}$ ohm (10^{-6} ohm). (The prefix "micro" means one-millionth.)

35. Resistance and the Geometry of Conductors.—The resistance of a body of given material depends on its size and shape, and on the direction in which the electric current flows through it.

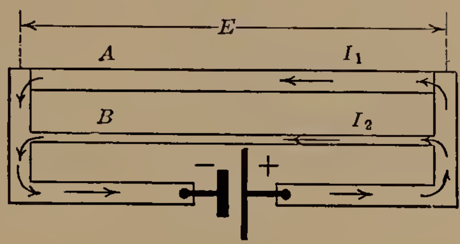


Fig. 41.—Flow of electricity through two conductors of equal lengths but having unequal cross-sections.

The flow of electricity through conductors is in many ways analogous to the flow of water through pipes.

Consider Fig. 40, in which is shown a reservoir to which two pipes A and B are connected. The two pipes are of the same length, but the cross-section of A is greater than that of B. There is the same difference of head (or pressure), h, between the ends of each pipe. It is obvious that pipe A discharges more water in a given time than pipe B, owing to its lesser frictional resistance to the flow of water.

The corresponding electrical analogy is given in Fig. 41. Two conductors A and B of equal lengths are connected to heavy copper bars. Conductor A has a larger cross-section than conductor B. The potential difference between the copper bars is maintained at E volts by the battery Ba. Therefore the same electrical pressure or head of E volts exists between the ends of each of the two conductors. The quantity of electricity per second (I_1 amp.) passing through conductor A is greater than the quantity of electricity per second (I_2 amp.) passing through conductor B, because of the lesser electrical resistance of conductor A. Therefore it may be said that the larger the cross-section of either water pipes or of electrical conductors, the less the resistance. With a homogeneous electrical conductor of given length the electrical resistance is *inversely* as its cross-

section. This is not strictly true, however, of the frictional resistance of pipes to the flow of water.

Consider Fig. 42, in which two equal reservoirs *A* and *B* are to be emptied through pipes *P* and *P′* respectively. The pipe *P* is twice as long as the pipe *P′* but of one-half the cross-section.

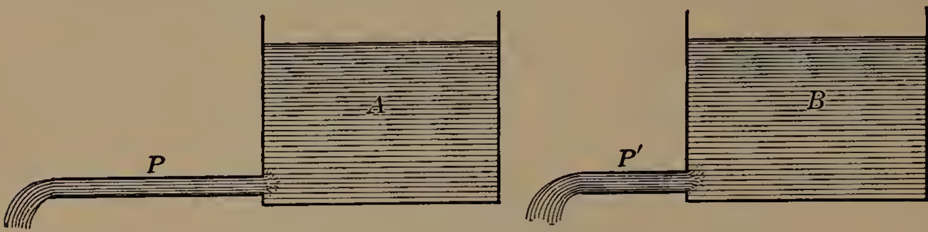


Fig. 42.—Water discharge through different-sized pipes.

Therefore both pipes have the same volume. It is evident that reservoir *B* will be emptied much quicker than *A*, because pipe *P′* has twice the cross-section of pipe *P*, and therefore offers less resistance per unit of length. Further, the length of *P′* is only half that of *P*, and this again makes the total friction of *P′* half that of *P*, even if the cross-sections were equal.

In Fig. 43 are shown two conductors *A* and *B* of the same material. Conductor *A* has twice the length of conductor *B*, but only one-half the cross-section. Therefore each conductor contains the same amount of material. It is evident, however, that the resistance per unit length of conductor *A* is twice that per unit length of *B*. Then, if conductors *A* and *B* were of the same length, conductor *A* would have twice the resistance of conductor *B*. However, conductor *A* has twice the length of *B*, and therefore must have 2 × 2, or 4 times the resistance of *B*.

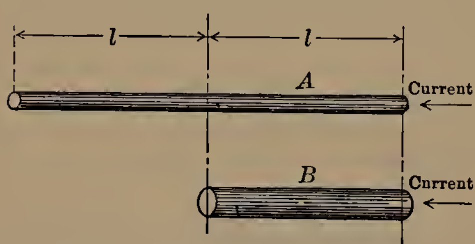


Fig. 43.—Two conductors of equal volume.

36. Specific Resistance or Resistivity.—The deductions of Par. 35 may be summarized in the following rule for resistance:

The resistance of a homogeneous body of uniform cross-section varies directly as its length, and inversely as its cross-section.

That is,

$$R = \rho\frac{L}{A} \tag{3}$$

where R is the resistance in ohms, L is the length in the direction of the current flow, A is the uniform area at right angles to the current flow, and ρ is a constant of the material known as its *resistivity* or *specific resistance.*

If L is 1 cm. and A is 1 cm. square, the body in question must have the form of a centimeter-cube, and

$$R = \rho\frac{1}{1 \times 1}$$

or

$$R = \rho.$$

In this case ρ is called the *specific resistance* or the *resistivity* of the substance per centimeter-cube. ρ may be expressed in terms of an inch-cube or in other units as well. The resistivity of copper is 1.724 microhms, or $\frac{1}{580,000}$ ohm, per centimeter-cube at 20° C. It is evident that the resistance of a cube gives a perfectly definite resistivity, since the resistance between any two opposite faces is the same. The resistivities of various substances are given in Par. 43. Knowing the specific resistance in terms of the centimeter-cube, the resistance of a wire, bar, etc. may be readily computed from formula (3).

Example.—Determine the resistance at 20° C. of 2 km. (1.24 miles) of solid aluminum wire having a cross-section of 4.0 sq. mm. (0.00620 sq. in.). The resistivity of aluminum is 2.828 microhms per centimeter-cube at 20° C.

$$A = \frac{4.0}{100} = 0.040 \text{ sq. cm.}$$
$$L = 2,000 \times 100 = 200,000 \text{ cm.}$$
$$\rho = 2.828 \times 10^{-6} = 0.000002828 \text{ ohm-cm.-cube}$$

Using formula (3)

$$R = 2.828 \times 10^{-6}\,\frac{200,000}{0.040} = 2.828 \times 10^{-6} \times 5,000,000 = 14.14 \text{ ohms. } \textit{Ans.}$$

Example.—Determine the resistance at 20° C. of 3,000 ft. (915 m.) of annealed 0000 copper wire, diameter of 0.460 in. (1.17 cm.), the specific resistance of copper being 1.724 microhms (= 0.000001724 ohm) per centimeter-cube (20° C.). (See Par. 43.)

$$3,000 \text{ ft.} = 3,000 \times 12 \times 2.54 = 91,500 \text{ cm.}$$

$$\text{Cross-section} = \frac{\pi}{4}(0.460 \times 2.54)^2 = 1.07 \text{ sq. cm.}$$

$$R = \rho\frac{L}{A} = (0.000001724) \times \left(\frac{91,500}{1.07}\right) = 0.1472 \text{ ohm. } \quad \textit{Ans.}$$

37. Volume Resistivity.—Since the volume of a body

$$V = LA$$

where L is its length and A its uniform cross-section, equation (3) may be written

$$R = \rho\frac{L}{A} = \rho\frac{L^2}{V} \tag{4}$$

$$= \rho\frac{V}{A^2}. \tag{5}$$

That is:

The resistance of a conductor varies directly as the square of its length when the volume is fixed.

The resistance of a conductor varies inversely as the square of its cross-section when the volume is fixed. (See Fig. 43, Par. 35.)

Example.—A kilometer (3,280 ft.) of wire having a diameter of 11.7 mm. (0.461 in.) and a resistance of 0.031 ohm is drawn down so that its diameter is 5.0 mm. (0.197 in.). What does its resistance become?

The original cross-section of the wire

$$A_1 = \frac{\pi}{4}(11.7)^2 = 107.5 \text{ sq. mm.}$$

The final cross-section

$$A_2 = \frac{\pi}{4}(5.0)^2 = 19.64 \text{ sq. mm.}$$

From equation (5)

$$R_1 = \rho\frac{V}{(107.5)^2} = 0.031 \text{ ohm}$$

$$R_2 = \rho\frac{V}{(19.64)^2}.$$

Since the volume of the wire does not change during the drawing process and the resistivity constant ρ remains the same,

$$\frac{R_2}{R_1} = \frac{R_2}{0.031} = \frac{\rho\frac{V}{(19.64)^2}}{\rho\frac{V}{(107.5)^2}}$$

$$R_2 = 0.031\,\frac{(107.5)^2}{(19.64)^2} = 0.031\,\frac{11{,}560}{386} = 0.93 \text{ ohm.} \quad \textit{Ans.}$$

Since the area A of a circle is proportional to the square of its diameter D, $\left(A = \frac{\pi}{4}D^2\right)$, the ratio of the resistances of two cylindrical conductors is directly proportional to their lengths and inversely proportional to the *squares* of their diameters.

Note: See Appendix F, page 310, for Greek symbols.

If the resistances, lengths, and diameters of two conductors of the same material are R_1, R_2, L_1, L_2, and D_1, D_2,

$$\frac{R_1}{R_2} = \frac{\frac{L_1}{D_1^2}}{\frac{L_2}{D_2^2}} = \frac{L_1 D_2^2}{L_2 D_1^2}. \qquad (6)$$

Example.—A 1,000-ft. (305 m.) length of No. 14 (A.W.G.) copper wire (diameter of 64.0 mils—1.625 mm.) has a resistance of 2.58 ohms at 25° C. Find the resistance at this same temperature of 1,200 ft. (366 m.) of No. 16 wire (diameter 51.0 mils—1.296 mm.).

Using equation (6) inverted,

$$\frac{R_2}{2.58} = \frac{1{,}200 \times \overline{64.0}^2}{1{,}000 \times \overline{51.0}^2}; \quad R_2 = 2.58\,\frac{1{,}200 \times 4{,}100}{1{,}000 \times 2{,}600} = 4.88 \text{ ohms.} \quad \textit{Ans.}$$

38. Conductance.—Conductance is the reciprocal of resistance and may be defined as that property of a circuit or of a material which causes it to permit the flow of an electric current. The unit of conductance is the reciprocal ohm or *mho*. Conductance is usually expressed by g.

$$g = \frac{1}{R}$$

also

$$g = k\tau'\frac{A}{L} \qquad (7)$$

where τ is the *specific conductance* or the *conductivity* of a substance, A the uniform cross-section, and L the length.

The conductivity of copper is 580,000 mhos per centimeter-cube at 20° C.

Example.—Determine the conductance of an aluminum bus-bar 0.5 in. (1.27 cm.) thick, 4 in. (10.16 cm.) wide, and 20 ft. (6.10 m.) long.

The conductivity of aluminum is 61 per cent that of copper, and copper has a conductivity of 580,000 mhos per centimeter-cube.

The conductivity of aluminum is:

$$\tau = 0.61 \times 580{,}000 = 354{,}000 \text{ mhos./cm.-cube}$$

The cross-section of the bus-bar:

$$A = 0.5 \times 4 \times 2.54 \times 2.54 = 12.9 \text{ sq. cm.}$$

The length $L = 20 \times 12 \times 2.54 = 610$ cm.

The conductance:

$$g = 354{,}000 \times \frac{12.9}{610} = 7{,}490 \text{ mhos.} \quad \textit{Ans.}$$

The resistance: $r = \dfrac{1}{7{,}490} = 0.0001335$ ohm.

39. Per Cent. Conductivity.—In order to establish a standard of conductivity, the Bureau of Standards made extensive resistivity measurements of commercial copper. Its recommendation that the standard of resistivity be 1.724 microhms per centimeter-cube at 20° C. has been adopted internationally. The per cent. conductivity for carefully refined copper may exceed 100.

Example.—The resistance of a copper rod measured at 20° C. gives a resistivity of 1.744 microhms per centimeter-cube. What is the per cent. conductivity of this copper?

$$\text{Per cent. conductivity} = \frac{1.724}{1.744}, \text{ or } 98.8 \text{ per cent.} \quad \textit{Ans.}$$

40. Resistances in Series and in Parallel.—If a number of resistances, r_1, r_2, r_3, etc., are connected in series, Fig. 44, that is, end to end, the total resistance of the combination is:

$$R = r_1 + r_2 + r_3 + \ldots \qquad (8)$$

That is:

In a series circuit, the total resistance is the sum of the individual resistances.

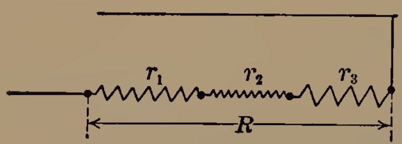


FIG. 44.—Resistances in series.

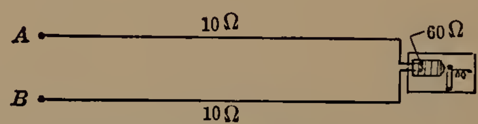


FIG. 45.—60-ohm telegraph relay in series with a 20-ohm line.

Example.—A 60-ohm telegraph relay is connected to the end of a telegraph line (Fig. 45) each conductor of which has a resistance of 10 ohms. What is the resistance of the entire circuit?

The two line conductors and the relay form a series circuit, the resistance of which,

$$R = 10 + 60 + 10 = 80 \text{ ohms.} \quad \textit{Ans.}$$

This is the resistance which would be measured between the ends A, B.

If a number of conductances, g_1, g_2, g_3, etc., are connected in parallel (Fig. 46) the total conductance of this portion of the circuit must be equal to the sum of the individual conductances; that is,

$$G = g_1 + g_2 + g_3 + \ldots \qquad (9)$$

Since,

$$G = \frac{1}{R},\ g_1 = \frac{1}{r_1}, \text{ etc., equation (9) may be written}$$

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdots \qquad (10)$$

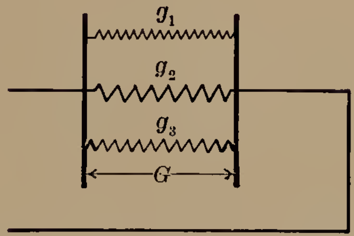


FIG. 46.—Conductances in parallel.

That is:

In a parallel circuit, the reciprocal of the total resistance is equal to the sum of the reciprocals of the individual resistances.

For a circuit of two resistances, r_1 and r_2, in parallel, the joint resistance

$$R = \frac{r_1 r_2}{r_1 + r_2}. \qquad (11)$$

Example.—A 12.0-ohm and a 7.00-ohm resistance are connected in parallel between points A and B (Fig. 47(a)). What is their joint resistance?

Using equation (10),

$$\frac{1}{R} = \frac{1}{12} + \frac{1}{7} = 0.0833 + 0.1429 = 0.2262 \ \text{℧ (mho)}$$

$$R = \frac{1}{0.2262} = 4.42\ \Omega \text{ (ohms).} \quad \textit{Ans.}$$

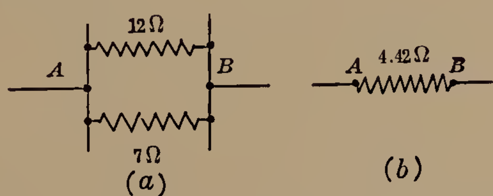


FIG. 47.—Two resistances in parallel and their equivalent.

Using equation (11),

$$R = \frac{12 \times 7}{12 + 7} = \frac{84}{19} = 4.42\ \Omega \text{ (check).} \quad \textit{Ans.}$$

This means that if the 12.0-ohm and the 7.00-ohm resistances were replaced by a single resistance of 4.42 ohms, (Fig. 47 (b)), the resistance between points A and B would remain unchanged.

Example.—Determine the total resistance of a circuit having 4 branches, the individual resistances of which are 3.0, 4.0, 6.0, and 8.0 ohms, respectively.

$$\frac{1}{R} = \frac{1}{3.0} + \frac{1}{4.0} + \frac{1}{6.0} + \frac{1}{8.0} = 0.333 + 0.250 + 0.167 + 0.125$$

$$= 0.875 \text{ mho}$$

$$R = \frac{1}{0.875} = 1.142 \text{ ohms.} \quad \textit{Ans.}$$

41. The Circular Mil.—In the English and American wire tables, the circular mil is the standard unit of wire cross-section.

The term "mil" means one-thousandth, for example, a millivolt is $\frac{1}{1,000}$ volt. A mil is one-thousandth of an inch (0.0254 mm.). A square mil is the area of a square, each side of which is one mil (0.001 in.), as shown in Fig. 48 (a). The area of a square mil is $0.001 \times 0.001 = 0.000001$ sq. in. (0.000645 sq. mm.).

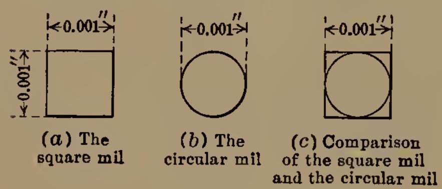


Fig. 48.

A circular mil is the area of a circle whose diameter is one mil (0.001 in.) (Fig. 48 (b)) and is usually written C.M. or cir. mil. As will be seen from an inspection of Fig. 48 (c), a circular mil is a smaller area than a square mil. The area in square inches of a circular mil is $\frac{\pi}{4}(0.001)^2 = 0.0000007854$ sq. in. (0.000506 sq. mm.).

The circular mil is the unit by which the cross-section of wires and cables is measured, just as the square foot is the unit by which larger areas, such as floors, land, etc., are measured. The advantage of the circular mil as a unit is that circular areas measured in terms of this unit bear a very simple relation to the diameters.

A, in Fig. 49 (a), represents the cross-section of a wire having a diameter of 1 in. Required: to determine its area in circular mils.

$$\text{The area, } A = \frac{\pi}{4}(1)^2 \text{ sq. in.}$$

$$\text{The area of a circular mil} = \frac{\pi}{4}(0.001)^2 \text{ sq. in.}$$

The ratio of these two areas obviously gives the number of circular mils in A.

Hence

$$\frac{\frac{\pi}{4}(1)^2}{\frac{\pi}{4}0.000001} = 1{,}000{,}000 \text{ cir. mils.}$$

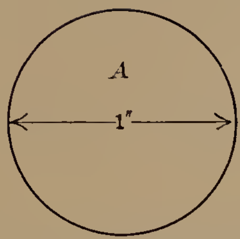


FIG. 49 (*a*).—Cross-section expressed in cir. mils.

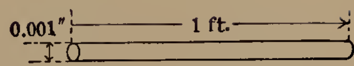


FIG. 49 (*b*).—The circular-mil-foot.

The general relation may be written:

$$\text{Cir. mils} = \frac{D_1^2}{(0.001)^2} = 1{,}000{,}000(D_1)^2 = D^2 \qquad (12)$$

where

D_1 is the diameter of the wire in *inches.*
D is the diameter of the wire in *mils.*

The matter may be summed up in two rules:

To obtain the number of circular mils in a solid wire of given diameter, express the diameter in mils and then square it.

To obtain the diameter of a solid wire having a given number of circular mils, take the square root of the circular mils and the result will be the diameter of the wire in mils.

Example.—00 wire (A.W.G.) has a diameter of 0.3648 in. (9.26 mm.). What is its circular milage?

$$0.3648 \text{ in.} = 364.8 \text{ mils}$$
$$(364.8)^2 = 133{,}100 \text{ cir. mils.} \quad \textit{Ans.}$$

Example.—A solid cylindrical wire has a cross-section of 52,640 cir. mils. What is its diameter?

$$\sqrt{52{,}640} = 229.4 \text{ mils} = 0.2294 \text{ in. } (5.83 \text{ mm.}). \quad \textit{Ans.}$$

42. The Circular-mil-foot.—Another convenient unit of resistivity, especially in the English system, is the resistance of a circular-mil-foot. This unit is the resistance of a wire having a cross-section of 1 cir. mil and a length of 1 ft., as shown in Fig. 49 (*b*). The resistance of a circular-mil-foot of copper at 20° C. is 10.37 ohms. (In practical work this resistance may frequently

be taken as 10 ohms.) Knowing this resistivity, the resistance of any length and size of wire may be determined by equation (3), page 35.

Example.—What is the resistance at 20° C. of a 750,000 C.M. copper cable, 2,500 ft. (762 m.) long?

If the cable had a cross-section of 1 cir. mil, it would have a resistance of $2{,}500 \times 10.37 = 25{,}900$ ohms. However, the cross-section is actually 750,000 cir. mils; therefore,

$$R = \frac{25{,}900}{750{,}000} = 0.0346 \text{ ohm.} \quad \textit{Ans.}$$

Or equation (3), page 35, may be used directly

$$R = 10.37 \frac{2{,}500}{750{,}000} = 0.0346 \text{ ohm.} \quad \textit{Ans.}$$

When applying equation (3), L must be expressed in *feet* and A in *circular mils.*

43. Table of Resistivities at 20° C.

Metals	Cm.-cube (microhms)	Cir.-mil-ft. (ohms)
Aluminum	2.828	17.0
Copper (drawn)	1.724	10.37
German silver	33.3 to 48.0	200 to 290
Iron:		
Electrolytic	9.96	59.8
Cast	75 to 95	450 to 570
Manganin	41.4 to 73.8	248 to 443
Mercury	94.07	565
Nichrome	100 to 110	600 to 660
Platinum	9.0 to 15.5	54 to 93
Silver	1.5 to 1.7	9.0 to 10.2
Steel, soft	15.9	95.4

44. Temperature Coefficient of Resistance.—The resistance of the non-alloyed metals increases very appreciably with the temperature. Since copper, as in the windings of electric machinery, often operates at temperatures much higher than that of the surrounding air, it is important to know the relation between temperature and resistance. The relation may be expressed as follows:

$$R_t = R_0(1 + at) \tag{13}$$

where R_t is the resistance at the temperature t, R_0 the resistance at 0° C. and a the *temperature coefficient of resistance* at 0° C.

For copper, a is 0.00427 and for most of the unalloyed metals a is sensibly of this value. This is equivalent to saying that the resistance increases 0.427 of 1 per cent for each degree Centigrade increase of temperature above 0°. For example, if a coil has a resistance of 100 ohms at 0° C., for every degree increase of temperature the resistance will increase

$$100 \times 0.00427 \text{ ohm, or } 0.427 \text{ ohm.}$$

At 40° C. the increase of resistance will be $40 \times 0.427 = 17.08$ ohms, and the resistance at 40° will be $100 + 17.08 = 117.08$ ohms.

If the resistance at some definite temperature other than 0° C. is known, ordinarily the resistance at 0° C. must first be found before the resistance at other temperatures can be determined.

For this purpose equation (13) may be transposed into the form

$$R_0 = \frac{R_t}{1 + at}. \tag{14}$$

Example.—The resistance of an electromagnet winding of copper wire at 20° C. is 30 ohms. What is its resistance at 80° C.?

The resistance at 0° C.

$$R_0 = \frac{30}{1 + 0.00427 \times 20} = \frac{30}{1.085} = 27.65 \text{ ohms}$$

$$R_{80} = 27.65\,(1 + 0.00427 \times 80) = 37.11 \text{ ohms. } \textit{Ans.}$$

This process of working back to 0° is a little inconvenient, although it is fundamental and easy to remember. Table 48 gives the temperature coefficients of resistance of copper at various initial temperatures. With this table available, the above problem involves but one computation.

Example.— From Table 48 the temperature coefficient of copper at 20° initial temperature is 0.00393. The rise in temperature = 80° − 20° = 60°. Then the resistance at 80° C.

$$R_{80} = 30(1 + 0.00393 \times 60) = 37.07 \text{ ohms. } \textit{Ans.}$$

45. Resistance and Inferred Absolute Zero.—If the resistance of copper at temperatures between 0° and 100° C. be plotted against temperature, the result is practically a straight line ab (Fig. 50). If this line be extended, it will intersect the zero resistance line at −234.5° C. (an easy number to remember), as shown in Fig. 50. This is equivalent to saying that, between ordinary limits of temperature, copper behaves as if it had zero resistance at −234.5° C. (Actually the curve bends at these

extremely low temperatures, as shown by the dotted line (Fig. 50). This gives a convenient method for determining temperature-resistance relations.

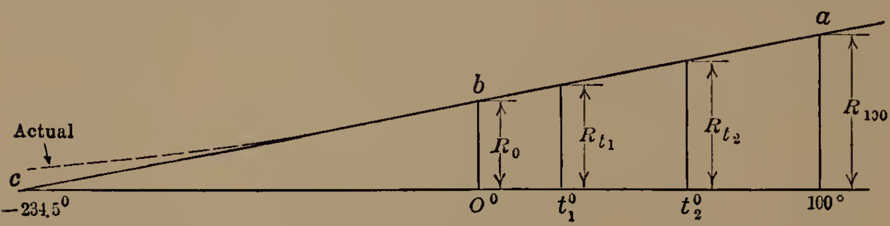


Fig. 50.—Variation of resistance of copper with temperature.

By the law of similar triangles (Fig. 50),

$$\frac{R_0}{234.5^0} = \frac{R_{t1}}{234.5^0 + t_1} \tag{15}$$

$$\frac{R_{t1}}{234.5^0 + t_1} = \frac{R_{t2}}{234.5^0 + t_2} \tag{16}$$

where R_{t1} is the resistance at $t_1°$ C. and R_{t2} is the resistance at $t_2°$ C.

Applying equation (16) to the previous example,

$$\frac{30}{234.5^0 + 20°} = \frac{R_{80}}{234.5^0 + 80°}$$

$$R_{80} = 30\frac{234.5° + 80°}{234.5° + 20°} = 30\,\frac{314.5°}{254.5°} = 37.1 \text{ ohms.} \quad \textit{Ans.}$$

46. Alloys.—Certain alloys, notably manganin and nickel-iron alloys, have practically zero temperature coefficients and are very useful as resistances for measuring instruments, where a change of resistance introduces error.

Table 47 gives the temperature coefficients at 20° C. for various materials. Table 48 gives the temperature coefficient of copper at various temperatures.

47. Temperature Coefficients of Resistance

	Per Degree C. at 20° C.
Aluminum	0.00388
Carbon (incandescent lamp)	−0.003
Graphite	−0.0006 to −0.0012
German silver	0.00031 to 0.00020
Iron and steel	0.00635
Manganin	0.000011 to 0.000039
Nichrome	0.00016 to 0.00044
Platinum	0.00367

Temperature Coefficients of Copper at Different Initial Temperatures

(From formula $\frac{1}{(234.5 + t)}$)

Initial Temperature	Increase in Resistance per 1° C.
0	0.00427
10	0.00409
20	0.00393
30	0.00378
40	0.00364

49. The American Wire Gage (A.W.G.).—The A.W.G. (formerly Brown & Sharpe gage) is based on a constant ratio of cross-section between wires of successive gage numbers. The following approximate relations make it a comparatively simple matter to determine the weight or resistance of any gage number without reference to the table: (1) No. 10 wire has a diameter of 0.1 in. and a resistance of 1 ohm per 1,000 ft. (2) The resistance of the wire doubles with every increase of 3 gage numbers. (3) Therefore the resistance increases $\sqrt[3]{2} = 1.26$ ($1\frac{1}{4}$) times for each successive gage number and $(1.26)^2 = 1.6$ times for every two numbers. (4) The resistance is multiplied or divided by 10 for every difference of 10 gage numbers. (5-*a*) The weight of 1,000 ft. of No. 2 wire is 200 lb. (90.7 kg.). (5-*b*) The weight of 1,000 ft. of No. 10 wire is 31.4, or 10π lb. (14.23 kg.). (For constant length the weight varies inversely as the resistance.)

Example.—What are the resistance and weight of 1,000 ft. (305 m.) of 0000 wire?

The resistances will decrease as follows:

Gage No.	10	7	4	1	000	
Resistance	1	0.5	0.25	0.125	0.0625	(rules 1 and 2)

Resistance of 0000 $= \frac{0.0625}{1.25} = 0.050$ ohm (rule 3). *Ans.*

Weight of 1,000 ft. No. 2 = 200 lb. (rule 5-*a*).

Weight of 1,000 ft. 00 = 400 lb. (rule 2)

Weight of 1,000 ft. 0000 = 400 × 1.6 = 640 lb. (290 kg.) (rule 3). *Ans.*

The example might have been worked more quickly by rule 4.

Resistance of 1,000 ft. of No. 10 = 1 ohm.

Resistance of 1,000 of 0 = 0.1 ohm (rule 4).

Resistance of 1,000 ft. of 0000 = 0.050 ohm (rule 3). *Ans.*

Example.—Give the resistance, the circular milage, the diameter, and the weight of 3,500 ft. (1,066 m.) of No. 28 copper wire.

Resistance of 1,000 ft. of No. 10 = 1 ohm (rule 1).

Resistance of 1,000 ft. of No. 20 = 10 ohms (rule 4).

Resistance of 1,000 ft. of No. 23 = 20 ohms (rule 2).
Resistance of 1,000 ft. of No. 26 = 40 ohms (rule 2).
Resistance of 1,000 ft. of No. 28 = 40 × 1.6 = 64 ohms (rule 3).

$$3.5 \times 64 = 224 \text{ ohms.} \quad \textit{Ans.}$$

No. 10 contains 10,000 C.M.
No. 20 contains 1,000 C.M.
No. 23 contains 500 C.M.
No. 26 contains 250 C.M.
No. 28 contains $\frac{250}{1.6} = 156$ C.M. *Ans.*

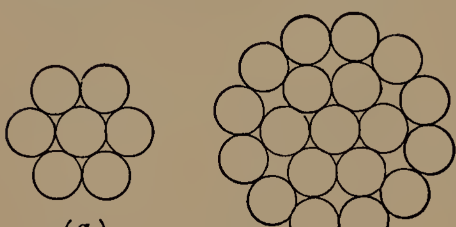
The diameter, $D = \sqrt{156} = 12.5$ mils (0.318 mm.). *Ans.*

Weight of 1,000 ft. of No. 10 = 31.4 lb.
Weight of 1,000 ft. of No. 20 = 3.14 lb.
Weight of 1,000 ft. of No. 23 = 1.57 lb.
Weight of 1,000 ft. of No. 26 = 0.785 lb.
Weight of 1,000 ft. of No. 28 = $\frac{0.785}{1.6} = 0.49$ lb.

$$0.49 \times 3.5 = 1.72 \text{ lb. (0.780 kg.).} \quad \textit{Ans.}$$

Tables 51 and 52 give the properties of copper wire. These tables were compiled by the Bureau of Standards and are accepted internationally as standards. Table 51 gives the properties of solid wires.

50. Stranded Wires and Cables.—Stranding increases the flexibility, and is necessary for wires and cables whose cross-section is greater than 0000 A.W.G. It is often desirable for conductors of smaller cross-section. In order that cables may make up properly, the following number of cylindrical conductors, all of the same cross-section, must be used.



(*a*) (*b*)

FIG. 51.—Make-up of a 7-strand and a 19-strand cable.

Six cylindrical conductors will just encircle a single one at the center (Fig. 51 (*a*)), making a 7-strand cable. Twelve more cylindrical conductors will just encircle the 7-strand cable (Fig. 51 (*b*)), making a total of 19 strands. The number of strands increases by six in each successive layer. Therefore, cables are ordinarily made up of 7, 19, 37, 61, 91, 127, . . . strands, respectively. Table 52 gives the properties of stranded wires and cables.

51. Working Table, Standard Annealed Copper Wire, Solid[1]
American Wire Gage (B. & S.). English Units

Gage No.	Diameter in mils	Cross-section		Ohms per 1,000 ft.		Ohms per mile	Pounds per 1,000 ft.
		Circular mils	Square inches	25° C. (77° F.)	65° C. (149° F.)	25° C. (77° F.)	
0000	460.0	212,000.0	0.166	0.0500	0.0577	0.264	641.0
000	410.0	168,000.0	0.132	0.0630	0.0727	0.333	508.0
00	365.0	133,000.0	0.105	0.0795	0.0917	0.420	403.0
0	325.0	106,000.0	0.0829	0.100	0.116	0.528	319.0
1	289.0	83,700.0	0.0657	0.126	0.146	0.665	253.0
2	258.0	66,400.0	0.0521	0.159	0 184	0.839	201.0
3	229.0	52,600.0	0.0413	0.201	0.232	1.061	159.0
4	204.0	41,700.0	0.0328	0.253	0.292	1.335	12[illegible].0
5	182.0	33,100.0	0.02[illegible]0	0.319	0.369	1.685	100.0
6	162.0	26,300.0	0.0206	0.403	0.465	2.13	79.5
7	144.0	20,800.0	0.0164	0.508	0.586	2.68	63.0
8	128.0	16,500.0	0.0130	0.641	0.739	3.38	50.0
9	114.0	13,100.0	0.0103	0.808	0.932	4.27	39.6
10	102.0	10,400.0	0.00815	1.02	1.18	5.38	31.4
11	91.0	8,230.0	0.00647	1.28	1.48	6.75	24.9
12	81.0	6,530.0	0.00513	1.62	1.87	8.55	19.8
13	72.0	5,180.0	0.00407	2.04	2.36	10.77	15.7
14	64.0	4,110.0	0.00323	2.58	2.97	13.62	12.4
15	57.0	3,260.0	0.00256	3.25	3.75	17.16	9.86
16	51.0	2,580.0	0.00203	4.09	4.73	21.6	7.82
17	45.0	2,050.0	0.00161	5.16	5.96	27.2	6.20
18	40.0	1,620.0	0.00128	6.51	7.51	34.4	4.92
19	36.0	1,290.0	0.00101	8.21	9.48	43.3	3.90
20	32.0	1,020.0	0.000802	10.4	11.9	54.9	3.09
21	28.5	810.0	0.000636	13.1	15.1	69.1	2.45
22	25.3	642.0	0.000505	16.5	19.0	87.1	1.94
23	22.6	509.0	0.000400	20.8	24.0	109.8	1.54
24	20.1	404.0	0.000317	26.2	30.2	138.3	1.22
25	17.9	320.0	0.000252	33.0	38.1	174.1	0.970
26	15.9	254.0	0.000200	41.6	48.0	220.0	0.769
27	14.2	202.0	0.000158	52.5	60.6	277.0	0.610
28	12.6	160.0	0.000126	66.2	76.4	350.0	0.484
29	11.3	127.0	0.0000995	83.4	96.3	440.0	0.384
30	10.0	101.0	0.0000789	105.0	121.0	554.0	0.304

[1] Tables of current-carrying capacity are given in Part II.

52. Bare Concentric Lay Cables of Standard Annealed Copper
English Units

A.W.G. No.	Circular mils	Ohms per 1,000 ft.		Pounds per 1,000 ft.	Standard concentric stranding		
		25° C. (77° F.)	65° C. (149° F.)		Number of wires	Diameter of wires, in mils	Outside diameter, in miles,
	1,000,000	0.0108	0.0124	3,090	61	128.0	1,152
	900,000	0.0120	0.0138	2,780	61	121.5	1,093
	850,000	0.0127	0.0146	2,620	61	118.0	1,062
	750,000	0.0144	0.0166	2,320	61	110.9	998
	650,000	0.0166	0.0192	2,010	61	103.2	929
	600,000	0.0180	0.0207	1,850	61	99.2	893
	550,000	0.0196	0.0226	1,700	61	95.0	855
	500,000	0.0216	0.0249	1,540	37	116.2	814
	450,000	0.0240	0.0277	1,390	37	110.3	772
	400,000	0.0270	0.0311	1,240	37	104.0	728
	350,000	0.0308	0.0356	1,080	37	97.3	681
	300,000	0.0360	0.0415	926	37	90.0	630
	250,000	0.0431	0.0498	772	37	82.2	575
0000	212,000	0.0509	0.0587	653	19	105.5	528
000	168,000	0.0642	0.0741	518	19	94.0	470
00	133,000	0.0811	0.0936	411	19	83.7	418
0	106,000	0.102	0.117	326	19	74.5	373
1	83,700	0.129	0.149	258	19	66.4	332
2	66,400	0.162	0.187	205	7	97.4	292
3	52,600	0.205	0.237	163	7	86.7	260
4	41,700	0.259	0.299	129	7	77.2	232

53. Conductors.—Although silver is a better conductor than copper, its use as a conductor is very limited because of its cost. In a few instances it is used where a delicate but highly conducting material is necessary, such as in the brushes and occasionally in the commutator of watthour meters. Copper, because of its high conductivity and moderate cost, is used more extensively as a conductor than any other material. It has many good qualities, such as ductility, high tensile strength, not easily abraided, not corroded by the atmosphere, and it is readily soldered.

Aluminum has only 61 per cent of the conductivity of copper, but, for the same length and weight, it has about twice the con-

ductance of copper. It is softer than copper, its tensile strength is much less, and it cannot be readily soldered. It is not affected by exposure to the atmosphere. The large diameter for a given conductance prohibits its use where an insulating covering is required. Aluminum is used extensively as a conductor for high-voltage transmission lines, where its lightness and large diameter are an advantage. It is used to some extent for low-voltage bus-bars, as it offers much greater radiating surface than copper of the same conductance. The price of aluminum is held about 10 per cent less than that of copper of the same conductance.

Iron and steel have about 9 times the resistance of copper for the same cross-section and length. The large cross-section for a given conductance prohibits their use where an insulating covering is necessary and the increased weight prevents their use in most cases where the conductors must be placed on poles. They are most commonly used as resistors in connection with rheostats and for third rails of electric railways. Iron and steel ordinarily must be protected from oxidation by galvanizing or other protective covering. Copper-clad steel consists of a steel wire coated or covered with a layer of copper, fused or welded to the steel. The advantages claimed for it are that it possesses the high tensile strength of steel, combined with the high conductivity of copper. Further, the copper protects the steel from corrosion. Its field is the transmission line conductor, where long spans make high tensile strength necessary. It is also used as an overhead ground wire on transmission lines.

CHAPTER IV

OHM'S LAW AND THE ELECTRIC CIRCUIT

The exact nature of electricity is not known, but recent investigations indicate that it consists of infinitesimal charges called electrons. When a difference of potential causes these electrons to travel in the same direction, an electric current results. The flow of electricity through a circuit resembles in many ways the flow of water through pipes, for it acts as an incompressible fluid would act, undergoes pressure drop, etc., as will be shown later.

54. Electromagnetic Units. *Current.*—The unit of electric current is the *ampere* and represents the *rate of flow* of electricity. It corresponds in hydraulics to the rate of flow of water, expressed as cubic feet per second, gallons per minute, etc.

A current is said to have the strength of one absolute unit (one *absampere*) when its value is such that if one centimeter length of the circuit is bent into an arc of one centimeter radius, the current in it exerts a force of one dyne on a magnetic pole of unit strength placed at the center of the arc. The *ampere* is only one-tenth of the absampere. In order that the value of the ampere may be readily determined by experiment, it is further defined by an act of Congress, 1894, as follows: The unit of current shall be what is known as the international ampere, which is one-tenth of the unit of current of the centimeter-gram-second system of electromagnetic units and is the practical equivalent of the unvarying current, which when passed through a solution of nitrate of silver in water in accordance with standard specifications, deposits silver at the rate of one thousand one hundred and eighteen millionths (0.001118) of a gram per second.

Quantity.—The unit of quantity is the *coulomb.* This is equal to the quantity of electricity conveyed by one ampere in one second. The coulomb is analogous to the unit quantity of water in hydraulics, such as the cubic foot, the gallon, etc.

From this definition it is evident that an electric current may be expressed in *coulombs per second* rather than in *amperes.*

Difference of Potential and Electromotive Force (e.m.f.).—Difference of potential and electromotive force tend to cause a flow of electricity. The unit of potential difference and of electromotive force is the *volt,* and is defined as that potential difference or electromotive force which, when impressed across the terminals of a resistance of one ohm, will cause a current of one ampere. The *international volt* is now more specifically defined as $\frac{1}{1.01830}$ of the voltage of a normal Weston cell (see page 94).

The mechanical analogy of potential is pressure. The difference in hydraulic pressure between the ends of a pipe causes or tends to cause the flow of water. The pressure of water behind the dam tends to cause water to flow through the penstock or through any leaks. The pressure in a boiler tends to cause steam to flow through the pipes, valves, etc. Likewise, electric pressure or difference of potential tends to cause a flow of electricity.

Resistance.—The *international ohm* is specifically defined as the resistance of a column of mercury at the temperature of melting ice (0° C.), 14.4521 grams in mass, of a constant cross-sectional area and of a length of 106.300 cm. (see page 32, Chap. III).

55. Nature of the Flow of Electricity.—The flow of electricity through a circuit resembles in many ways the flow of water through a closed system of pipes. For example, in Fig. 52, water enters the mechanically-driven centrifugal pump P at a pressure h_1 (represented by the length of a column of mercury) above the point of zero pressure shown by the line h_0. In virtue of the action of the pump blades, its pressure through the pump is increased from h_1 to h_2, representing a net increase of pressure H_1. The water then flows out along pipe F_1 to the hydraulic motor W. Because of the friction loss in the pipe F_1, the pressure at the motor terminals h_3 is slightly less than h_2. In other words, a pressure of $h_2 - h_3$ is required to overcome the frictional resistance of the pipe F_1. The distance between the horizontal line ac and the line ab shows the pressure drop along the pipe, this pressure drop being uniform.

In Fig. 53, the mechanically driven electrical generator G raises the potential of the current entering its negative terminal

from v_1 to v_2 where v_1 and v_2 are measured from the earth, whose potential is ordinarily assumed as zero. (The various voltages are measured with voltmeters v'_1, v'_2, etc.) The generator, in raising the potential of this portion of the circuit from v_1 to v_2, produces a net increase in pressure $v_2 - v_1 = V_1$. The current

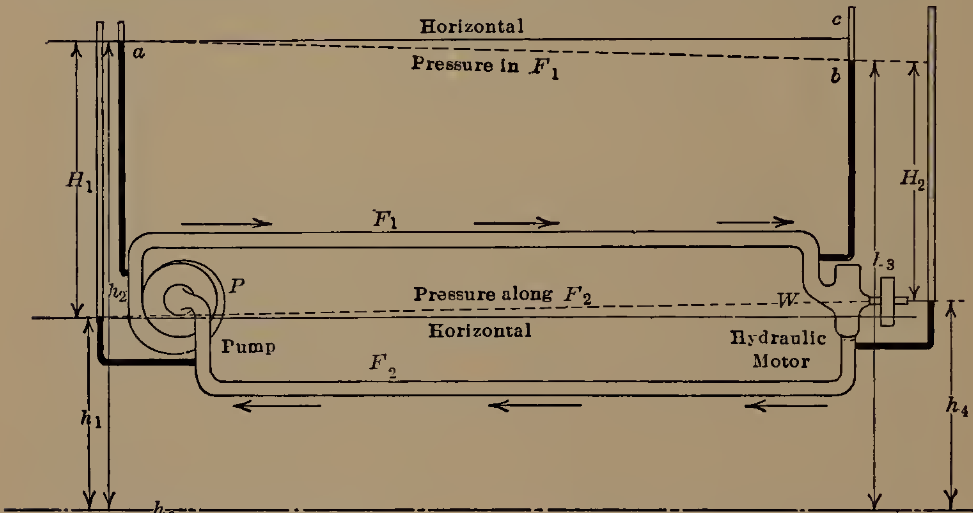


FIG. 52.—Flow of water through a hydraulic motor and pipe system.

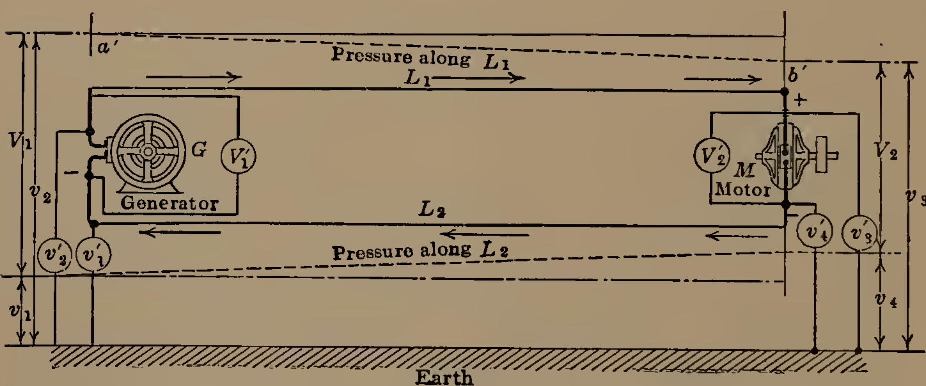


FIG. 53.—Flow of electric current through an electric motor and feeder system.

now flows out through the line L_1 to the + terminal of the motor M. Because of the line resistance, the potential drops from v_2 at the generator to v_3 at the motor in practically the same manner that the water pressure drops in pipe F_1 (Fig. 52). A voltage $v_2 - v_3$ is necessary to force the current through the line L_1. The

line $a'b'$ shows the actual voltage at each point along the wire, the distance of $a'b'$ from the ground line being proportional to the voltage at each point. The voltage drop is uniform.

Referring again to Fig. 52, the water enters the hydraulic motor W and, in overcoming the back pressure of the revolving blades, its pressure drops from h_3 to h_4, representing a net drop in pressure H_2. Pressure h_4 must necessarily be greater than h_1 in order that the water may flow back through the pipe F_2. The pressure $h_4 - h_1$ is necessary to overcome the friction loss in the pipe F_2. It is to be noted that H_2, the net pressure at the motor terminals, is less than the pressure H_1 at the pump, by the sum of the pressures necessary to overcome the friction in the *outgoing and return pipes* F_1 and F_2.

In a similar manner, the pressure of the electric current in passing through the motor M drops from v_3 to v_4, representing a net drop in pressure V_2. A large percentage of this voltage V_2 is necessary to overcome the back electromotive force of the motor (see page 264). v_4 is necessarily greater than v_1, or the current could not flow along L_2 back to the negative terminal of the generator. It is to be noted that, as in the case of Fig. 52, the net potential difference V_2 at the motor M is less than the potential difference V at the generator G by the drop in potential due to the resistance of both the *outgoing* and the *return* wire.

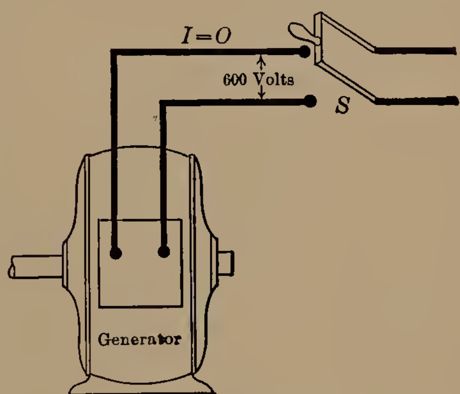


FIG. 54.—Illustrating the existence of potential difference without current.

Difference of potential is, therefore, the equivalent of pressure and *tends* to send current through a circuit; current is quantity of electricity per second. Potential difference may exist with no current flow, in the same manner that a boiler may have a very high steam pressure with no steam flow, due to all the valves being closed. Likewise a generator, Fig. 54, may have a very high potential difference at its terminals, yet, because the switch S is open, no current flows.

56. Difference of Potential.—In order that current may flow between two points, there must be a *difference of potential* between the two points, as shown in Fig. 53. This is further illustrated in Fig. 55. A large reservoir and a small tank are connected by a

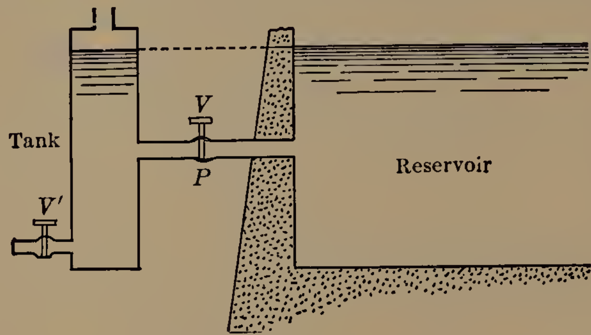


FIG. 55.—Tank and reservoir at the same pressure.

pipe P. The water level in the tank and in the reservoir is the same, that is, there is pressure in each but there is no *difference in pressure* between them. When the valve V is opened, no water flows from the reservoir to the tank. However, if the valve V is opened, allowing the water level in the tank to fall, a difference of pressure results and water flows from the reservoir to the tank.

Figure 56 shows two batteries A_1 and A_2 each having an electromotive force of 2 volts. The positive terminal of A_1 has a potential of +2 volts above its negative terminal; likewise the + terminal of A_2 has a potential of +2 volts above its negative terminal. The negative terminals of both batteries are at the same potential because they are connected by a copper wire through which no current flows, and consequently there can be no potential difference between the ends of this copper wire. Therefore points a and b must each be at the same potential of +2 volts. If now the switch S be closed, *no* current will flow from a to b, because there is no *difference of potential* between a and b.

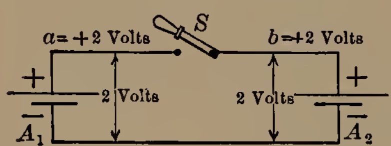


FIG. 56.—Two batteries having equal electromotive forces.

In Fig. 57 the electromotive force of battery B_1 is 3 volts and therefore the potential of its positive terminal is 3 volts above

that of its negative terminal. The electromotive force of battery B_2 is 2 volts and therefore the potential of its positive terminal is 2 volts above that of its negative terminal. The negative terminals are at the same potential. If this potential be assumed as zero, the point c is at a potential of $+3$ volts, whereas the potential of d is $+2$ volts. Therefore the point c is at a potential of $3 - 2$, or 1 volt higher than d. When switch S' is closed, a current will flow from c to d, in virtue of c being at a higher potential than d.

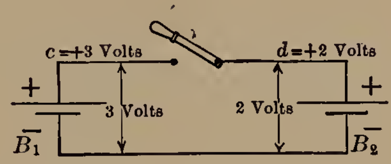


FIG. 57.—Two batteries having unequal electromotive forces.

57. Measurement of Voltage and Current.—Voltage or potential difference is ordinarily measured with a voltmeter. The voltmeter therefore should be connected *across* or *between* the

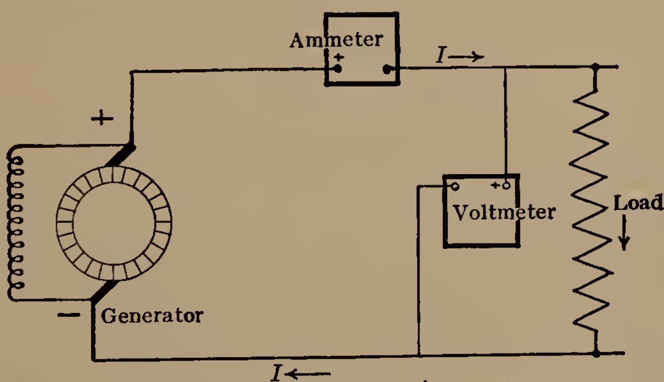


FIG. 58.—Proper method of connecting a voltmeter and an ammeter.

wires whose difference of potential is to be measured, as shown in Fig. 58.

Current is ordinarily measured with an ammeter. As current is the *quantity* of *electricity* per second passing in the wire, the ammeter must be so connected that only the current to be measured passes through it. This is accomplished by opening one of the wires of the circuit and inserting the ammeter, just as a water meter is inserted in a pipe when it is desired to measure the flow of water in the pipe. When the ammeter is so connected, the current passing through to the load is measured by the ammeter (see Fig. 58). *Never connect an ammeter across the line.*

58. Ohm's Law.—Ohm's law states that, for a steady current, the current in a circuit is *directly* proportional to the total electromotive force acting in the circuit and is *inversely* proportional to the total resistance of the circuit.

The law may be expressed by the following equation if the current I is in *amperes*, the electromotive force E is in *volts*, and the resistance R is in *ohms*.

$$I = \frac{E}{R}. \tag{17}$$

That is, the current in amperes in a circuit is equal to the electromotive force of the circuit in volts divided by the resistance of the circuit in ohms. Potential difference may be represented by either the letter "V" or "E," V usually signifying terminal voltage and E electromotive force or induced voltage.

Example.—The resistance of the field winding of a shunt motor is 41 ohms. What current flows through the winding when it is connected across 220-volt mains?

$$I = \frac{E}{R} = \frac{220}{41} = 5.37 \text{ amp.} \quad \textit{Ans.}$$

By transformation, equation (17) becomes

$$E = IR \tag{18}$$

That is, the voltage across any part of a circuit is equal to the product of the current in amperes and the resistance in ohms, provided the current is steady and there are no sources of e.m.f. within this part of the circuit.

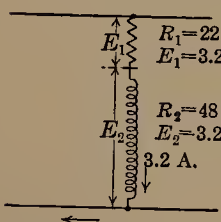


Fig. 59.—Voltage-drops across a generator field and its rheostat.

Example.—The resistance of the field winding of a shunt generator is 48.0 ohms and the resistance of its rheostat is 22.0 ohms (see Fig. 59). If the field current is 3.2 amp., what is the voltage across the field winding, the voltage across the rheostat, and the voltage across the generator terminals?

$E_1 = IR_1 = 3.2 \times 22.0 = 70.4$ volts across rheostat.
$E_2 = IR_2 = 3.2 \times 48.0 = 153.6$ volts field winding.

Total $= 224.0$ volts at generator terminals.

Also

$$E = I(R_1 + R_2) = 3.2\ (22.0 + 48.0) = 224.0 \text{ volts (check).} \quad \textit{Ans.}$$

Again, if equation (18) be solved for the resistance, the result is

$$R = \frac{E}{I}. \tag{19}$$

That is, the resistance of a circuit or part of a circuit is equal to the voltage divided by the current, provided the current is steady and there are no sources of e.m.f. within the part of the circuit under consideration. This formula is very useful in resistance measurements (see page 127).

Example.—The voltage across the terminals of a generator field is 230 volts and the field current is 7.0 amp. What is the resistance of the field circuit?

$$R = \frac{E}{I} = \frac{230}{7} = 32.9 \text{ ohms.} \quad \textit{Ans.}$$

59. The Series Circuit.—As was stated in Par. 40, if several resistances are connected in series, the total resistance is the sum of the individual resistances. That is,

$$R = r_1 + r_2 + r_3 + \text{etc.} \tag{20}$$

Therefore the current in such a series circuit

$$I = \frac{E}{R} = \frac{E}{r_1 + r_2 + r_3 + \ . \ . \ .} \tag{21}$$

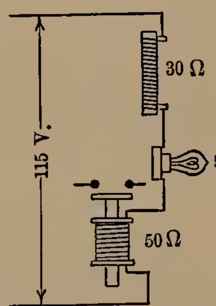


FIG. 60.—Relay circuit.

Example.—A 50-ohm relay is connected in series with a 30-ohm resistance tube and with a small pilot lamp having a resistance of 5.0 ohms, as shown in Fig. 60. The operating voltage is 115 volts. What current flows in this relay circuit?

$$I = \frac{115}{50 + 30 + 5.0} = \frac{115}{85} = 1.35 \text{ amp.} \quad \textit{Ans.}$$

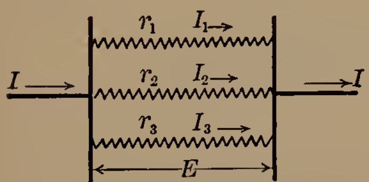


FIG. 61.—A parallel circuit.

60. The Parallel Circuit.—In Par. 40, the relation of total resistance to the component resistances in a parallel circuit was proved by transforming conductances into resistances. This equation may be proved by Ohm's law as follows: Consider the circuit of Fig. 61, consisting of resistances r_1, r_2, and r_3 in parallel across the voltage E. Let I_1 be the current in resistance r_1, I_2 the current in r_2 and I_3 the current in r_3.

Then

$$I_1 = \frac{E}{r_1}, \qquad I_2 = \frac{E}{r_2}, \qquad I_3 = \frac{E}{r_3} \qquad \text{(equation 17)}$$

Adding,

$$I_1 + I_2 + I_3 = \frac{E}{r_1} + \frac{E}{r_2} + \frac{E}{r_3} = E\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right).$$

Let the total current be $I = I_1 + I_2 + I_3$.
Let the equivalent resistance be R.

$$I = \frac{E}{R}.$$

Substituting I for $I_1 + I_2 + I_3$

$$I = \frac{E}{R} = E\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}\right)$$

or

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}. \tag{22}$$

That is, *the reciprocal of the equivalent resistance of a parallel circuit is the sum of the reciprocals of the individual resistances.*

If but two resistances are involved,

$$R = \frac{r_1 r_2}{r_1 + r_2} \tag{23}$$

Example.—Four lamps, having hot resistances of 50, 40, 25, and 15 ohms, respectively, are connected in multiple across the mains of a 32-volt lighting system (Fig. 62). What is the equivalent resistance of the combination and what is the total current?

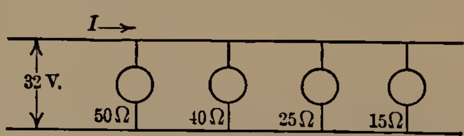


FIG. 62.—Circuit consisting of four lamps in parallel.

Applying equation (22),

$$\frac{1}{R} = \frac{1}{50} + \frac{1}{40} + \frac{1}{25} + \frac{1}{15} = 0.0200 + 0.0250 + 0.0400 + 0.0667$$
$$= 0.1517 \text{ mho}$$

$$R = \frac{1}{0.1517} = 6.59 \text{ ohms.} \quad \textit{Ans.}$$

$$I = \frac{32}{6.59} = 4.86 \text{ amp.} \quad \textit{Ans.}$$

61. Division of Current in a Parallel Circuit.—In Fig. 63, two resistances, R_1 and R_2, are connected in parallel across the voltage E. The current I_1 flows in the resistance R_1 and the current I_2 flows in the resistance R_2. It is desired to obtain the ratio of I_1 to I_2.

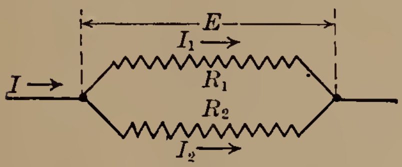


FIG. 63.—Division of current in a two-branch parallel circuit.

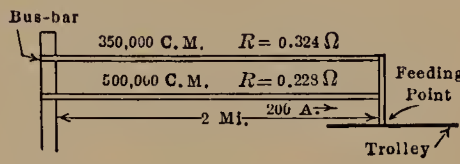


FIG. 64.—Trolley feeders in parallel.

$$I_1 = \frac{E}{R_1}, \; I_2 = \frac{E}{R_2}$$

or

$$\frac{I_1}{I_2} = \frac{\dfrac{E}{R_1}}{\dfrac{E}{R_2}} = \frac{R_2}{R_1}. \tag{24}$$

That is, *in a parallel circuit of two branches, the currents are inversely as the resistances.* (This does *not* apply to the division of current between the field and armature of a shunt motor *when the motor is running.*)

Example.—A 2-mile length of 350,000 C.M. trolley-feeder having a total resistance of 0.324 ohm and a 2-mile length of 500,000 C.M. feeder having a total resistance of 0.228 ohm are connected in parallel between the station bus-bars and the trolley feeding point, Fig. 64. If a current of 200 amp. flows in the 500,000 C.M. feeder, what current flows in the 350,000 C.M. feeder?

Let I_1 be the current in the 350,000 C.M. feeder and I_2 be the current in the 500,000 C.M. feeder. Applying equation (24)

$$\frac{I_1}{200} = \frac{0.228}{0.324}, \; I_1 = 200\frac{0.228}{0.324} = 141 \text{ amp. } \textit{Ans.}$$

Since both feeders are connected across the same difference of potential, $I_1R_1 = I_2R_2$.

Hence, $141 \times 0.324 = 45.6$ volts. $200 \times 0.228 = 45.6$ volts (check).

That is, 45.6 volts are lost in overcoming the joint resistance of the two feeders or the resistance of either separately.

Example.—A current of 12.0 amp. divides between two paths in parallel, part passing through a branch having a resistance of 8.0 ohms, the other branch having a resistance of 12.0 ohms. How much current flows in each branch?

Let I_1 be the current in the 8.0-ohm branch and I_2 be the current in the 12.0-ohm branch.

$$\frac{I_1}{I_2} = \frac{12.0}{8.0} \text{ (equation (24))} \tag{1}$$

$$I_1 + I_2 = 12 \tag{2}$$

$$I_1 = I_2 \frac{12.0}{8.0} = I_2 \frac{3}{2} \text{ from (1).}$$

Substituting in (2),

$$I_2 \frac{3}{2} + I_2 = 12.0$$

$$\frac{5I_2}{2} = 12.0,\ I_2 = 4.8 \text{ amp. } \textit{Ans.}$$

$$I_1 = 4.8\frac{3}{2} = 7.2 \text{ amp. } \textit{Ans.}$$

62. The Series-parallel Circuit.—A circuit may consist of groups of parallel resistances in series with other resistances or groups of resistances, as shown in Figs. 65 and 66. When such is the case, each group of parallel resistances is first combined into its equivalent single resistance by equation (22) and the whole is then treated as a series circuit.

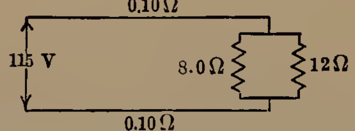


FIG. 65.—Heating units in parallel and in series with line resistance.

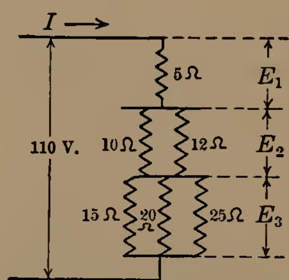


FIG. 66.—Series-parallel circuit.

Example.—Two heating units having resistances of 8.0 and 12 ohms respectively are connected in multiple and are supplied from constant-voltage, 115-volt bus-bars over a 100-ft. run of No. 10, A.W.G. wire, each conductor of which has a resistance of 0.10 ohm, Fig. 65. What is the total current and what is the current in each heating unit?

The 8.0-ohm and 12-ohm resistances are combined into an equivalent resistance R_1 by equation (22).

$$\frac{1}{R_1} = \frac{1}{8.0} + \frac{1}{12} = 0.1250 + 0.0833 = 0.2083 \text{ mho}$$

$$R_1 = \frac{1}{0.2083} = 4.80 \text{ ohms}$$

$$I = \frac{115}{0.10 + 0.10 + 4.80} = \frac{115}{5.00} = 23.0 \text{ amp. } \textit{Ans.}$$

The voltage across the units

$$V = 23.0 \times 4.80 = 110.4 \text{ volts}$$

$$\text{The current in 8.0 ohms, } I_1 = \frac{110.4}{8.0} = 13.8 \text{ amp} \quad \textit{Ans.}$$

$$\text{The current in 12 ohms, } I_2 = \frac{110.4}{12} = 9.2 \text{ amp.} \quad \textit{Ans.}$$

Example.—Determine the total current in the circuit shown in Fig. 66; the voltage across each portion of the circuit; the current in each resistance.

Combine first the 10-ohm and 12-ohm resistances into a resistance R_1

$$\frac{1}{R_1} = \frac{1}{10} + \frac{1}{12} = 0.10 + 0.0833 = 0.1833 \text{ mho}$$

$$R_1 = \frac{1}{0.1833} = 5.45 \text{ ohms.}$$

Likewise, combining the group of three resistances into R_2

$$\frac{1}{R_2} = \frac{1}{15} + \frac{1}{20} + \frac{1}{25} = 0.0667 + 0.050 + 0.040$$

$$= 0.1567 \text{ mho}$$

$$R_2 = \frac{1}{0.1567} = 6.39 \text{ ohms}$$

$$I = \frac{110}{5 + 5.45 + 6.39} = \frac{110}{16.84} = 6.54 \text{ amp.}$$

$$E_1 = 6.54 \times 5.0 = 32.7 \text{ volts}$$

$$E_2 = 6.54 \times 5.45 = 35.6 \text{ volts}$$

$$E_3 = 6.54 \times 6.39 = 41.7 \text{ volts}$$

$$\text{Total } 110.0 \text{ volts (check).}$$

$$\text{Current in } 10\ \Omega = \frac{35.6}{10} = 3.56 \text{ amp.}$$

$$\text{Current in } 12\ \Omega = \frac{35.6}{12} = 2.97 \text{ amp.}$$

$$\text{Total } 6.53 \text{ amp. (check).}$$

$$\text{Current in } 15\ \Omega = \frac{41.7}{15} = 2.78 \text{ amp.}$$

$$\text{Current in } 20\ \Omega = \frac{41.7}{20} = 2.09 \text{ amp.}$$

$$\text{Current in } 25\ \Omega = \frac{41.7}{25} = 1.67 \text{ amp.}$$

$$\text{Total } 6.54 \text{ (check).}$$

63. Electrical Power.—The unit of electrical power is the *watt* and is defined as follows:

"The *International Watt* is the energy expended per second by an unvarying electric current of one international ampere under the pressure of one international volt." The power in watts is, therefore, equal to the product of the volts and the amperes. Thus the power

$$P = EI \text{ watts.} \tag{25}$$

Since $E = IR$ in a circuit containing resistance only, (page 56, equation (18)), equation (25) may be written

$$P = (IR)I = I^2R. \tag{26}$$

Substituting for I its value $\left(I = \frac{E}{R}\right)$ in equation (25),

$$P = E\left(\frac{E}{R}\right) = \frac{E^2}{R}. \tag{27}$$

Equation (25) is useful when the volts and the amperes are known; equation (26) is useful when the current and the resistance are known; and equation (27) is useful when the voltage and the resistance are known.

Example.—The resistance of a 150-scale voltmeter is 12,000 ohms. What power is consumed by this voltmeter when it is connected across a 125-volt circuit?

Since the voltage and the resistance are known, equation (27) is most convenient:

$$P = \frac{(125)^2}{12{,}000} = 1.30 \text{ watts. } \textit{Ans.}$$

This may be checked by equation (25)

$$I = \frac{125}{12{,}000} = 0.0104 \text{ amp.}$$

$$P = 125 \times 0.0104 = 1.30 \text{ watts (check).}$$

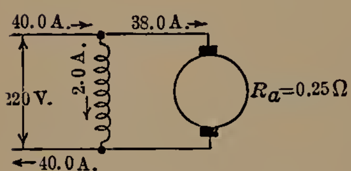


FIG. 67.—Power taken by a shunt motor.

Example.—A motor takes 40 amp. at 220 volts (Fig. 67). The field resistance is 110 ohms and the armature resistance, R_a, is 0.25 ohm. (*a*) How much power does the motor take? (*b*) How much power is consumed by the field? (*c*) How much power does the armature take? (*d*) What power is lost as heat in the armature?

(*a*) Applying equation (25), $P_1 = 220 \times 40 = 8{,}800$ watts. *Ans.*

(*b*) Applying equation (27), $P_2 = \frac{(220)^2}{110} = 440$ watts. *Ans.*

(*c*) The field current $I_f = \frac{220}{110} = 2.0$ amp.

The armature current $I_a = 40.0 - 2.0 = 38.0$ amp.
The armature power $P_a = 220 \times 38.0 = 8{,}360$ watts. *Ans.*

(*d*) The armature heating loss is found by using equation (26).

$$P_c = (38.0)^2 \times 0.25 = 361 \text{ watts. } \textit{Ans.}$$

The watt is often too small a unit for commercial use and the *kilowatt* (equal to 1,000 watts) is used when large amounts of

power are being considered. It is often necessary to transform from mechanical horsepower to electrical power and conversely, and a knowledge of the relation of the two is, therefore, useful:

$$746 \text{ watts} = 1 \text{ h.p.} \tag{28}$$

$$0.746 \text{ kw.} = 1 \text{ h.p.} \tag{29}$$

and

$$1 \text{ h.p.} = \tfrac{3}{4} \text{ kw., very nearly} \tag{30}$$

$$1 \text{ kw.} = \tfrac{4}{3} \text{ h.p., very nearly} \tag{31}$$

(Also see Appendix, page 308.)

Example.—An electric motor takes 28 amp. at 550 volts and has an efficiency of 89 per cent. What horsepower does it deliver?

$$\text{Input} = 28 \times 550 = 15{,}400 \text{ watts}$$

$$\text{Output} = 15{,}400 \times 0.89 = 13{,}700 \text{ watts}$$

$$\frac{13{,}700}{746} = 18.35 \text{ h.p. at the pulley.} \quad \textit{Ans.}$$

64. Electrical Energy.—Power is the *rate of doing work,* or is the *rate of expenditure of energy.* For example, the definition of the international watt, page 61, states that a watt is equal to the energy expended *per second* by an unvarying electric current of one international ampere under the pressure of one international volt. That is, power is *rate* of expenditure of energy. The energy expended in one second by one international ampere under the pressure of one international volt is one *joule.* Hence a joule is equal to a *watt-second.* Therefore, electrical energy must be equal to the product of electrical power and time. The unit of electrical energy is therefore the *watt-second* or *joule.*

$$W = EIt \text{ watt-sec.} \tag{32}$$

if t is in seconds, E is in volts, and I is in amperes.

The watt-second is ordinarily too small a unit for commercial purposes, so the larger unit, the *kilowatt-hour* (kw-hr.) is commonly used. 1 kilowatt-hour $= 1{,}000 \times 60 \times 60 = 3{,}600{,}000$ $= (3.6 \times 10^6)$ joules or watt-seconds.

The distinction between power and energy (or work) should be clearly kept in mind. Power is *rate* of doing work, just as velocity is rate of motion. On the other hand, energy is the total work done and is equal to the power multiplied by the time during which it acts, just as distance covered is the velocity or rate of motion multiplied by the time. To speak of a train traveling at a rate of 40 miles per hour gives no information as to the total

distance which the train travels. Likewise, to speak of 50 kw. does not state the amount of energy that is involved. The statement "electricity is sold for so many cents per kilowatt" is incorrect. The correct expression is "electrical *energy* is sold for so many cents per kilowatt-*hour*." To illustrate:

Example.—If energy is sold for 10c. per kilowatt-hour, how many kilowatts may be purchased for 20c.?

This question as it stands cannot be answered, since the *time* is not given. If, however, it is assumed that the power is to be used for 1 hr.:

$$\frac{20\text{c.}}{10\text{c.}} = 2 \text{ kw-hr. available}$$

$$\frac{2 \text{ kw-hr.}}{1 \text{ hr.}} = 2 \text{ kw.} \quad \textit{Ans.}$$

$$\text{If used in 0.5 hour, } \frac{2 \text{ kw-hr.}}{0.5 \text{ hour}} = 4 \text{ kw.} \quad \textit{Ans.}$$

$$\text{If used in 0.001 hour, } \frac{2 \text{ kw-hr.}}{0.001 \text{ hour}} = 2{,}000 \text{ kw.} \quad \textit{Ans.}$$

so that the 20c. could purchase *any number* of *kilowatts*, depending on the time during which the power is supplied.

In a similar way, horsepower is *rate of doing work* and is equivalent to 33,000 ft.-lb. *per minute* and *not* to 33,000 ft.-lb. A motor developing $\frac{1}{8}$ h.p. could do 33,000 ft.-lb. of work if allowed 8 min. in which to do it. When speaking of *work* in connection with horsepower, the *horsepower-hour* is the unit ordinarily used.

Example.—How many watt-seconds are supplied by a motor developing 2 hp. for 5 hr.?

$$2 \times 5 = 10 \text{ h.p.-hr.}$$

$$10 \text{ hp.-hr.} \times 746 = 7{,}460 \text{ watt-hr.}$$

$$7{,}460 \times 3{,}600 = 2.68 \times 10^7 \text{ watt-sec.} \quad \textit{Ans.}$$

65. Heat and Energy.—It is well known that heat may be converted into mechanical and electrical energy, and, conversely, that electrical and mechanical energy may be converted into heat. The complete cycle of energy transformation is well illustrated by a steam power plant. The energy is brought to the plant in the coal, as *chemical energy*. The ingredients of the coal combine with the oxygen of the air, thus converting the chemical energy into *heat energy*. A certain percentage of this heat is transferred to the boiler and produces steam. The expansion of the steam in the engine cylinders, or through the buckets and blades of the turbine, converts the heat energy of the steam into *mechan-*

ical energy. This mechanical energy drives the generator, which converts a large percentage of this energy into *electrical energy.* A portion of this electrical energy is transformed into heat in the wires, bus-bars, etc. Finally, the remainder is used to supply lamps, propel electric cars, operate motors, and some may be used for chemical processes. Ultimately all the energy appears again as heat or else is converted into chemical or other forms of energy.

The following table shows approximately what becomes of each 100 heat units, existing initially in the coal, in the most efficient modern power plants, using superheaters, condensers, and large generating units.

EFFICIENCY OF ENERGY CONVERSION

	Form of energy	Efficiency (per cent.)	Heat units converted
Coal	Chemical	..	100.0
Boiler	Heat	80	80.0
Turbine	Mechanical	25	20.0
Generator	Electrical	95	19.0
Distribution system (to point of utilization)	Electrical	85	16.2
Small motors	Mechanical	65 (av.)	10.5
Lamps	Light	2	0.32

Figure 68[1] (by R. A. Philip) shows graphically the flow of energy from the boiler to the point of utilization. It is apparent that even in the most modern power plants the overall efficiency is very low.

66. Thermal Units.—The unit of heat in the English system is the B.t.u. (British thermal unit) and is equal to the amount of heat required to raise one pound of water 1° F. It is equal to 778 ft.-lb. (called the "mechanical equivalent" of heat). That is, if all the heat energy of a B.t.u. could be converted into useful work it would raise 1 lb. a distance of 778 ft.

[1] *Trans.*, Am. Inst. Elec. Eng., vol. XXXIV, p. 781, 1915.

In the C.G.S. system the heat unit is the gram-calorie[1] and is the amount of heat required to raise one gram of water 1° C. A gram-calorie is equal to 4.184 watt-sec. or joules.

By Joule's law the heat developed in a circuit is:

$$W = \frac{1}{4.184} I^2Rt = 0.2390\ I^2Rt \text{ cal.} \tag{33}$$

where t is in seconds, I in amperes, and R in ohms.

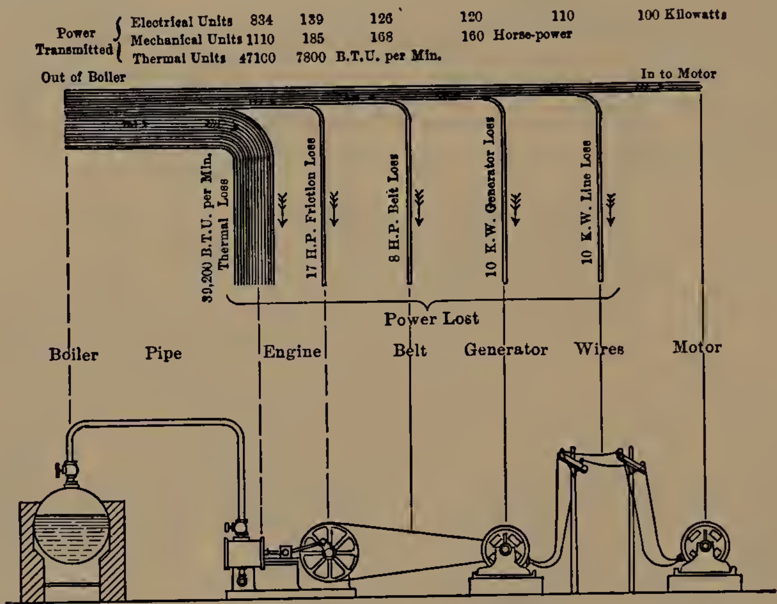


FIG. 68.—Energy flow—thermal, mechanical and electrical transmission.

Example.—It is desired to raise the temperature of 1 liter of water from 20° C. to the boiling temperature, 100° C., with an immersion heater which takes 3.50 amp. at 110 volts. Neglecting losses: (*a*) How much time is required? (*b*) At 10 cts. per kilowatt-hour, what does it cost to heat this water?

Since 1 liter = 1,000 c.c., 1,000 c.c. × 80° = 80,000 gram-cal. required.

$80{,}000 \times 4.18 = 334{,}000$ joules or watt-sec.

Rate of energy supply, $110 \times 3.50 = 385$ watts.

$$\frac{334{,}000}{385} = 868 \text{ sec. or } 14.5 \text{ min.} \quad \textit{Ans.} \tag{a}$$

$$\frac{334{,}000}{3{,}600{,}000} = 0.093 \text{ kw-hr.}$$

0.093×10 cts. $= 0.93$ cts. *Ans.*

Example.—10 hp. is delivered by a pump circulating 400 gal. of water per minute through a certain cooling system. How many degrees Fahren-

[1] See Appendix, page 308.

heit per minute is the temperature of the water raised by the action of the pump?

$$10 \text{ hp.} = 10 \times 33{,}000 = 330{,}000 \text{ ft.-lb. per minute}$$

$$\frac{330{,}000}{778} = 424 \text{ B.t.u. per minute}$$

$$400 \text{ gal.} = 400 \times 8.34 = 3{,}336 \text{ lb.}$$

$$\frac{424}{3{,}336} = 0.13° \text{ F.} \quad \textit{Ans.}$$

67. Potential-drop in a Feeder Supplying One Concentrated Load.—Figure 69 shows a feeder (consisting of a positive and a negative wire) supplying a motor load. The feeder is connected

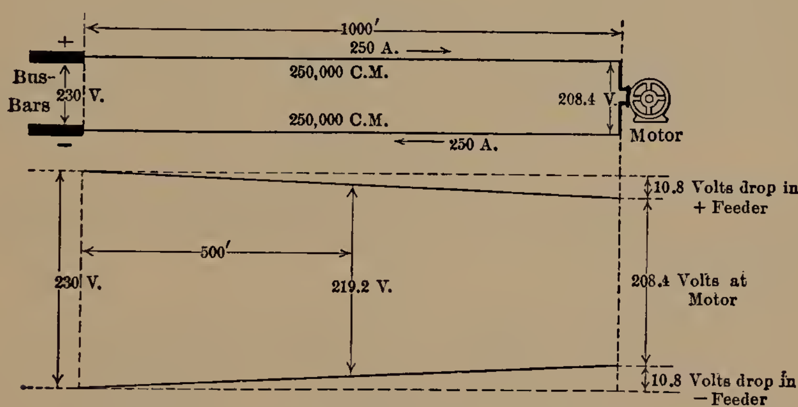


FIG. 69.—Voltage-drop in a feeder due to a single load.

to bus-bars having a constant potential difference of 230 volts. The feeder is 1,000 ft. long and consists of two 250,000 C.M. conductors. The maximum load on the feeder is 250 amp. It is required to determine the voltage at the motor terminals and the efficiency of transmission.

As was stated on page 51, Par. 55, the voltage at the motor must be less than that at the bus-bars because of the voltage lost in supplying the resistance-drop in the feeder.

From Table 52, the resistance of 1,000 ft. of 250,000 C.M. cable is 0.0431 ohm. As was shown in Par. 55, the net voltage at the receiving end of the line is less than the voltage at the sending end by the voltage loss in both the *outgoing* and the *return* wire. Therefore, the drop in 2,000 ft. of cable must be taken, the total resistance being 0.0862 ohm.

The current is 250 amp.

By equation (18), page 56, the voltage-drop in the line:

$$E' = 250 \times 0.0862 = 21.55 \text{ volts.}$$

Therefore, the voltage at the motor terminals is

$$230 - 21.6 = 208.4 \text{ volts.} \quad \textit{Ans.}$$

In Fig. 69 the voltage-drop along the line is shown graphically. The voltage at the sending end of the line is 230 volts, and there is a uniform drop in each wire, this drop increasing uniformly to 10.8 volts, making a total voltage loss of 21.6 volts. The potential difference between the two wires 500 ft. from the sending end will be $230 - 10.8 = 219.2$ volts as shown.

The power delivered to the motor $= 208.4 \times 250$ watts.

The power delivered to the line $= 230 \times 250$ watts.

$$\text{The efficiency of transmission} = \frac{\text{output}}{\text{input}} = \frac{208.4 \times 250}{230 \times 250} = \frac{208.4}{230}$$

or 90.6 per cent.

With one concentrated load, the efficiency of transmission is given by the voltage at the load divided by the voltage at the sending end of the line.

The power loss in each wire,

$$P_c = I^2R = (250)^2 \times 0.0431 = 2{,}690 \text{ watts.}$$

$$\text{Total power loss} = 2 \times 2{,}690 = 5{,}380 \text{ watts.}$$

$$\text{Percentage power loss} = \frac{5{,}380}{230 \times 250}\,100 = \frac{5{,}380}{57{,}500}\,100, = 9.4$$

per cent.

The percentage power loss also equals

$$\frac{I^2R}{EI}\,100 = \frac{IR}{E}\,100 = \frac{21.55}{230}\,100 = 9.4 \text{ per cent.}$$

That is, with a single concentrated load, the percentage power loss is equal to the ratio of the *IR* drop in the line to the sending-end voltage.

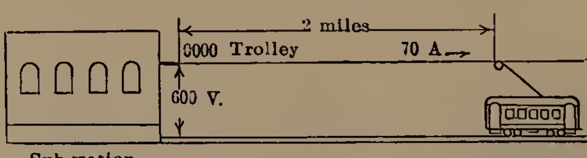


Fig. 70.—Sub-station supplying a trolley car.

Example.—A trolley car, 2 miles from the sub-station (Fig. 70), takes 70 amp. over a 0000 hard-drawn trolley wire having a resistance of 0.270 ohm

per mile. The rail and ground return have a resistance of 0.06 ohm per mile. If the sub-station voltage is 600 volts: (*a*) What is the voltage at the car? (*b*) What is the efficiency of transmission?

Resistance of trolley $= 2 \times 0.270 = 0.540$ ohm.

Resistance of rail return $= 2 \times 0.06 = 0.120$ ohm.

Total resistance $= 0.660$ ohm.

Total IR drop $= 70 \times 0.660 = 46.2$ volts.

(*a*) Voltage at car $= 600 - 46.2 = 553.8$ volts. *Ans.*

(*b*) Efficiency of transmission $= \dfrac{553.8}{600} = 0.923$, or 92.3 per cent. *Ans.*

CHAPTER V

BATTERY ELECTROMOTIVE FORCES—KIRCHHOFF'S LAWS

68. Battery Electromotive Force and Resistance.—If a voltmeter be connected across the terminals of a battery (Fig. 71), the switch S being open, the instrument will indicate a certain voltage E. If now the switch S be closed, allowing the current I to flow, the instrument will indicate a voltage V which is less than E.

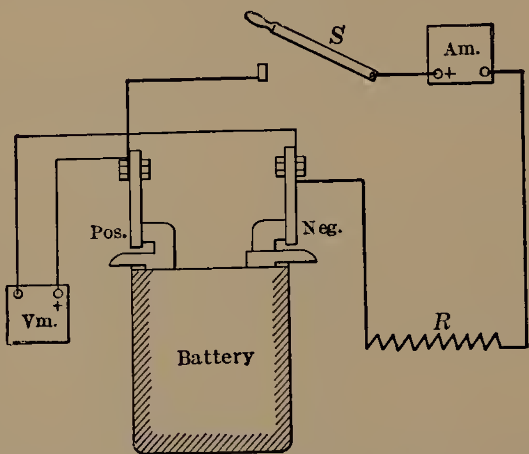


FIG. 71.—Connections for measuring battery resistance.

The voltage E, measured when the battery delivers no current, is the *internal voltage* or the *electromotive force* of the battery; the voltage V, measured when a current I flows, is known as the *terminal voltage* of the battery for that particular current value.

The difference between the open-circuit voltage E and the voltage V, measured when current is being taken from the battery, is the *voltage-drop* in the battery due to the passage of current through the battery resistance. Every cell has resistance, lying for the most part in the electrolyte, but partly in the battery plates and terminals. When the external circuit is closed so that

current flows, voltage is required to send this current through the battery resistance, just as voltage is required to send current through an external resistance.

If the voltage E, measured at the battery terminals when the circuit is open, drops to V when the circuit is closed, the voltage $e = (E - V)$ is the voltage-drop through the cell due to the passage of the current I. Let the cell resistance be r. Then, by Ohm's law,

$$E - V = e = Ir \text{ (by equation (18), page 56)}$$

or

$$r = \frac{e}{I} = \frac{E - V}{I} \text{ (by equation (19), page 57).} \tag{34}$$

That is, the internal resistance of the battery is equal to the open-circuit voltage minus the closed-circuit terminal voltage divided by the closed-circuit current.

Also by transformation,

$$E = V + Ir \tag{35}$$
$$V = E - Ir. \tag{36}$$

When a battery delivers current the terminal voltage is equal to the electromotive force minus the resistance-drop within the battery.

Example.—The open-circuit voltage of a storage cell is 2.20 volts. The terminal voltage measured when a current of 12 amp. flows is found to be 1.98 volts. What is the internal resistance of the cell?

The voltage-drop through the cell

$$E - V = 2.20 - 1.98 = 0.22 \text{ volt.}$$

Then $$r = \frac{0.22}{12} = 0.0183 \text{ ohm. } \textit{Ans.}$$

In making a measurement of this character, it must be remembered that under open-circuit conditions even the ordinary voltmeter takes some current. If the cell capacity is small (as in a Weston cell) the voltmeter current alone may reduce the terminal voltage to a value one-half, or even less, of the open-circuit voltage. Under these conditions the ordinary voltmeter cannot be used to measure the electromotive force of the cell.

Moreover, it is impossible to measure directly the internal voltage of the battery when the battery delivers current, for the voltage-drop occurs *within* the cell itself. Figure 72 represents these conditions so far as their effect on the external circuit is concerned. A battery cell B is enclosed in a sealed box. Its resistance r is considered as removed from the cell itself and connected external to the cell, but within the sealed box. The cell may be considered as having no resistance, its resistance having been replaced by r. The connections are brought through bushings in the box to terminals a and b. When no current is being delivered by the cell, if a voltmeter be connected across the two terminals a and b, the instrument will measure the internal electromotive force, E. If, however, a current I flows, the terminal voltage will drop from E to V, due to the voltage-drop in the resistance r. Under these conditions it is impossible to measure E directly when current is flowing, since the voltmeter can only be connected outside the resistance through which the voltage-drop occurs.

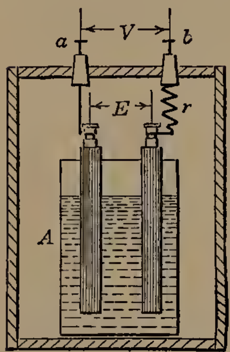


FIG. 72.—The internal resistance of a cell.

Example.—A voltmeter, connected across the terminals of a dry cell, reads 1.40 volts when the cell is open-circuited. The voltmeter reads 1.02 volts when the cell delivers a current of 3.0 amp. What is the internal resistance of the cell?

The voltage-drop within the cell, $e = 1.40 - 1.02 = 0.38$ volt. By Ohm's law,

$$r = \frac{0.38}{3.0} = 0.127 \text{ ohm.} \quad \textit{Ans.}$$

Or, by applying equation (34) directly,

$$r = \frac{1.40 - 1.02}{3.0} = \frac{0.38}{3.0} = 0.127 \text{ ohm.}$$

The voltage E and the resistance r are seldom constant, but are more or less dependent on the current. They are also affected by temperature, change in specific gravity of the electrolyte, polarization, etc.

69. Battery Resistance and Current.—As was shown in Par. 68, the resistance within the battery tends to reduce the flow of

current. If, in Fig. 71, the switch be closed, the cell electromotive force E will be acting upon a circuit consisting of the internal resistance r of the cell and the resistance R of the external circuit. The resistances r and R being in series, the total resistance in the circuit is their sum. The current is

$$I = \frac{E}{r + R} \tag{37}$$

The total power developed within the battery,

$$P = EI. \tag{38}$$

The power lost in the battery,

$$p = I^2r.$$

The power delivered to the load by the battery,

$$P' = VI \tag{39}$$

where V is the terminal voltage.

If the cell is short-circuited, R becomes zero and $I = \frac{E}{r}$. Under these conditions all the electrical energy developed by the cell is converted into heat within the cell itself.

Example.—A battery cell having an electromotive force of 2.2 volts and an internal resistance of 0.03 ohm is connected to an external resistance of 0.10 ohm. What current flows and what is the efficiency of the battery as used? If the battery is short-circuited, what current flows?

$$I = \frac{2.2}{0.03 + 0.10} = \frac{2.2}{0.13} = 16.9 \text{ amp.} \quad \textit{Ans.}$$

Power lost in the battery,

$$p = (16.9)^2 \times 0.03 = 8.57 \text{ watts.}$$

The useful power,

$$P' = (16.9)^2 \times 0.10 = 28.6 \text{ watts.}$$

P' is equal to the total power developed by the battery minus the battery loss.

$$P = 2.2 \times 16.9 = 37.2 \text{ watts}$$

$$P' = 37.2 - 8.6 = 28.6 \text{ watts (check).}$$

$$\text{Eff.} = \frac{28.6}{28.6 + 8.6} \text{ or } 76.9 \text{ per cent.} \quad \textit{Ans.}$$

The short-circuit current,

$$I' = \frac{2.2}{0.03} = 73.3 \text{ amp.} \quad \textit{Ans.}$$

From the foregoing, the following rule may be deduced: *The current in a circuit is equal to the total electromotive force acting in the circuit divided by the total resistance of the circuit.*

70. Batteries Receiving Energy.—If a resistance load be connected across a battery, current will immediately flow from the positive terminal of the battery through the load resistance and will return to the battery through its negative terminal (see Fig. 85, page 89). As has already been pointed out, the battery terminal voltage will be less than its open-circuit value, due to the current flowing through the internal resistance of the battery. Under these conditions, the battery is a source of energy and is acting as a generator, that is, it *delivers* energy.

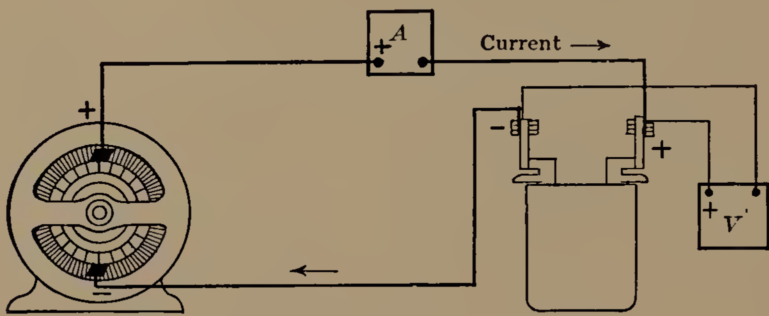


FIG. 73.—Generator charging a battery.

If current is forced to *enter* the battery at its positive terminal, the battery will no longer be supplying energy but will be receiving energy. This energy must be supplied from some other source, as from another battery, or, as is more common, from a generator. The cell shown in Fig. 73 has an electromotive force of 2 volts, and a voltmeter V, connected across its terminals, indicates 2 volts when no current flows. If another source of electrical energy, such as a direct-current generator, supplies a potential difference of just 2 volts and its + terminal is connected to the + terminal of the battery and its − terminal connected to the − terminal of the battery, as shown in the figure, the voltmeter V will still read 2 volts and the ammeter A will read zero. That is, the battery neither delivers nor receives energy and no effect is noted other than those observed when the battery is open-circuited. Under these conditions the battery is said to be "floating." If, however, the voltage of the generator be raised slightly, the ammeter A will indicate a current flowing from the + terminal of the generator *into* the + terminal of the battery, a direction just opposite to that which the current has when the

battery *supplies* energy. The voltmeter will no longer read 2 volts, but will indicate a potential difference somewhat in excess of 2 volts.

The foregoing may be illustrated by a mechanical analogy (Fig. 74) which shows a car standing on the track. A force of 400 lb. is necessary to overcome the standing friction of the car on the track. At one end of the car a force F is applied. Before the force F can move the car its value must at least equal 400 lb. When F is exactly 400 lb. the car will not move, just as no current flows into the battery when the generator voltage

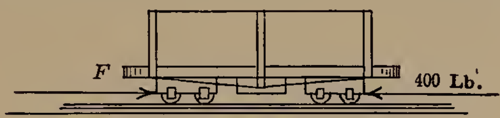


FIG. 74.—Force necessary to start a car.

is just equal to the electromotive force of the battery. When the force F exceeds 400 lb., however, the car will move, the force effective in producing this motion being the amount by which F exceeds 400 lb. Thus, if $F = 450$ lb., 400 lb. is utilized in overcoming the 400 lb. opposing force due to friction and 50 lb. is effective in moving the car.

With the battery no current flows until voltage in excess of the 2 volts is produced by the generator. Thus, if the generator voltage be raised to 2.4 volts, 2.0 volts of this is utilized to "buck" the 2.0 volts of the cell and 0.4 volt is effective in sending current into the cell. If the resistance of the leads be negligible and the cell resistance be 0.1 ohm, the current will be

$$I = \frac{0.4}{0.1} = 4.0 \text{ amp.}$$

Therefore, if E is the electromotive force of a battery, r its resistance, and V the terminal voltage when current flows *in* at its positive terminal,

$$I = \frac{V - E}{r} \tag{40}$$

$$V = E + IR \tag{41}$$

$$E = V - Ir. \tag{42}$$

That is, the electromotive force of the cell is less than the terminal voltage by the amount of the resistance drop in the cell itself.

These equations should be compared with equations (34), (35), and (36), (page 71).

Under these conditions, the cell is *receiving* electric energy, as is the case when a storage battery is being charged.

Example.—A 6-volt ignition battery, consisting of three lead storage cells in series, has a total e.m f. of 6.2 volts and an internal resistance of 0.08 ohm. What is the terminal voltage of the battery when it is being charged at the 15-amp. rate? How much energy is stored chemically per hour?

The resistance drop in the battery,

$$e = 15 \times 0.08 = 1.2 \text{ volts.}$$

The terminal voltage,

$$V = 6.2 + 1.2 = 7.4 \text{ volts.} \quad \textit{Ans.}$$

The stored energy,

$$15 \times 6.2 \times 1.0 = 93.0 \text{ watt-hr.} \quad \textit{Ans.}$$

71. Battery Cells in Series.—Strictly speaking, a battery consists of more than one unit or cell. However, the term battery has come also to mean a single cell, when this cell is not acting in conjunction with others.

When cells are connected in series, their electromotive forces are added together to obtain the total electromotive force of the battery, and their resistances are added together to obtain the total resistance of the battery.

Thus, if several cells, having electromotive force E_1, E_2, E_3, E_4, etc., and resistances r_1, r_2, r_3, r_4, etc., are connected in series to form a battery, the total electromotive force of the combination is

$$E = E_1 + E_2 + E_3 + E_4 + \text{etc.} \tag{43}$$

and the total resistance is

$$r = r_1 + r_2 + r_3 + r_4, + \text{etc.} \tag{44}$$

Equation (43) assumes that the cells are all connected + to – so that their electromotive forces add. If any cell be connected so that its electromotive force opposes the others, its voltage in equation (43) must be preceded by a minus sign.

If an external resistance R is connected across the terminals of this battery, then by equation (37), page 73, the current is

$$I = \frac{E}{r + R} = \frac{E_1 + E_2 + E_3 + E_4, + \text{etc.}}{r_1 + r_2 + r_3 + r_4, + \text{etc.}, + R}. \tag{45}$$

Example.—Four dry cells having electromotive forces of 1.30, 1.30, 1.35, and 1.40 volts and resistances of 0.3, 0.4, 0.2, and 0.1 ohm, are connected in series to operate a relay having a resistance of 10 ohms. What current flows in the relay?

$$I = \frac{1.30 + 1.30 + 1.35 + 1.40}{0.3 + 0.4 + 0.2 + 0.1 + 10} = \frac{5.35}{11.0} = 0.486 \text{ amp.} \quad \textit{Ans.}$$

A battery consisting of n equal cells in series has an electromotive force n times that of one cell, but has the current capacity of one cell only.

72. Equal Batteries in Parallel.—To operate satisfactorily in parallel, all the batteries should have the same electromotive force. The behavior of batteries having unequal electromotive forces can be treated as a special problem (see Par. 74).

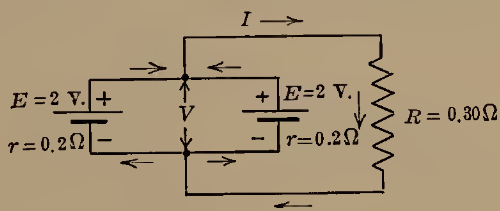


FIG. 75.—Parallel connection of two equal cells.

Figure 75 shows two cells, each having an electromotive force of 2.0 volts and a resistance of 0.2 ohm. It is clear that the electromotive force of the entire battery is no greater than the electromotive force of either cell. The current, however, has two paths through which to flow. Therefore, for a fixed external current, the voltage drop in each cell is one-half that occurring when the entire current passes through one cell. If the internal resistance of each cell is 0.2 ohm, the resistance of the battery must be $\frac{0.2}{2} = 0.10$ ohm.

Example.—Find the external current in Fig. 75, when a resistance of 0.30 ohm is connected across the battery terminals.

$$\text{The battery resistance} = \frac{0.20}{2} = 0.10 \text{ ohm.}$$

$$I = \frac{2.0}{0.10 + 0.30} = 5.0 \text{ amp.} \quad \textit{Ans.}$$

The current in each cell is 2.5 amp.

The terminal voltage $V = 2.0 - 2.5 \times 0.2 = 1.5$ volts.

$$\text{The current } I = \frac{1.5}{0.30} = 5.0 \text{ amp. (check).}$$

73. Kirchhoff's Laws.—By means of Kirchhoff's laws it is possible to solve many circuit networks that would otherwise be difficult of solution.

1. *In any branching network of wires, the algebraic sum of the currents in all the wires that meet at a point is zero.*

2. *The sum of all the electromotive forces acting around a complete circuit is equal to the sum of the resistances of its separate parts multiplied each into the strength of the current that flows through it,* or *the total change of potential around any closed circuit is zero.*

The first law is obvious. It states that the total current *leaving* a junction is equal to the total current *entering* the junction. If this were not so, electricity would either accumulate or disappear at the junction, an obvious impossibility.

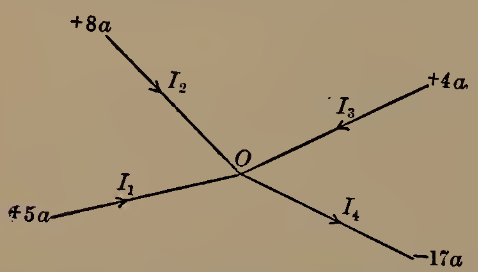


FIG. 76.—Illustrating Kirchhoff's first law.

The law is illustrated by Fig. 76. Four currents, I_1, I_2, I_3, and I_4 meet at the junction *O*. The first three currents flow *toward* the junction and so have plus signs, as they *add* to the quantity at the point *O*. The current I_4 flows away from the junction and so has a minus sign, as it subtracts from the quantity at the point *O*. Then

$$I_1 + I_2 + I_3 - I_4 = 0 \text{ (first law).} \tag{45}$$

Assume that $I_1 = 5$ amp., $I_2 = 8$ amp., and $I_4 = 17$ amp.

Then $$5 + 8 + I_3 - 17 = 0$$

and $I_3 = +4$ amp., the plus sign indicating that the current flows toward the junction.

The second law is but another application of Ohm's law (equation (18), page 56). The basis of the law is obvious; if one starts at a certain point in a circuit, and follows continuously around the paths of the circuit until the starting point is again reached, he must again come to the same potential with which he started. Therefore, the sources of electromotive force encountered in this passage must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign.

This second law is illustrated by the following example.

Two batteries (Fig. 77), having electromotive forces of 10 and 6 volts and internal resistances of 1 and 2 ohms are connected in series opposing (their + terminals connected together) and in series with an external resistance of 5 ohms. Determine the current and the voltage at each part of the circuit.

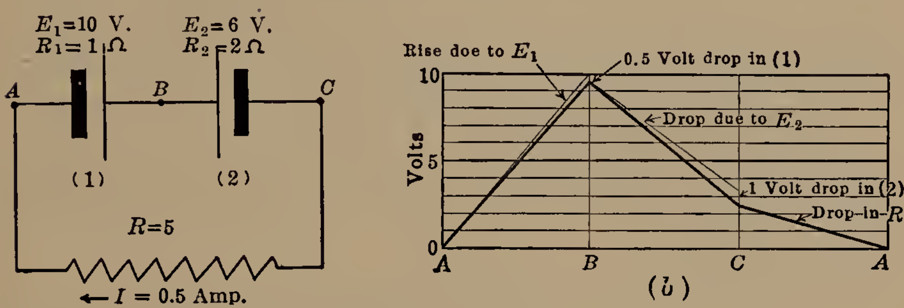


FIG. 77.—Voltage relations in an electric circuit.

Since the two batteries act in opposition, the net electromotive force of the two batteries is 10 − 6 = 4 volts.

The current is

$$I = \frac{10 - 6}{1 + 2 + 5} = \frac{4}{8} = 0.5 \text{ amp.}$$

When a current flows through a resistance, there is always a *drop* of potential in the direction of current flow. This is illustrated by Fig. 53, page 52, where the line $a'b'$ shows that the voltage drops in the direction of the flow of current. This is further illustrated by the analogy of a stream of water, such as a river. When one travels in the direction of the flow of water, he finds that the surface of the water is continually falling with respect to some fixed level on the earth's surface.

It also follows that when a current flows through resistance, there is always a *rise* of potential in the direction opposite to that of the current flow. This again may be illustrated by a hydraulic analogy. When one travels along a river against the current, he finds that the surface of the water is continually rising with respect to some fixed level on the earth's surface.

Referring again to Fig. 77, consider the point A as being at reference or zero potential. In passing from A to B one goes from the negative to the positive terminal of battery (1). There-

fore, there is a 10-volt *rise* in potential due to the electromotive force of battery (1). Since the current flows in the direction A to B, there occurs a simultaneous 0.5-volt drop of potential due to the current of 0.5 amp. flowing through the 1-ohm resistance of cell (1). Therefore, the net potential at B is but 9.5 volts greater than that at A, as is shown in Fig. 77 (*b*). In passing from B to C there is a drop of 6 volts, due to passing from the + to the − terminal of battery (2), and there is also a further drop of 1 volt, due to the current of 0.5 amp. flowing through the 2-ohm resistance of battery (2). This makes the net potential at $C = 9.5 - 6 - 1 = +2.5$ volts. In passing from C to A there is a drop in potential of 2.5 volts, due to the current of 0.5 amp. flowing through the 5-ohm resistance. When point A is reached, the potential has dropped to zero.

Therefore, the sum of all the electromotive forces in the circuit, each taken with its proper sign is equal to the sum of the Ir drops. This is illustrated as follows:

Electromotive Forces		Ir Drops	
Cell (1) =	+ 10 volts	Cell (1) = -0.5×1 =	− 0.5 volt
Cell (2) =	− 6 volts	Cell (2) = -0.5×2 =	− 1.0 volt
Total	+ 4 volts	5-ohm res = -0.5×5 =	− 2.5 volts
		Total	− 4.0 volts

$$+4 + (-4) = 0$$

74. Simple Applications of Kirchhoff's Laws.—In the application of Kirchhoff's second law to specific problems, the question of algebraic signs may be troublesome and is a frequent source of error. If the following rules be kept in mind, no difficulties should occur.

A rise in potential should be preceded by a + sign.

A drop in potential should be preceded by a − sign.

For example, in passing *through a battery* from the − to the + terminal, the potential due to the battery electromotive force *rises* so that this voltage should be preceded by a + sign. On the other hand, when passing *through the battery* from the + terminal to the − terminal, the potential due to the battery electromotive force *drops*, so that a − sign should precede this voltage. These points are illustrated by Fig. 77.

When going through a resistance in the *same* direction as the current flow, the voltage drops, so that this voltage should be preceded by a − sign. When going through a resistance in the direction *opposite* to the current flow the voltage rises, so that this voltage should be preceded by a + sign. These two rules apply to the internal resistances of batteries as well as to external resistances.

The foregoing rules are further illustrated by the electric circuit shown in Fig. 78. Two batteries having electromotive forces E_1, E_2, and internal resistances r_1, r_2, are connected in parallel, with positive terminal to positive terminal and negative terminal to negative terminal. An external resistance R is connected across the battery terminals *ab*. Assume the current in the left-hand battery to be I_1, the current in the right-hand battery I_2, and the external current I. The arrows show the *assumed* direction of current flow. Since there are three unknown quantities, I_1, I_2, and I, three simultaneous equations are necessary.

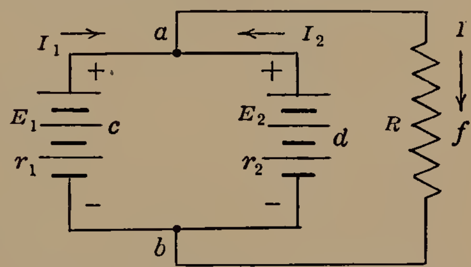


FIG. 78.—Application of Kirchhoff's laws to battery circuits.

First, considering the path *bcafb*, and applying Kirchhoff's second law

$$+E_1 - I_1r_1 - IR = 0. \quad (1)$$

Another equation may be obtained by applying Kirchhoff's second law to path *bdafb*.

$$+E_2 - I_2r_2 - IR = 0. \quad (2)$$

A third equation may be obtained by considering path *bcadb*, but when this equation is combined with (2) it gives (1), and when combined with (1) it gives (2). Hence this third equation, when combined with either (1) or (2), will not give a solution. The third equation must involve Kirchhoff's *first* law. For example, Kirchhoff's first law may be applied to junction *a*.

$$+I_1 + I_2 - I = 0. \quad (3)$$

I_1 and I_2 are preceded by + signs, since they add to the quantity of electricity at *a*, and I is preceded by a − sign, since it subtracts from the quantity at *a*.

Equations (1), (2), and (3), when solved simultaneously, give the three currents, I_1, I_2, and I. This may be illustrated by giving numerical values to the electromotive forces and resistances shown in Fig. 79 (*a*).

For path *bcafb*,

$$+6.0 - I_1(0.30) - I(2.0) = 0. \tag{1}$$

For path *bdafb*,

$$+5.0 - I_2(0.10) - I(2.0) = 0. \tag{2}$$

At junction *a*,

$$I_1 + I_2 - I = 0 \quad \text{or} \quad I_1 + I_2 = I. \tag{3}$$

Substituting the value of I from (3) in (1) and (2),

$$+ 6.0 - 0.30I_1 - 2.0(I_1 + I_2) = 0 \quad \text{or}$$

$$+ 6.0 - 2.3I_1 - 2.0I_2 = 0. \tag{4}$$

$$+ 5.0 - 0.10I_2 - 2.0(I_1 + I_2) = 0 \text{ or}$$

$$+ 5.0 - 2.0\,I_1 - 2.1\,I_2 = 0. \tag{5}$$

Multiplying (4) by 2.1, (5) by 2.0, and subtracting,

$$\begin{array}{l} 12.6 - 4.83\,I_1 - 4.2\,I_2 = 0 \\ 10.0 - 4.00\,I_1 - 4.2\,I_2 = 0 \\ \hline 2.6 - 0.83\,I_1 \qquad\qquad = 0 \end{array}$$

$$I_1 = +3.13 \text{ amp.} \quad \textit{Ans.}$$

Substituting the value of I_1 in (5),

$$5.0 - 6.26 - 2.1\,I_2 = 0$$

$$2.1\,I_2 = 5.0 - 6.26 = -1.26$$

$$I_2 = -0.60 \text{ amp.} \quad \textit{Ans.}$$

The negative sign means that the current I_2 flows in a direction *opposite* to that *assumed.*

From (3)

$$I = 3.13 - 0.60 = 2.53 \text{ amp.} \quad \textit{Ans.}$$

As a check,

$$V_{ab} = 6.0 - 3.13 \times 0.3 = 6.0 - 0.94 = 5.06 \text{ volts.}$$

$$V_{ab} = 5.0 - (-0.60 \times 0.1) = 5.06 \text{ volts.}$$

$$V_{ab} = 2.53 \times 2.0 = 5.06 \text{ volts.}$$

Figure 79 (*b*) shows the same circuit as that of Fig. 79 (*a*), but gives the numerical values of voltage and current and the actual directions of the currents as well. It will be noted that battery (1) supplies 3.13 amp. Of this current, 2.53 amp. flow to the external resistance R and 0.60 amp. flow into battery (2).

Battery (2) therefore, is, receiving energy. If it is a storage battery, it is being charged.

75. Assumed Direction of Current Flow.—In solving for the three unknown currents in the problem of Par. 74, a negative value of the currents I_2 was required to satisfy the three simultaneous equations. This negative value of the current I_2 means

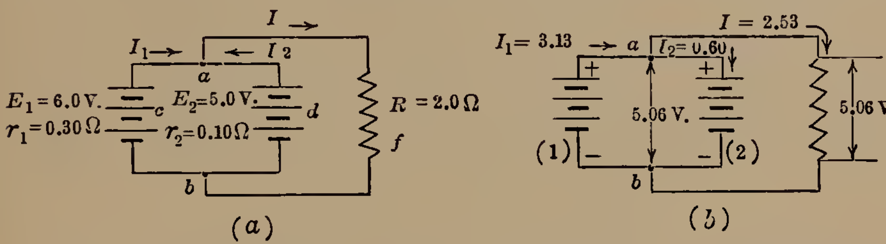


FIG. 79.—Determination of currents in batteries by Kirchhoff's Laws.

that the current I_2 flows in a direction opposite to the *assumed* direction, as is shown by comparing the directions of the arrows in Figs. 79 (*a*) and (*b*). Consequently, in setting down the equations for a circuit, it is immaterial whether or not the arrows are shown as pointing in the actual direction of flow of the current. The signs preceding the answers show whether or not the assumed directions are correct. If the value of current found by solving the equations is preceded by a positive sign, the assumed direction is correct. If the value of current found by solving the equations is preceded by a negative sign, the direction of flow of the current is opposite to the direction assumed. After having once assumed the directions of current flow, however, it is necessary that the quantities in the equations be preceded by signs consistent with these assumed directions.

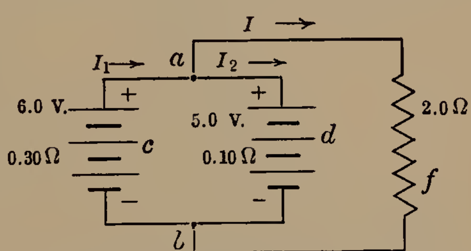


FIG. 80.—Applications of Kirchhoff's Laws to parallel batteries.

As an example, let it be required to solve the problem of Par. 74, with the flow of the current I_2 taken in the direction *adb*, its actual direction of flow as determined by solving the equations in Par. 74. The circuit is shown in Fig. 80.

For path *bcafb*,

$$+6.0 - 0.30\,I_1 - 2.0\,I = 0 \tag{1}$$

For path *bdafb*,

$$+5.0 + 0.10\, I_2 - 2.0\, I_1 = 0 \tag{2}$$

At junction (a),

$$I_1 - I_2 - I = 0 \text{ or } I = I_1 - I_2. \tag{3}$$

Substituting this value of I in (1) and (2),

$$6.0 - 0.30\, I_1 - 2.0\,(I_1 - I_2) = 0 \text{ or}$$

$$6.0 - 2.3\, I_1 + 2.0\, I_2 = 0. \tag{4}$$

$$5.0 + 0.10\, I_2 - 2.0\,(I_1 - I_2) = 0 \text{ or}$$

$$5.0 - 2.0\, I_1 + 2.1\, I_2 = 0. \tag{5}$$

Multiplying (4) by 2.1, (5) by 2.0 and subtracting,

$$12.6 - 4.83\, I_1 - 4.2\, I_2 = 0$$
$$10.0 - 4.00\, I_1 - 4.2\, I_2 = 0$$

$$2.6 - 0.83\, I_1 = 0$$

$$I_1 = +\frac{2.6}{0.83} = +3.13 \text{ amp.} \quad \textit{Ans.}$$

Substituting the value of I_1 in (5),

$$5.0 - 6.26 + 2.1 I_2 = 0$$

$$2.1\, I_2 = +1.26 \quad I_2 = +0.60 \text{ amp.} \quad \textit{Ans.}$$

The + sign indicates that the *assumed* direction is the *actual* direction of current flow.

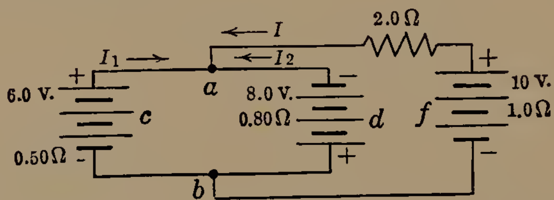


Fig. 81.—Application of Kirchhoff's Laws to a general network.

The fact that it is immaterial whether or not the assumed direction of the current be correct when writing the simultaneous equations is again illustrated by the network shown in Fig. 81. The three unknown currents, I_1, I_2 and I, are all assumed to be flowing *toward* the junction a, a condition which obviously cannot exist. The fact that the assumed direction of at least one of these currents is incorrect is shown by a negative sign appearing before one of the currents in the solutions of the equations.

Example.—Find the magnitude and direction for each of the three currents I_1, I_2, and I shown in the network, Fig. 81.

Applying Kirchhoff's second law to path *bcafb*,

$$+6.0 - 0.50\,I_1 + 2.0\,I - 10 + 1.0\,I = 0. \qquad (1)$$

For path *bcadb*,

$$+6.0 - 0.50\,I_1 + 8.0 + 0.80\,I_2 = 0. \qquad (2)$$

Applying Kirchhoff's first law to the junction *a*,

$$I_1 + I_2 + I = 0 \qquad (3)$$

$$I = -I_1 - I_2.$$

Substituting this value of I in (1) and solving,

$$+6.0 - 0.50\,I_1 - 3.0\,I_1 - 3.0\,I_2 - 10 = 0$$

$$-4.0 - 3.50\,I_1 - 3.0\,I_2 = 0 \qquad (4)$$

(2) $$+14.0 - 0.50\,I_1 + 0.80\,I_2 = 0$$

$$-4.0 - 3.50\,I_1 - 3.0\,I_2 = 0(4)$$

Multiplying (2) by 7, and subtracting from (4) (4)

$$+98.0 - 3.50\,I_1 + 5.60\,I_2 = 0$$

$$-102.0 - 8.60\,I_2 = 0$$

$$I_2 = -11.86 \text{ amp.} \quad \textit{Ans.}$$

Substituting this value of I_2 in (4),

$$-4.0 - 3.50I_1 + 35.58 = 0$$

$$3.50\,I_1 = 31.58$$

$$I_1 = +9.02 \text{ amp.} \quad \textit{Ans.}$$

From (3),

$$+9.02 - 11.86 + I = 0$$

$$I = +2.84 \text{ amp.} \quad \textit{Ans.}$$

Therefore, I_2 flows away from the junction *a*, being opposite to the direction assumed. The currents I_1 and I flow toward the junction, as assumed.

The foregoing results may be checked by computing the voltage across *ab* (V_{ab}) using each value of current.

$$V_{ab} = +6.0 - (9.02 \times 0.50) = 1.49 \text{ volts}$$

$$V_{ab} = -8.0 - (-11.86 \times 0.80) = 1.49 \text{ volts}$$

$$V_{ab} = +10 - (2.84 \times 3.0) = 1.48 \text{ volts (check).}$$

76. Further Applications of Kirchhoff's Laws.—In solving problems involving networks, the number of unknowns, and hence of equations, may usually be reduced by writing one of the currents at a junction as the sum or difference of the others, as the case may be, rather than designating this current by a separate symbol. For example, in Fig. 79, page 83, the current in the external circuit may be denoted by $(I_1 + I_2)$. This reduces the number of unknowns, and equations, therefore, from three to two. Likewise in Fig. 80, page 83 $(I_1 - I_2)$ may be used to designate the current in the branch *f*, and in Fig. 81 $(-I_1 - I_2)$ may be used to designate the current I. That is, Kirchhoff's

first law is applied to the diagram, rather than expressing the condition at a junction in a separate equation. This principle is illustrated by the following example:

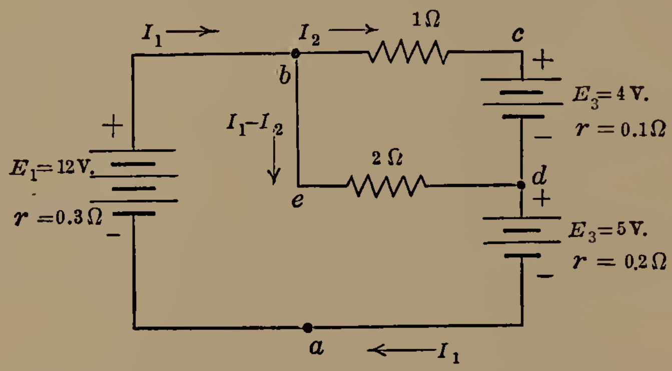


FIG. 82.—Further application of Kirchhoff's Laws.

Example.—Find the currents in all three branches of the network shown in Fig. 82, the assumed directions of currents being as shown.

The current in branch *bed* is designated as $I_1 - I_2$, rather than by a distinct symbol such as I_3.

Applying Kirchhoff's second law to path *abcda*,

$$+12 - 0.3\, I_1 - 1\, I_2 - 4 - 0.1\, I_2 - 5 - 0.2\, I_1 = 0$$

or

$$+3 - 0.5\, I_1 - 1.1\, I_2 = 0. \qquad (1)$$

For path *abeda*,

$$+12 - 0.3\, I_1 - (I_1 - I_2)\, 2 - 5 - 0.2\, I_1 = 0$$

or

$$+7 - 2.5\, I_1 + 2\, I_2 = 0. \qquad (2)$$

Multiplying (1) by 2.0 and (2) by 1.1,

$$+6 - I_1 - 2.2\, I_2 = 0 \qquad (3)$$

$$+7.7 - 2.75\, I_1 + 2.2\, I_2 = 0. \qquad (4)$$

Adding (3) and (4),

$$+13.7 - 3.75\, I_1 = 0$$

$$I_1 = +3.65. \quad \textit{Ans.}$$

Substituting the value of I_1 in (3),

$$+6 - 3.65 - 2.2\, I_2 = 0$$

$$2.2\, I_2 = 2.35$$

$$I_2 = +1.07. \quad \textit{Ans.}$$

To check, voltage V_{ba} is found,

$$12 - (3.65 \times 0.3) = 12 - 1.095 = 10.905 \text{ volts.}$$

For path *adcb*

$$+5 + (3.65 \times 0.2) + 4 + (1.07 \times 1.1) = 9 + 1.907 = 10.907 \text{ volts (check).}$$

CHAPTER VI

PRIMARY AND SECONDARY BATTERIES

77. Principle of Electric Batteries.—If two copper strips or plates be immersed in a dilute sulphuric acid solution (Fig- 83), and be connected to the terminals of a voltmeter, no appreciable deflection of the voltmeter will be observed. This shows that no appreciable difference of potential exists between the, copper strips. If, however, one of the copper strips (Fig. 84)

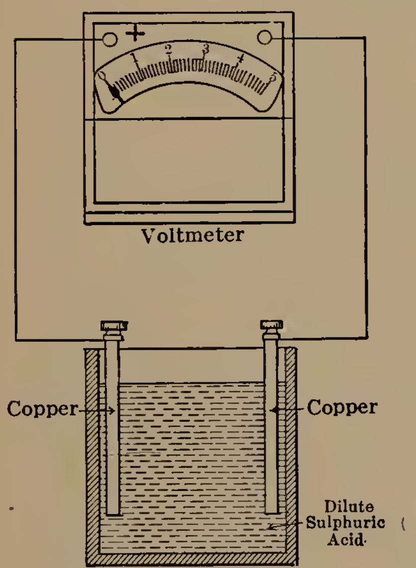


Fig. 83.—Primary cell with copper electrodes.

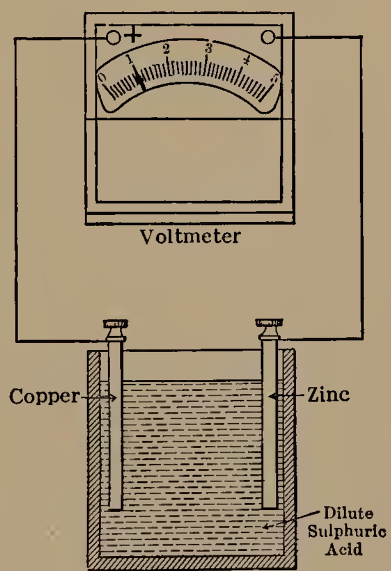


Fig. 84.—Primary cell with copper and zinc electrodes.

be replaced by a zinc strip, the voltmeter needle will deflect and will indicate approximately 1 volt, showing that a potential difference now exists. It will be necessary to connect the copper to the + terminal of the voltmeter and the zinc to the − terminal in order that the voltmeter may read up scale. This shows that so far as the *external* circuit is concerned, the copper is positive to the zinc.

The above experiment may be repeated with various metals. For example, carbon or lead may be substituted for the copper and a potential difference will be found to exist between each of these and the zinc, although it will not be of the same value as it was for the copper-zinc combination. Likewise, other metals may be substituted for the zinc, and potential differences will be found to exist.

Furthermore, it is not necessary that sulphuric acid be used for the solution. Other acids, such as hydrochloric, chromic, etc., may be substituted for the sulphuric; or salt solutions, such as common salt (sodium chloride), ammonium chloride (sal ammoniac), copper sulphate, zinc sulphate, etc., may be used.

In order to obtain a difference of potential between the two metal plates, but two conditions are necessary:

1. The plates must be of different metals.
2. They must be immersed in some electrolytic solution, such as an acid, alkali, or salt.

Again, if current be taken from the cell shown in Fig. 84, by connecting a resistance across its terminals (see Fig. 85), current will flow from the copper through the resistance AB and into the cell through the zinc. Inside the cell, however, the current will flow *from* the *zinc* through the solution *to* the *copper*, as shown in Fig. 85. Since current flows *from zinc to copper within the cell*, zinc is said to be electrochemically positive to copper. Therefore, when considering such an electrochemical cell, the copper is positive to the zinc when the external circuit is considered, but the zinc is electropositive to the copper when the plates and the solution alone are considered.

78. Definitions.—An electrochemical cell which delivers electrical energy in virtue of the difference of potential between its electrodes is called a *galvanic cell.* An electrochemical cell in which electrolytic and chemical reactions are caused to take place by electrical energy received from an external source is called an *electrolytic cell* (see page 112, Par. 105). The metal strips or plates of a cell are called *electrodes.* The electrode at which current enters the solution (as the zinc, Fig. 85) is the *anode*, and the electrode at which current leaves the solution (as the copper, Fig. 85) is the *cathode.*

The solution used in a cell is called the *electrolyte.*

When the cell delivers energy, the zinc plate diminishes in weight. This is true not only for this particular cell, but in practically all *primary* cells the flow of current is accompanied either by a loss in weight of at least one of the plates, or by a reduction of the plate materials to a compound of lesser chemical energy. Energy is stored in the cell *chemically*, and the electrical energy is delivered at the expense of the plate which goes into solution.

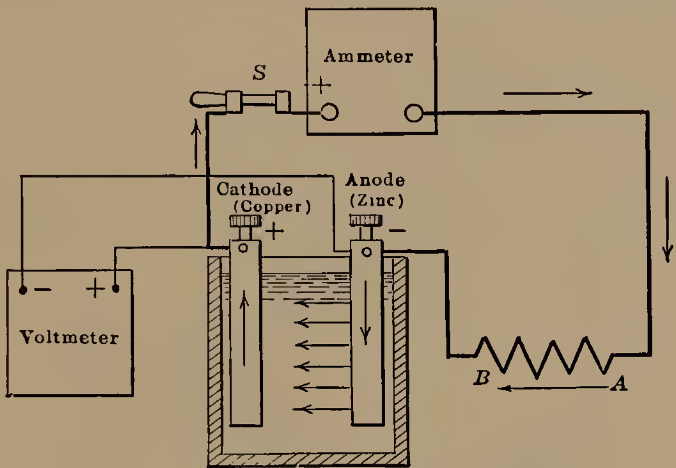


FIG. 85.—Current-flow in a single cell.

That is, one plate is converted into another chemical compound, this change being accompanied by a decrease in the available chemical energy of the system. Therefore, *chemical energy* is converted into *electrical energy* when the cell delivers current.

Hence:

A galvanic cell or battery is a device for transforming chemical energy into electrical energy.

Such cells or batteries are divided into two classes: *primary cells* and *secondary cells.*

In a *primary cell,* it is necessary from time to time to renew the electrolyte and the electrode which goes into solution, by fresh solution and new plates.

In a *secondary cell* the electrolyte and the electrodes which undergo change during the process of supplying current are restored electrochemically by sending a current through the cell in the reverse direction.

79. Primary Cells.—Although it was stated in Par. 77 that there are many combinations of metals and solutions capable

of generating an electromotive force and so forming a cell, only a limited number of such combinations are commercially practicable. The general requirements of a good cell are as follows:

1. There must be little or no wastage of the materials when the cell is not delivering current.
2. The electromotive force must be of such a magnitude as to enable the cell to deliver a reasonable amount of energy with a moderate current flowing.
3. Frequent replacement of materials must not be necessary and such materials must not be expensive.
4. The internal resistance and the polarization effects must not be excessive, otherwise the battery cannot supply even moderate values of current, at least for any appreciable time.

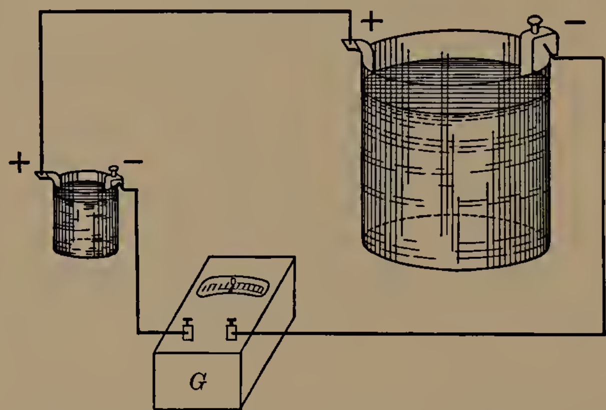


FIG. 86.—Equality of electromotive forces in cells of unequal sizes.

As an illustration, the cell shown in Fig. 84 would not be practicable, because both the copper and the zinc would waste away even were the battery delivering no current. Polarization (see Par. 81) would be excessive, and hence the battery would be capable of delivering only a comparatively small current.

80. Internal Resistance.—As was pointed out in Chap. V, every cell or battery has internal resistance, which reduces the magnitude of the current and causes the terminal voltage to drop when current is taken from the cell. A small portion of this resistance lies in the electrodes, but the greater portion lies in the electrolyte itself and at the contact surface between the electrolyte and the electrodes. The resistance of a cell may be re-

duced, therefore, by increasing the areas of the electrodes in contact with the electrolyte and by reducing the distance between the positive and negative electrodes.

Increasing the size of the cell *does not increase its electromotive force.* This electromotive force depends only on the material of the two electrodes and the electrolyte. Figure 86 shows two gravity cells, made up of the same materials, but differing in size. The cells are in opposition, that is, their + terminals are joined and their − terminals are joined. A galvanometer G connected in one of the leads reads zero, indicating that no current flows from the larger to the smaller cell. Hence, their electromotive forces must be equal.

81. Polarization.—When a fixed resistance is connected across a galvanic cell (Fig. 85) the current decreases in value with time. This is due both to a decrease in the electromotive force of the cell and to an increase in its internal resistance. These changes are caused by *polarization.* Small bubbles of hydrogen form on the positive plate or cathode. These bubbles, acting in conjunction with the cathode, set up an electromotive force in opposition to that of the cell, thus decreasing the net electromotive force of the cell. They also increase the resistance of the cell, thus decreasing the current.

A common method of reducing polarization is to bring some oxidizing agent, such as chromic acid or manganese peroxide, into intimate contact with the cathode. The hydrogen readily combines with the oxygen of these compounds to form water (H_2O).

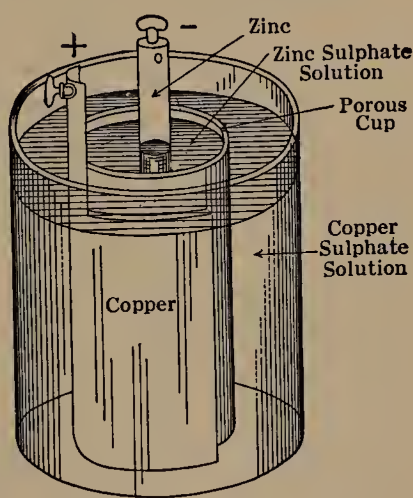


FIG. 87.—Daniell cell.

82. Daniell Cell.—This cell (Fig. 87) is a two-fluid cell having copper and zinc as electrodes. It consists of a glass jar, inside of which is a porous cup containing zinc sulphate solution or a solution of zinc sulphate and sulphuric acid. The zinc anode or negative electrode is immersed in this electrolyte. The porous cup is placed in a solution of copper sulphate with

copper sulphate crystals in the bottom of the jar. The copper plate, which is cathode, surrounds the porous cup. The porous cup keeps the two solutions separated. As the copper is in a copper sulphate solution, there is no polarization. This cell is designed for use in a circuit which is kept continually closed. If left idle, the electrodes waste away. When the cell is taken out of service for some time, the electrodes should be removed and the porous cup should be thoroughly washed. The electromotive force of this cell is about 1.1 volts.

83. Gravity Cell.—The gravity cell (Fig. 88) is similar to the Daniell cell, except that gravity, rather than a porous cup, is depended upon to keep the electrolytes separated. The cathode, which is of copper, is made of strips riveted together and placed in the bottom of the cell together with copper-sulphate crystals. A solution of copper sulphate is then poured to within a few inches of the top of the jar. The connection to the copper is usually an insulated copper wire fastened to the copper and carried out through the solution to the top of the jar.

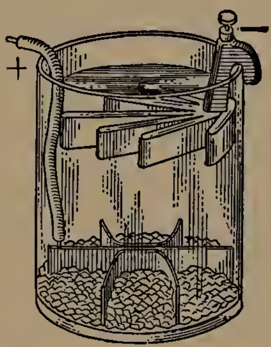


Fig. 88.—Gravity cell.

The anode is zinc, is usually rather massive and is cast in the form of a crow's foot and hung on the top of the jar. This is surrounded by a zinc-sulphate solution. The solutions are kept separated by gravity. The copper sulphate, being the heavier of the two, tends to remain at the bottom. In the operation of the cell, the zinc goes into solution as zinc sulphate, and metallic copper comes out of the copper-sulphate solution and is deposited on the copper electrode. The cathode therefore gains in weight, whereas the anode loses in weight.

The gravity cell is a *closed circuit* battery and the circuit should be kept closed, therefore, for the best results. The cell has been found very useful in connection with railway signals, fire-alarm systems, and telephone exchanges, all closed-circuit work, although the storage battery has replaced it in many instances. Its electromotive force is approximately 1.1 volts.

84. Edison-Lalande Cell.—The Edison-Lalande cell (Fig. 89) has a cathode of copper oxide suspended between two zinc plates

which form the anode. The electrolyte is caustic soda (NaOH), one part by weight of soda to three of water. To prevent the soda being acted on by the air, the electrolyte is covered with a layer of mineral oil. The electromotive force is about 0.95 volt, and when delivering current the terminal voltage drops to 0.75 volt. There is little or no local action in this cell and it can be used to advantage on both open-circuit and closed-circuit work.

85. Le Clanché Cell.—The Le Clanché cell is perhaps the most familiar type of primary battery. The cathode is molded carbon and the anode is amalgamated zinc. The electrolyte is sal ammoniac (ammonium chloride). This type of cell is suited only

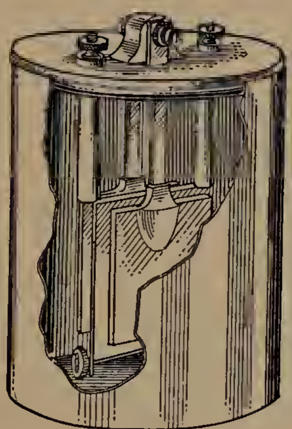


FIG. 89.—Edison-Lalande cell or Edison primary battery.

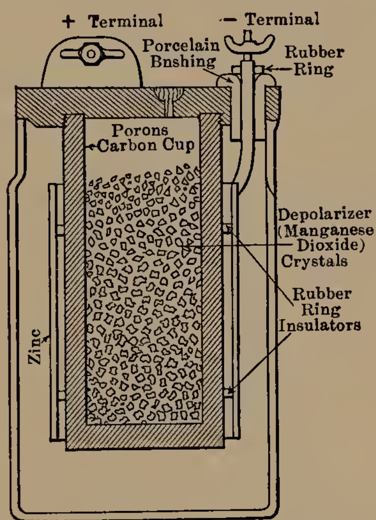


FIG. 90.—Porous cup Le Clanché cell.

for open-circuit work because of the rapidity with which it polarizes. The electromotive force is 1.4 volts, and the terminal voltage is approximately 1 volt when the cell is in operation. The most common method of reducing polarization is to bring manganese dioxide into intimate contact with the carbon. This gives up oxygen readily, which unites with the hydrogen bubbles to form water.

In one type of Le Clanché cell, a pencil zinc is suspended in the center of a hollow cylinder of carbon and manganese dioxide. An improved type, the porous cup cell, is shown in Fig. 90. In this form, a hollow carbon cylinder is filled with maganese dioxide,

and the zinc, bent into cylindrical form, surrounds the carbon cylinder.

The solution should consist of 3 oz. (85 gram) of sal ammoniac to 1 pt. (0.47 liter) of water. To prevent the solution "creeping," the top of the cell is dipped in paraffin and the top of the carbon is covered with a black wax.

This cell is used for intermittent work, such as ringing door-bells, telephone work, and open-circuit telegraph work.

86. Weston Standard Cell.—It is essential that reproducible standards of current, resistance and voltage, be available. The ampere may be determined by weighing the amount of silver deposited in a standard voltameter (see page 50, Par. 54). This method is not satisfactory as a working method, since it requires skill and care in making the measurement, and the cell materials must have a high degree of purity. The standard or International Ohm involves a column of mercury of fixed dimensions (see page 32, Par. 34), and extremely accurate secondary standards, usually of manganin, are readily obtainable. The International Volt is defined as $\frac{1}{1.0183}$ of the voltage of a Weston *normal* cell at 20° C. This standard is readily reproducible to two parts in 100,000. Since the working standards of resistance and voltage are readily obtainable and do not require unusual skill in their use, these two are almost universally used, and the current is obtained by Ohm's law (see page 56).

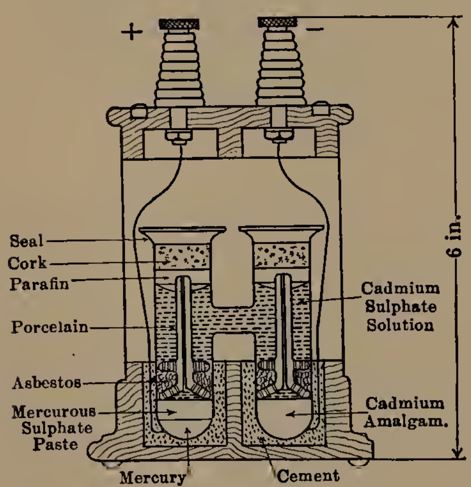


FIG. 91.—Weston standard cell.

A cross-section of the portable form of Weston cell is shown in Fig. 91. The cathode is mercury, located at the bottom of one leg of an H-tube. Above this is mercurous-sulphate paste. A porcelain tube extends to the top of the cell and acts as a vent for any gases that are formed. In the bottom of the other leg of the H-tube is the anode, of cadmium

amalgam. Another porcelain tube acts as a vent. The electrolyte is cadmium sulphate. The leads from the cathode and the anode are sealed into the tubes at the bottom. The top of the cell is sealed with cork, paraffin, and wax. The entire cell is mounted in a wood and metal case with binding posts at the top.

In the *unsaturated cell*, or working standard (Fig. 91), the solution is saturated at 4° C. and, as no crystals are left in the solution, its concentration is substantially constant at other temperatures. Such cells have practically no temperature coefficient. A certificate should accompany each one, giving its electromotive force, which usually is about 1.0186 volts.

As the resistance of a Weston cell is about 200 ohms, its electromotive force cannot be measured by any method requiring appreciable current, such as the use of a voltmeter. By means of the Poggendorff method, described on page 141, Par. 119, the cell is used without delivering current.

87. Dry Cells.—Dry cells are a modification of the Le Clanché cell and, as they are very light, portable, and convenient, they are rapidly replacing other types of cells. The word "dry cell" is really a misnomer, for no cell that is actually dry will deliver an appreciable current.

A cross-section of a typical dry cell is shown in Fig. 92. The anode is sheet zinc, made in the form of a cylinder with an open top, and acts as the container of the cell. The zinc is lined with some non-conducting material, such as blotting paper. The anode consists of a carbon rod with the mixture of coke, carbon, etc., which surrounds this rod. The rod is located axially in the zinc container. The depolarizing agent, powdered manganese dioxide, is mixed with finely crushed coke and pressed solidly into the container between the carbon and the non-conducting material which lines the zinc. Sal ammoniac, with perhaps a little zinc sulphate, is added and the cell then sealed with wax or some tar compound. The cell is always set in a close-fitting cardboard container.

The electromotive force of a dry cell is about 1.5 volts when new, but this drops to about 1.4 volts with time, even though the cell remains idle. A cell is practically useless after a year to 18 months, even if not used at all during this time. The internal resistance of the cell is about 0.1 ohm when new and increases to

several times this value with time. A method for testing the condition of a cell is to short-circuit it through an ammeter, when it should deliver an instantaneous current of $\frac{1.5}{0.1}$ or 15 amp., if in good condition. When delivering appreciable current, the terminal voltage is very nearly 1 volt.

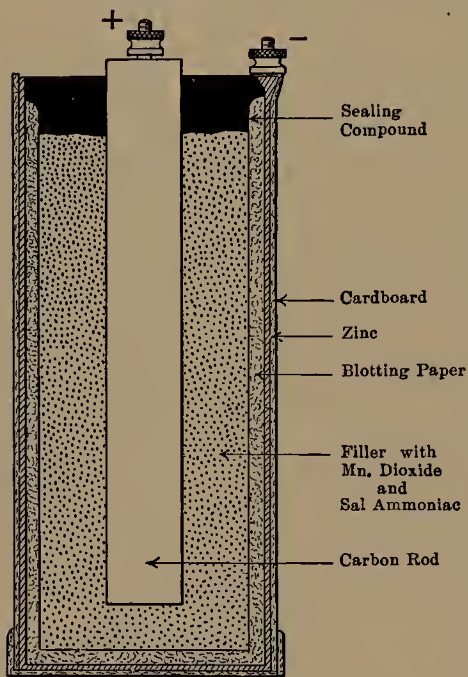


FIG. 92.—Sectional view—dry cell.

One of the chief causes of a cell's becoming useless is the using up of the zinc as a result of electrochemical actions in the cell. This allows the solution to leak out and to dry up and the cell then becomes worthless.

Dry cells can supply moderate currents only intermittently, but they are capable of supplying very small currents of the magnitude of 0.1 amp. continuously. They are used extensively for electric bells, buzzers, telephones, telegraph instruments, gas-engine ignition, flash lamps, "A" and "B" batteries for radio receiving sets, and for many other purposes.

STORAGE BATTERIES

88. Storage Batteries.—A storage or secondary cell (sometimes called an accumulator) involves the same principles as a primary cell, but the two differ from each other in the manner in which they are renewed. The materials of a primary cell which are used up in the process of delivering current are replaced by new materials, whereas, in the storage cell, the cell materials are restored to their initial condition by sending a current through the cell in a reverse direction. There are but two types of storage cells in common use, the *lead-lead-acid* type and the *nickel-iron-*

alkali type. In both of these cells *the active materials do not leave the electrodes.*

THE LEAD-LEAD-ACID BATTERY

89. The Lead Cell.—The principle underlying the lead cell may be illustrated by a simple experiment. Two plain lead strips (Fig. 93) are immersed in a glass of dilute sulphuric acid.

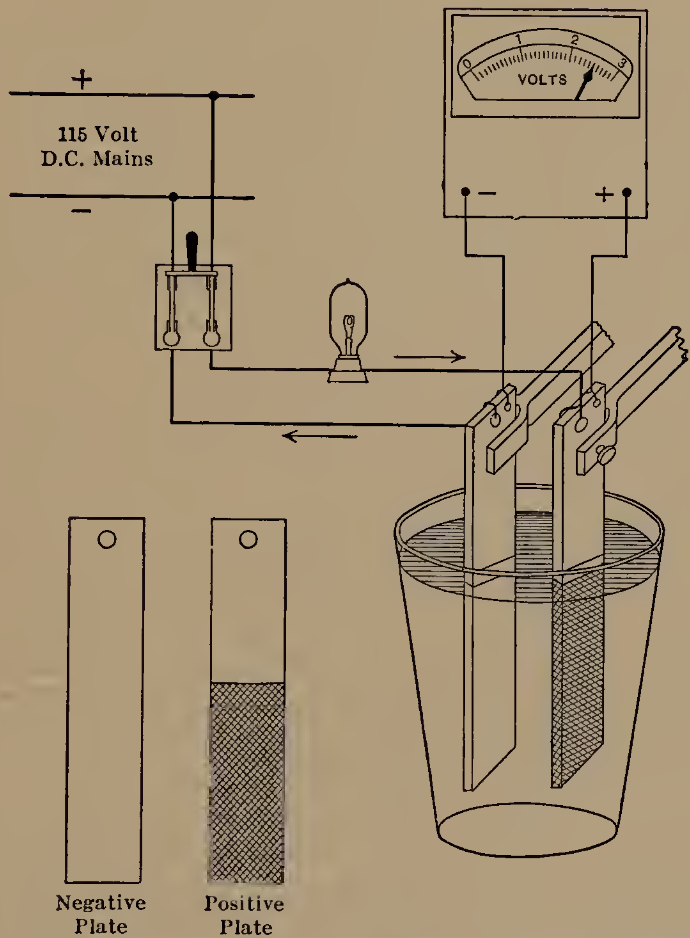


FIG. 93.—Forming the plates of an elementary lead storage cell.

These are connected in series with an incandescent lamp supplied from 115-volt direct-current mains, or from a battery. When current flows through this cell, bubbles of gas will be given off from each plate, but it will be found that a much greater number

come from one plate than from the other. After a short time one plate will be observed to have changed to a dark chocolate color, and the other apparently will not have changed its appearance. A careful examination, however, will show that the metallic lead at the surface of the latter plate has started to change from solid metallic lead to spongy lead.

When the current is flowing as shown in Fig. 93, the voltmeter connected across the cell indicates about 2.5 volts. If the current be interrupted by opening the switch, the voltmeter reading will fall to about 2.1 volts, and the cell will now be found to be capable of delivering a small current, but the amount of energy that such a cell can deliver is very limited. As the cell discharges, the voltage drops off slowly to about 1.75 volts, after which it drops more rapidly until it becomes zero and the cell is apparently exhausted. The color of the dark brown plate will now have become lighter. After a short rest the cell will recover slightly and will again deliver current for a very brief period.

The plate, which is a dark chocolate color in this experiment, is the positive plate or cathode, and the one which is partially converted to spongy lead is the negative plate or anoide. The bubbles which were noted come mostly from the negative plate and are free hydrogen gas. When current is passed through such a cell, the metallic lead of the positive plate is converted into lead peroxide, whereas the negative plate, though not changed chemically, is converted from solid lead into the spongy or porous form. When the cell is discharged, the lead peroxide of the positive plate is changed to lead sulphate and the spongy lead of the negative plate is changed to lead sulphate, so that both plates tend to become electrochemically equivalent.

The principle of the cell is the same as that of the primary cell. When the two lead plates are the same electrochemically, that is, when both are lead sulphate, there is practically no electromotive force. When the positive is converted to the peroxide and the negative to spongy lead by the action of an electric current, the two plates become dissimilar and an electromotive force results. This electromotive force is about 2.1 volts, the excess of 0.4 volt observed in charging the cell being necessary to overcome the internal resistance and the polarization effects

This simple experiment illustrates the principle underlying the operation of lead storage cells.

The chemical reactions which take place in a storage cell are as follows:

Battery discharged				Battery charged		
(+ plate)	(− plate)			(+ plate)	(− plate)	
$PbSO_4$	$+ PbSO_4$	$+ 2H_2O$	$\leftrightarrows$	PbO_2	$+ Pb$	$+ 2H_2SO_4$
Lead sulphate	+ lead sulphate	+ water	is changed to	Lead peroxide	+ lead	+ sulphuric acid

It will be noted that, when the battery is being charged, the change that takes place in the electrolyte is that water is converted into sulphuric acid. This accounts for the rise of specific gravity on charge. On discharge, the sulphuric acid is dissociated and reacts with the lead peroxide to form water. Therefore, the specific gravity of the electrolyte decreases when the cell is discharging. When charging, free hydrogen is given off at the negative plate and oxygen at the positive plate. Because of the explosive nature of hydrogen, *no flame should be allowed to come near a storage battery while the battery is charging.*

It would not be practicable to construct storage cells of plain lead sheets, such as were used in this experiment. The current capacity of the cell would be so small that the cell could not deliver currents of commercial value for any length of time, unless the cell were made prohibitively large in order to secure the necessary plate area.

If the charging of the elementary cell (Fig. 93) were carried further, the dark lead peroxide of the positive plate would be observed to fall off in flakes and drop to the bottom of the tumbler. Therefore, in a commercial cell provision must be made to minimize this flaking of the active material.

90. Planté Plates.—There are two methods of increasing the active plate area, the Planté process and the Faure process. In the Planté process, the active material on the plates is formed from the metallic lead by passing a current through the cell first in one direction and then in the reverse direction, which procedure works the lead on the surface of the plates into active material.

The Manchester plate, shown in Fig. 94, is made by this process. A grid made of lead and antimony is perforated. The active material consists of a corrugated lead ribbon, which is

coiled in spirals and pressed into the perforations of the grid. The peroxide has a greater volume than the lead from which it is

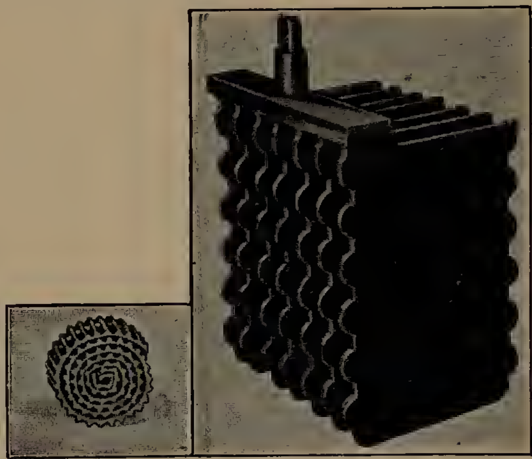


FIG. 94.—Planté (Manchester) positive group and button.

derived. Hence, when the cell is charged, these spirals expand and become more firmly embedded in the plate.

FIG. 95.—Skeleton for pasted plate.

91. Faure or Pasted Plate.—This type of plate consists of a lead-antimony lattice work or skeleton (Fig. 95) into which lead oxide is applied in the form of a paste. The battery is then charged. The paste on the positive grid is converted into peroxide and that on the negative grid into spongy lead.

The chief advantage of the pasted plate is its high overload capacity, especially for short periods, together with its lesser size, cost, and weight for a given discharge rate. It is, therefore, very useful where lightness and compactness are necessary, such as in electrical-vehicle batteries, ignition and starting batteries for gasoline cars, etc. The pasted type of positive has a much shorter life than the Planté type, due to a more rapid shedding of the active material.

In all batteries there is one more negative than positive plate. This allows all the positives to be worked on *both* sides. Were any of the positives to be worked on one side only, the expansion of the active material, which occurs when it is converted to the peroxide on charge, would be unequal on the two sides of the plate and buckling would result.

Storage batteries are divided into two general classes, *stationary batteries* and *portable batteries.*

92. Stationary Batteries.—The plates of this type of battery may be either of the Planté type or of the pasted type, depending on the nature of the service. For merely regulating duty, involving only moderate, though continual, charging and discharging, the Planté plate is preferable. Where a battery is installed for emergency service, to carry an enormous overload for a very short period during a temporary shut-down of the generating apparatus, the pasted plate, because of its high discharge rate, is preferable. As such batteries are usually located in congested city districts where floor area is very valuable, the high discharge rate of the pasted plate is of great importance.

93. Tanks.—The containing tanks for stationary batteries are of three general types: glass, earthenware, and lead-lined wooden tanks. Glass jars are used only for cells of small capacity, as they are expensive and have not the requisite mechanical strength in the larger sizes. Earthenware tanks have been used more as an experiment and will probably not come into general use. Wooden tanks are lined with sheet lead. The seams of the lead lining must be sealed by burning the lead with a non-oxidizing flame. Solder should never be used. The wood should be painted with an acid-resisting paint, such as asphaltum.

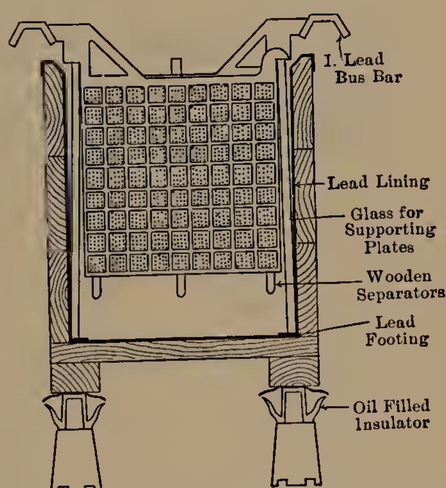


FIG. 96.—Lead-lined wooden tank storage cell.

When glass jars are used, the plates are suspended by projecting lugs which rest on the edges of the jar (see page 105, Fig.

100). In the lead-lined tanks, the plates are similarly suspended on two glass slabs, which rest on the bottom of the tank (Fig. 96). The plates of like polarity are burned to a heavy lead strip or bus-bar to which the current-carrying lug is either burned or bolted. There should always be a liberal space between the plates and the bottom of the tank to allow the red lead-peroxide to accumulate without short-circuiting the plates. All types of stationary batteries should have a glass cover to reduce evaporation and to intercept the fine acid spray which occurs during the charging period.

94. Separators.—To prevent the positive and negative plates from coming in contact with one another, several types of separators have been tried. Glass rods inserted between plates have been used as separators, and thin perforated hard rubber is also used occasionally in small cells. The most satisfactory separators are made of thin wood. They act as complete barriers between plates and do not add appreciably to the resistance of the cell. The separators are usually grooved vertically to permit the circulation of the electrolyte (see Fig. 101, page 106). The wood is specially treated to remove ingredients that would be detrimental to the electrolyte. These separators should never be allowed to become dry, as they then decompose very readily (see page 105, Fig. 100).

95. Electrolyte.—The electrolyte should be chemically pure sulphuric acid. When fully charged, the specific gravity should be 1.210 for Planté plates and not higher than 1.300 for pasted plates. This solution may be made from concentrated acid (oil of vitriol, sp. gr. 1.84) by *pouring the acid into water* in the following ratios:

	Parts Water to One Part Acid	
Specific Gravity	Volume	Weight
1.200	4.3	2.4
1.210	4.0	2.2
1.240	3.4	1.9
1.280	2.75	1.5

A large amount of heat is evolved when acid and water are mixed. This results in the generation of a large amount of steam, if the water is added to the acid. This should be avoided, as it

may scatter the acid, break the container, and may cause personal injury.

The specific gravity of a solution may be determined directly by the use of a hydrometer (Fig. 97). This consists of a weighted bulb and graduated tube which floats vertically in the liquid whose specific gravity is to be measured. The specific gravity is read at the point where the surface of the liquid intercepts the tube. Such a tube may be left floating permanently in stationary batteries in a representative cell called a *pilot cell.*

The small amount of liquid and the inaccessibility of vehicle and starting batteries make the use of such a hydrometer impossible. To determine the specific gravity with such batteries, the syringe hydrometer is used (Fig. 98). The syringe contains a small hydrometer and when sufficient liquid is drawn into the syringe tube, the small hydrometer floats and may be read directly.

FIG. 97.—Simple hydrometer.

FIG. 98.—Syringe hydrometer.

Figure 99 shows the change in specific gravity during charge and discharge.

96. Specific Gravity.—When the battery is charged, hydrogen is given off at the negative plate and oxygen is given to the positive plate to convert it into the peroxide. The electrolyte gives up water, which means that the solution becomes more and more concentrated (see chemical equation, page 99). The specific gravity will rise from the complete discharge value of 1.160 to 1.210 when fully charged, as shown in Fig. 99. At *a*, the gassing point, the specific gravity drops slightly, due to the presence of hydrogen bubbles in the electrolyte. On *discharge,*

the specific gravity decreases even after the battery has ceased to deliver current, as the dilute acid in the pores of the active material diffuses into the solution. The specific gravity is such a good indicator of the state of charge of the battery that the hydrometer reading is generally used to determine how nearly charged or discharged the battery is at the time.

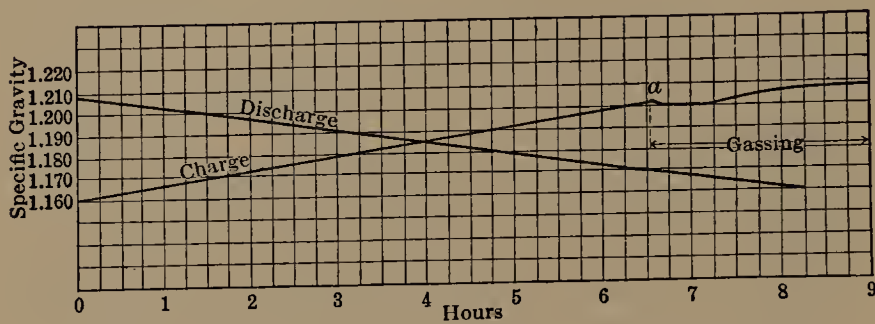


FIG. 99.—Variation of specific gravity in a stationary battery.

As the hydrogen and oxygen which escape from the battery during the charging and discharging periods are, ordinarily, only dissociated water, nothing but water, need be added to replace the electrolyte. Acid need be added only when an actual loss of electrolyte takes place, such as occurs with a leaky tank. As a rule, distilled water is used to replace the evaporation of the electrolyte.

The freezing temperature of the electrolyte is considerably reduced with increasing specific gravity. For example, the freezing temperature is $-6°$ F. when the specific gravity is 1.180, and $-90°$ F. when the specific gravity is 1.280. Hence, there is little danger of a battery's freezing in the temperate zone, if it is well charged.

97. Installing a Stationary Battery.—When installed for service, a stationary cell should rest in either a glass tray or a wooden tray, painted with asphaltum, and filled with dry sand (Fig. 100). The tray should be set on insulators. Before the battery is ready for service, the active material on the plates, which is more or less converted into lead salts during exposure to the atmosphere, must be reduced electrically. Therefore, the battery should be given an initial charge at the normal charging rate for about 40 hr. or more.

If the battery stands for a long time without being used, the active material becomes more or less converted into inactive lead sulphate. Therefore, a battery if idle should be charged occasionally.

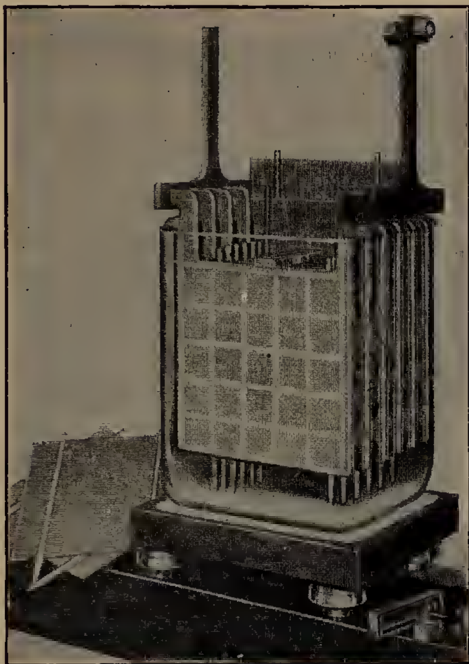


FIG. 100.—Stationary battery in position.

98. Vehicle and Starting Batteries.—In the design of batteries for propelling vehicles and for automobile starting, it is necessary to obtain a very high discharge rate with minimum weight and size. Therefore, pasted plates are used for both positives and negatives. These are made extremely thin and are insulated from one another by very thin wooden separators. They are then packed tightly into a hard-rubber jar (Fig. 101). This jar is sealed with an asphaltum compound to prevent the liquid splashing out. There is a hole in the top of the jar which is closed with a cap. This permits the replenishing of the electrolyte. A vent in the cap allows the gases to escape. Because of the high discharge rates which occur where this type of battery starts a gasoline engine, and because of the necessity for a high ampere capacity for the weight, the specific gravity of the electrolyte is as high as 1.280 to 1.300. Further, the

amount of electrolyte is very small and it is necessary to work it between wide limits, the lower limit being 1.185 and the upper 1.280 to 1.300.

The individual cells are mounted beside one another in boxes or crates and are connected together on top by lead connectors which may be burned or held by lead nuts. The number of cells in such a unit depends on the voltage which is desired.

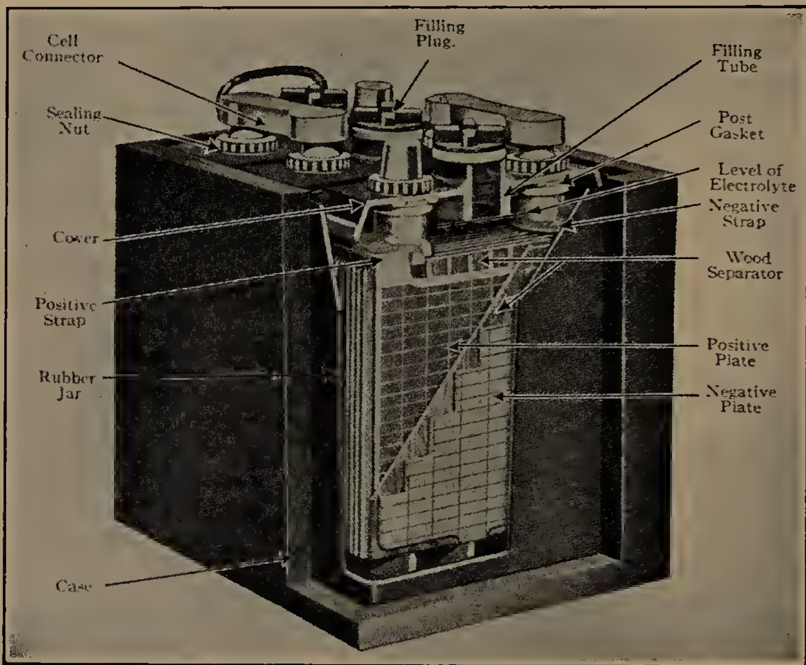


FIG. 101.—Exide 6-volt starting battery.

As the space for the electrolyte is very limited in vehicle batteries, the level of the electrolyte falls quite rapidly, so that frequent additions of water are necessary.

99. Rating of Batteries.—Practically all batteries have a nominal rating based on the 8-hr. rate of discharge. Thus, if a Planté battery can deliver a current of 40 amp. continuously for 8 hr., the battery will have a rating of $40 \times 8 = 320$ amp-hr. The normal charging rate of such a battery would be 40 amp. Even if the battery is capable of delivering 40 amp. for 8 hr., it will not be able to deliver 64 amp. for 5 hr. (= 320 amp-hr.), but only 88 per cent of this, or 56.4 amp. for 5 hr.; 56.4 amp. is called the 5-hr. rate.

This falling off in capacity with higher rates of discharge is due to the inability of the free solution to penetrate the pores of the active material. After such a battery has stood a short time, it will be found to have recovered to some extent and is therefore capable of delivering more current, even after apparently having become exhausted (see page 98). This is due to the free solution finally penetrating the pores of the active material (see page 104).

Batteries are able to discharge at very high rates for very short intervals. For example, a starting battery having an 8-hr. rating of 10 amp. is often called upon to supply 150 amp. when doing starting duty.

100. Charging.—There are three methods in general use for charging storage batteries. When batteries of comparatively small capacity, such as starting and ignition batteries, are to be

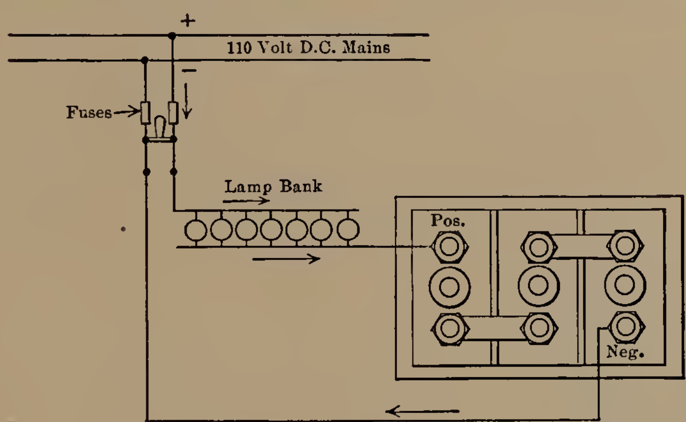


FIG. 102.—Charging a starting battery from 110-volt mains.

charged, and 110-volt *direct-current* service is available, the battery may be connected across the line in series with resistance (Fig. 102). Lamps which are no longer efficient as illuminants are desirable for the series resistance, since they indicate visually the approximate current, are not expensive, and are readily renewed. This method, although convenient, is inefficient from the energy point of view, as will be seen from the following example:

Example.—When a 6-volt starting battery is charged from 110-volt d.c. mains at the 15-amp. rate, its terminal voltage is 7.4 volts. (*a*) How much

power is being delivered to the battery? (b) How much power is consumed in the series resistance? (c) What percentage of the power delivered by the line is received by the battery?

(a) $7.4 \times 15 = 111$ watts. *Ans.*

(b) $(110 - 7.4)15 = 1{,}539$ watts. *Ans.*

(c) $\frac{7.4 \times 15}{110 \times 15}100 = \frac{7.4}{110}100 = 6.7$ per cent. *Ans.*

The efficiency of this method is increased by increasing the number of batteries connected in series. Therefore it is desirable to charge as many batteries as possible at one time.

Example.—What percentage of the power delivered by the line is utilized if two batteries are charged at the 15-amp. rate in the foregoing problem?

$$\frac{2 \times 7.4 \times 15}{110 \times 15}100 = \frac{14.8}{110}100 = 13.4 \text{ per cent.} \quad \textit{Ans.}$$

The percentage of power utilized is doubled by charging two batteries in series.

A separate generator may be used for charging. The generator voltage may be controlled by its field rheostat, and thus the current to the battery can be adjusted to its proper value without using series resistance. Since the battery voltage rises during charge (see Fig. 103), the current will decrease

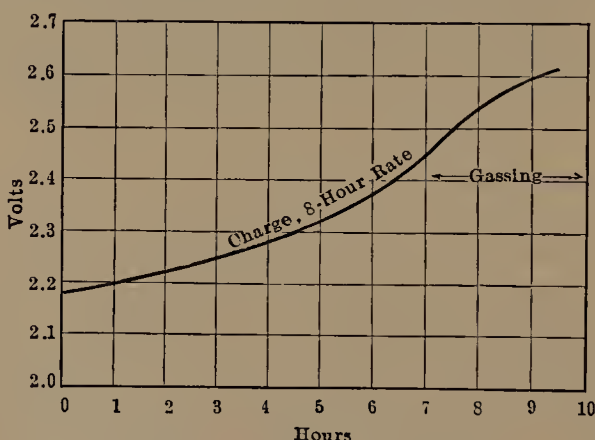


Fig. 103.—Variation of cell voltage during charge.

unless the generator voltage is raised from time to time. This method of charging is commonly employed in farm-lighting units.

The terminal voltage of a cell rises on charge, as shown in Fig. 103. The terminal voltage is about 2 volts at the beginning of charge and rises slowly to about 2.4 volts, after which it rises very rapidly to 2.6 volts. This last rise occurs in the gassing period. This final rise of voltage also indicates that the cell is nearing the completion of charge.

With batteries of the pasted-plate type, the current may be maintained at either the 5- or the 8-hr. rate (see page 106) until gassing begins, when the rate should be reduced. The charging rate with Planté plates is usually in excess of these. The charge may be started at the 3-hr. rate and end at the 8-hr. rate. The rate should always be reduced when gassing begins, as gassing represents a waste of energy and causes active material to become loosened from the positive plates.

A battery cannot be charged from *alternating-current* supply without the use of some rectifying device.[1] The battery must be so connected that current enters its positive terminal from the positive line (Fig. 102). If doubt exists as to the polarity of either the line or the battery, use a voltmeter, if one is available. If a voltmeter is not available, dip the ends of two wires which are connected to either the battery or the circuit into slightly acidulated water. Bubbles form about the negative wire (also see page 97).

FIG. 104.—Positive and negative plates of an Edison storage cell.

101. The Nickel-iron-alkaline Battery. Instead of using acid as an electrolyte, the Edison cell uses an alkali, consisting of a 21 per cent. potassium-hydrate solution. The positive plate consists of nickel pencils about ¼ in. (0.64 cm.) diameter and 4½ in. (11.4 cm.) long, filled with green nickel oxide. As the nickel oxide is a very poor electrical conductor, very fine metallic nickel flakes are mixed with it to produce sufficient conductivity. The negative plate consists of flat perforated nickel-plated steel stampings, containing iron in a very finely divided form. These flat pockets are mounted on a

[1] See "Tungar," and "Rectifiers," Part II.

nickel-plated steel frame for support. Both the positive and the negative plates are shown in Fig. 104.

The chemical reaction in the cell is complex, but its nature is indicated by the following chemical equation:

$$\overbrace{8KOH + 6NiO_2 + 3Fe}^{\text{Positive Plate}} = \overbrace{Fe_3O_4 + 2Ni_3O_4 + 8KOH.}^{\text{Negative Plate}}$$

The equation read from left to right indicates discharge, and read from right to left indicates charge. It is to be noted that the same quantity of potassium hydrate solution (KOH) appears on both sides of the equation. This indicates that ultimately all the reaction occurs between the electrodes themselves, and also that no water is formed. Hence the specific gravity of the solution does not change during charge or discharge.

The plates all have a perforated lug by which they are fastened together with a steel bolt and to a binding post. The positive

Fig. 105.—Five Edison storage cells mounted in a tray.

and negative plates are insulated from one another by hard-rubber grids. The positive and negative assembly is placed in a corrugated, nickel-plated, welded steel tank. In the top is a valve which allows the gases to escape during charge and through which water may be added to the electrolyte. This valve should never be allowed to become so encrusted with a potash deposit that it sticks, because the internal pressure may become sufficient to cause the sides of the container to bulge.

The individual cells are usually mounted in wooden racks, as shown in Fig. 105, the cells being connected together by nickel-plated steel connectors.

102. Charge and Discharge.—The Edison cell is rated on the basis of a 5-hr. charging rate. Figure 106 shows typical charge and discharge curves for the Edison battery. The average

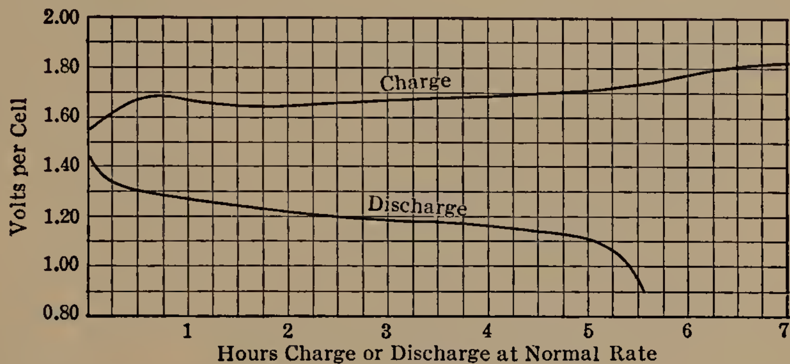


Fig. 106.—Voltage changes during the charge and discharge of an Edison cell.

voltage on discharge is about 1.2 volts per cell. As the specific gravity of the electrolyte changes but slightly, it cannot be used to indicate the condition of charge, as with the lead cell. Moreover, there is no sharp voltage rise near the completion of charge. If doubt exists as to the condition of charge, it is advisable to give an overcharge in order to be on the safe side.

The electrolyte in an Edison cell changes to potassium carbonate very readily, so that only freshly distilled water should be used in replacing the electrolyte, as tap water usually contains carbonates in solution. The electrolyte should be replaced by fresh electrolyte every 250 complete cycles of charge and discharge.

The Edison cell has many advantages. It is light, rugged, and can safely stand for a long time in a discharged condition without being given an occasional charge. The plates do not buckle and the active material does not "flake" or drop from the plates.

103. Applications.—Edison cells are used for vehicle lighting and ignition, and are also much used in motor boats. They are used in various types of electric trucks and for battery street cars. In automobiles they are not generally used for

starting, as their comparatively large internal resistance does not permit a sufficiently high discharge rate.

104. Efficiency of Storage Batteries.—The efficiency of a storage battery is the ratio of the watt-hour output to the watt-hour input.

For example, a normally discharged cell is charged at a uniform rate of 40 amp. for 6 hr. at an average voltage of 2.3 volts. The cell is then completely discharged at a uniform rate of 38 amp. for 6 hr., the average voltage being 1.95 volts. What is the efficiency of this cell?

$$\text{Watt-hours output} = 38 \times 1.95 \times 6 = 445$$
$$\text{Watt-hours input} = 40 \times 2.3 \times 6 = 552$$
$$\text{Efficiency} = \tfrac{445}{552}, \text{ or } 80.6 \text{ per cent.}$$

The *ampere-hour efficiency* of a storage battery is the ratio of the ampere-hours output to the ampere-hours input. In the preceding example the ampere-hour efficiency may be found as follows:

$$\text{Ampere-hours output} = 38 \times 6 = 228$$
$$\text{Ampere-hours input} = 40 \times 6 = 240$$
$$\text{Ampere-hour efficiency} = \tfrac{228}{240}, \text{ or } 95 \text{ per cent.}$$

The much lower watt-hour efficiency is due to the difference between the voltage of charge and that of discharge, as shown in Figs. 103 and 106.

105. Electroplating.[1]—Electroplating is a very important industry and is closely related to the subject of batteries. The principle is very simple. Assume that it is desired to copper plate a carbon dynamo-brush. The portions of the brush to be plated are immersed in a solution of copper sulphate (Fig. 107). A copper strip is also immersed in the solution and is connected to the + terminal of a dynamo or some other source of direct-current supply. The article to be plated is connected to the negative terminal of this supply. Under these conditions the current will carry copper from the solution and deposit it on the carbon brush. The copper which leaves the solution is replaced

[1] See "Standard Handbook," 5th ed., Sec. 19, Pars. 185 to 206, for a more complete discussion.

by copper which is carried from the copper strip (the anode) into solution, so there is no change in the solution itself. The current should be such that the density is about 0.02 amp. per square inch of the surface to be plated.

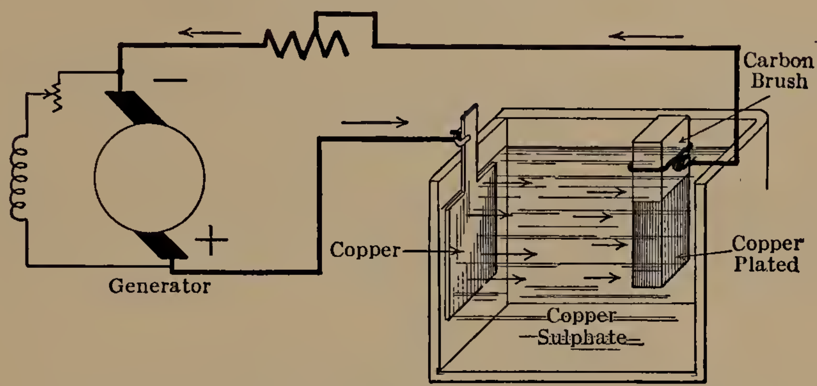


FIG. 107.—Copper plating bath.

It is not absolutely necessary that the anode be of the metal which it is desired to deposit. Other metals may be used. Under these conditions, however, the solution in time becomes contaminated by the going into solution of the anode. If an inert substance such as carbon be used as anode, acid is formed in the solution.

The *only opposing* electromotive force in the bath just described is the *IR* drop in the solution. Hence, electroplating baths are naturally low-voltage devices, and, when practicable, several are connected in series. A low-voltage and high-current generator is generally used for plating purposes. In practice, there are many refinements to be observed.

Electrotyping is another common example of electroplating. An impression is made in wax with the type or object to be reproduced. The surface of the wax is made conducting by applying a thin coating of graphite. Copper is then plated on this surface. It is later backed by type metal to give it the necessary mechanical strength.

CHAPTER VII

ELECTRICAL INSTRUMENTS AND ELECTRICAL MEASUREMENTS

106. Principle of Direct-current Instruments.—Figure 108 (*a*) shows a coil pivoted so that it may swing about an axis perpendicular to the plane of the paper. This coil is placed in a

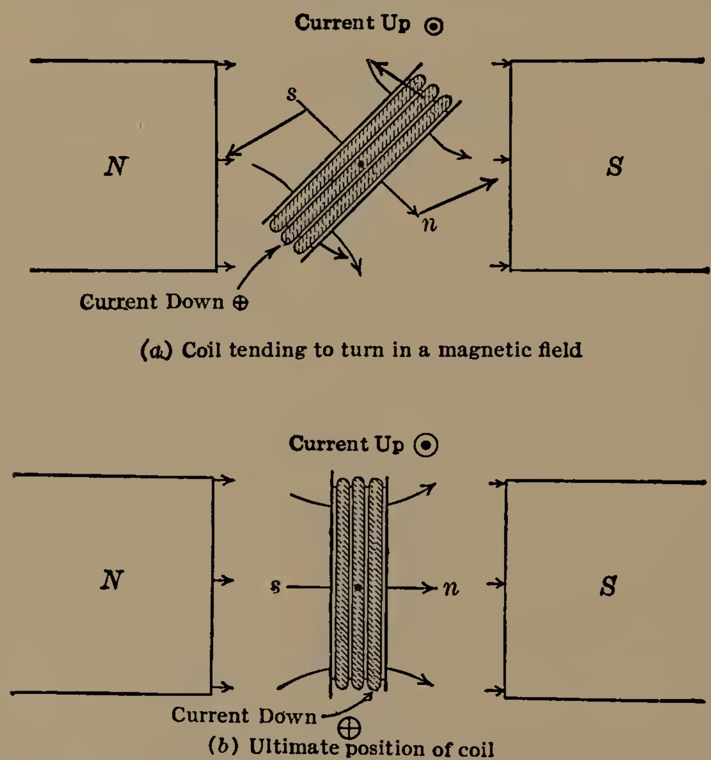


Fig. 108.—Turning moment of an instrument coil.

uniform magnetic field. Current is led into the coil through flexible connections. The direction of current flow in the coil is downwards in the lower left-hand side and upwards in the upper right-hand side, as shown by the small circles with cross and dot. The coil will tend to assume such a position that the

flux of the system is a maximum (see Par. 16, page 12). This position is shown in Fig. 108 (*b*), in which the coil has taken a position such that its plane is at right angles to the magnetic field. The ampere-turns of the coil produce flux whose direction is from left to right and hence in conjunction with the field due to the magnet. For a given current in the coil, the flux of the system is obviously a maximum when the coil is in this position.

Again, the tendency of the coil to turn may be explained by the laws of attraction of magnetic poles. The coil itself has a north and a south pole, *N* and *S*. The *S*-pole of the coil is attracted by the *N*-pole of the magnet, and the *N*-pole of the coil is attracted by the *S*-pole of the magnet, as shown by the heavy arrows.

Under the conditions shown in Fig. 108, the coil tends to turn in a counter-clockwise direction until its plane is perpendicular to the magnetic field in which it finds itself, and its own field acts in conjunction with the magnetic field in which it finds itself.

This behavior of a coil carrying a current and placed in a magnetic field should be thoroughly understood, for it is the underlying principle of most current-measuring instruments and, in addition, is the principle upon which all electric motors operate.

107. The D'Arsonval Galvanometer.—A galvanometer is a sensitive instrument used for detecting and measuring small electric currents. The D'Arsonval galvanometer, which is based on the principle of a coil turning in a magnetic field, is the most common type of galvanometer. Figure 109 shows the principle of its construction. A coil of very fine wire is suspended between the poles of a permanent magnet by means of a filament, usually a flat strip of phosphor-bronze.

Between the poles of the magnet a soft-iron core is usually placed (Fig. 109). This core, by reducing the length of the air path, increases the flux linking the coil. The flux also tends to enter the core radially, which makes the deflections of the galvanometer almost directly proportional to the current flowing in the galvanometer coil.

Electrical connections to the coil are made through the phosphor-bronze suspension and a very flexible spiral filament fastened to the bottom of the coil (Fig. 109).

Any turning of the coil produces torsion in the filament, which opposes the turning of the coil and hence is called the *restoring force.* When the moment of the restoring force and the turning moment due to the current are equal, the galvanometer assumes a steady deflection.

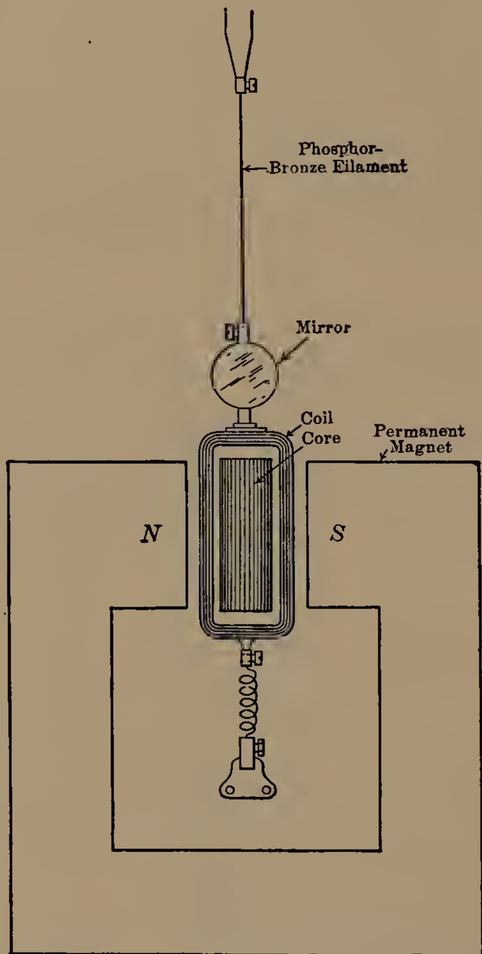


FIG. 109.—Principle of the D'Arsonval galvanometer.

There are two common methods of reading the deflection of a galvanometer. A plane mirror is mounted on the coil system (Fig. 109), and a scale and telescope are mounted about 0.5 m. from the galvanometer (Fig. 110). The reflection of the scale in the mirror can be seen with the telescope. The deflection is read by means of a cross-hair in the telescope.

Another method is to use a concave mirror on the galvanometer moving system. A lamp filament is placed some distance from the mirror and its image is focused on a ground glass to which a scale, graduated in centimeters, is fastened.

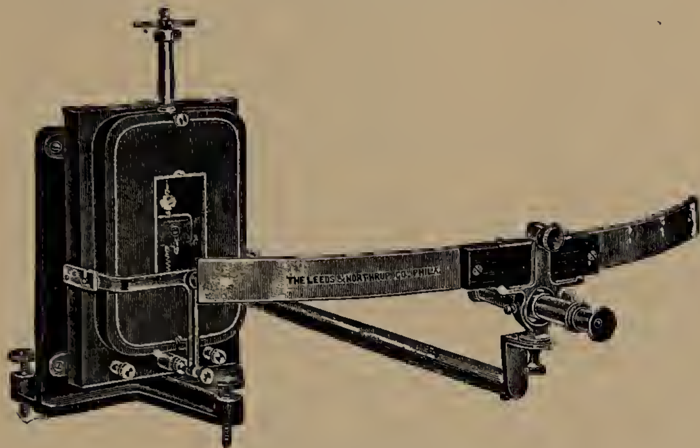


FIG. 110.—Telescope and scale method of reading a galvanometer.

Damping.—If a galvanometer coil, which is hung freely, starts to swing, it will continue swinging for some time unless it is retarded or damped. If the coil be wound on a metal bobbin, the motion of the bobbin through the magnetic field induces eddy currents in itself, and these are in such a direction as to put an electric load on the moving coil as in an electric generator. This opposes the motion of the coil. The same result may be obtained by binding short-circuited copper coils on the main coil, or by shunting the galvanometer externally with a resistance (see "Ayrton Shunt"), or even by short-circuiting the galvanometer.

108. Galvanometer Shunts.—In order to measure currents greater than the rated currents of the galvanometer and to protect the galvanometer from excessive currents, galvanometers are provided with shunts. There are two common types of shunt. One type (Fig. 111) consists of three or four separate resistances which are plugged in parallel with the galvanometer one at a time. These are so adjusted in value that, with a given current to be measured, the successive galvanometer currents are in the ratio of 10 to 1. Referring to Fig. 111, when the plug is entirely removed, no shunt is connected across the galvanometer and the entire line current I flows through the galvanom-

eter. When the plug is in the uppermost ($\frac{1}{9}$) position, one-tenth of the line current goes through the galvanometer and nine-tenths goes through the shunt. When the plug is in the position shown in the figure, one one-hundredth of the line current I goes through the galvanometer; when in the second position from the bottom ($\frac{1}{999}$), one one-thousandth of the line current goes through the galvanometer; when in the lowest position (0), the galvanometer is short-circuited and practically none of the line current goes through the galvanometer. The values of the shunt resistances necessary for obtaining the foregoing shunting ratios are determined as follows:

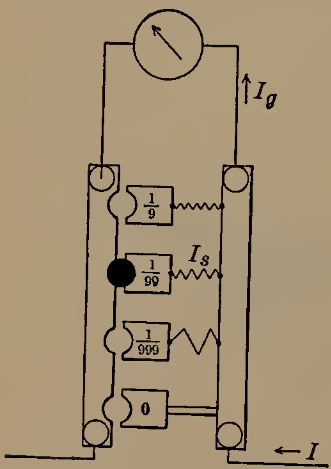


FIG. 111.—Galvanometer shunt.

If, for example, it is desired that the galvanometer current be one-tenth of the line current, nine-tenths of the current must be shunted, and the shunt resistance must be one-ninth of the galvanometer resistance, as shown by the upper resistance (Fig. 111). If it is desired that the galvanometer current be one one-hundredth of the line current, the shunt resistance must be one ninety-ninth of the galvanometer resistance. This may be proved as follows:

Let R_g and I_g be the galvanometer resistance and current, respectively, I_s the shunt current, and I the line current. By equation 24, page 59,

$$\frac{I_g}{I_s} = \frac{I_g}{I - I_g} = \frac{R_s}{R_g}.$$

If $I = 10\ I_g$,

$$\frac{R_s}{R_g} = \frac{I_g}{10\ I_g - I_g} = \frac{1}{9}.$$

If $I = 100\ I_g$,

$$\frac{R_s}{R_g} = \frac{I_g}{100\ I_g - I_g} = \frac{1}{99}.$$

Example.—A galvanometer has a resistance of 600 ohms. What resistances should shunt it in order to reduce its deflections in the ratio of 10 to 1 and 100 to 1?

$$R_1 = \frac{600}{9} = 66.7 \text{ ohms.} \quad \textit{Ans.}$$

$$R_2 = \frac{600}{99} = 6.06 \text{ ohms.} \quad \textit{Ans.}$$

Ayrton Shunt.—The *Ayrton shunt* is shown in Fig. 112. A permanent resistance AB is connected across the galvanometer terminals. One line terminal is permanently connected to one end of this resistance and the other line terminal, C, is movable and can be connected to various points along AB. With a fixed line current the maximum galvanometer current is obtained when C is at B. If point C be moved to a, where resistance Aa is one one-thousandth of the total resistance AB, the galvanometer current will be one one-thousandth of its maximum value. If C be moved to b, where Ab is one one-hundredth of the resistance AB, the galvanometer current will be one one-hundredth of its maximum value, etc.

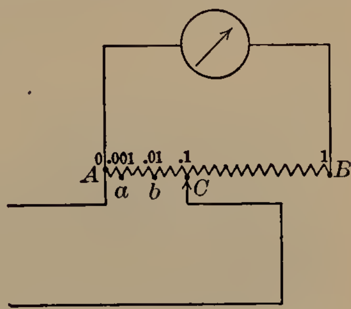


FIG. 112.—Ayrton shunt.

The advantages of the Ayrton shunt are:

1. The shunt is applicable to any galvanometer, regardless of the galvanometer resistance.

2. A fixed resistance is shunted across the galvanometer, which gives a constant value of damping in open-circuit ballistic measurements (see page 202).

The maximum sensitivity of the galvanometer is reduced by the addition of the shunt. The maximum sensitivity of the galvanometer *in combination with its shunt* is obtained when the contactor C is at B. The galvanometer is then shunted by the resistance AB, so that the entire line current does not go through the galvanometer. That is, the sensitivity of the galvanometer in combination with its shunt is less than its sensitivity when used alone. If the shunt has a resistance of only five times that of the galvanometer, the sensitivity will be reduced only in the ratio of 6 to 5, which is not usually objectionable.

The *multiplying power* of a shunt is the ratio of the line current to the galvanometer current. The product of the galvanometer current and the multiplying power gives the line current.

109. Ammeters.—An ammeter is an electrical instrument which measures the current flowing in an electric circuit.

For direct-current measurements, the Weston instrument, developed by Edward Weston, has come into almost universal

use. The instrument is based on the principle of the D'Arsonval galvanometer, but it is so constructed that it is easily portable and it is provided with a pointer and scale for indicating the deflections of the moving coil.

The essential parts of the instrument are shown in Fig. 113. As in the D'Arsonval galvanometer, a permanent magnet is necessary, being made in horseshoe form. Two soft-iron pole-pieces are fitted to the magnet poles and a soft-iron, cylindrical core is held between these pole-pieces by a strip of brass. This gives a short, uniform air-gap and a radial field. The moving coil consists of very fine, silk-covered, copper wire wound on an aluminum bobbin. The aluminum bobbin, besides supporting the coil mechanically, makes the instrument highly damped. This damping is due to the currents set up in the aluminum, because of its cutting the magnetic field.

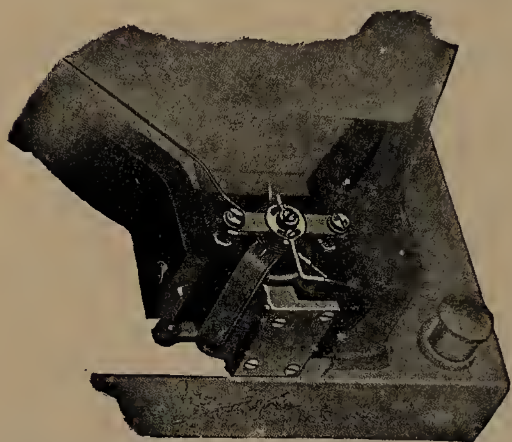


FIG. 113.—Movement of a Weston instrument.

The moving system is supported at the top and bottom by hardened steel pivots turning in cup-shaped jewels, usually sapphire. This method of supporting the moving coil is almost frictionless and makes the instrument portable, whereas the D'Arsonval galvanometer is not readily portable. The current is led in and out of the coil by two flat spiral springs, one at the top of the coil and the other at the bottom. These springs also serve as the controlling device for the coil. That is, any tendency of the coil to turn is opposed by these two springs. The top and bottom springs are coiled in opposite directions so that the effect of change of temperature, which causes a spiral spring to coil or uncoil, will not cause the needle to change its zero position. A very light and delicate aluminum pointer is attached to the moving element, to indicate the deflection of the coil. The pointer moves over a graduated scale, which may be marked

in volts or in amperes, as the case may be. Because of the radial field, the deflection of the moving coil in this type of instrument is practically proportional to the current in the moving coil, so that the scale of the instrument has substantially uniform graduations, which is desirable. The internal connections of a Weston instrument are shown in Fig. 114.

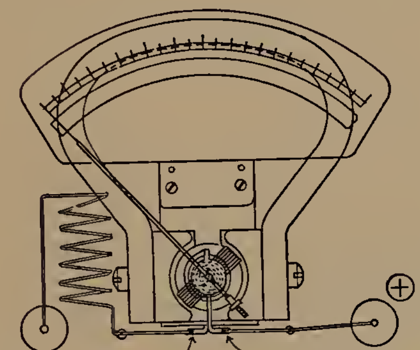


Fig. 114.—A typical Weston direct-current millivoltmeter.

Fig. 115.—Weston portable galvanometer.

Instruments of this construction having very weak springs are often used for portable galvanometers (Fig. 115). Although lacking the extreme sensitivity of the suspended type, they can be made sufficiently sensitive for many classes of work and their ruggedness and portability make them very useful.

The moving coil of Weston portable instruments deflects to the full-scale value with about 0.01 amp. in the coil. Therefore, to measure currents greater than this, the larger portion of the current must be diverted from the moving coil by a *shunt*. The shunt is merely a low resistance, usually made of manganin strip M brazed to comparatively heavy copper blocks c (Fig. 116.) Two sets of binding nuts are fastened to the copper blocks. The heavy wing nuts BB are for carrying the main current through the shunt. The small posts bb are used to connect the

ammeter leads. The ammeter is, in reality, a voltmeter reading the voltage drop across a resistance.

The voltage drop across the shunt is

$$V_{sh} = I_{sh}R_{sh},$$

where I_{sh} and R_{sh} are the shunt current and the shunt resistance respectively. If R_{sh} is constant, the voltage drop across the shunt is proportional to the current in the shunt, so that the

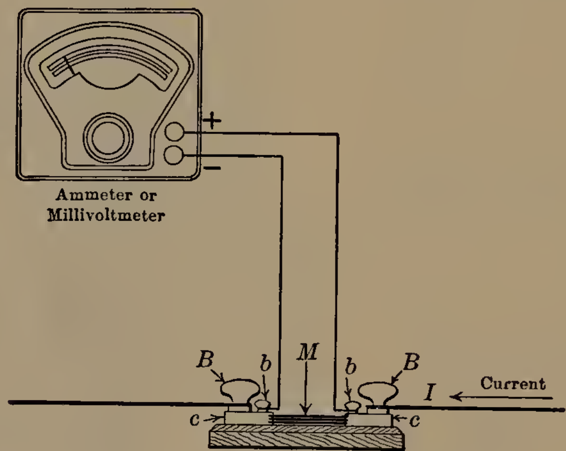


FIG. 116.—Ammeter with an external shunt.

instrument readings are proportional to the current in the shunt. For this reason the ammeter itself (Fig. 114) is often marked "Millivoltmeter." For full-scale deflection the drop across a shunt is about 50 millivolts. The current taken by the instrument

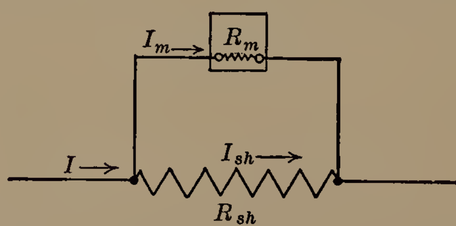


FIG. 117.—Division of current between an ammeter and its shunt.

ment itself is usually about 0.01 amp., so that it is almost always negligible as compared with the main current. Therefore, in most cases the line current equals the shunt current, practically.

An ammeter and its shunt may also be considered as a divided circuit. In Fig. 117, let R_{sh} and I_{sh} be the shunt resistance and

the shunt current respectively, and let R_m and I_m be the instrument resistance and the instrument current respectively. By the law of divided circuits (page 59):

$$\frac{I_{sh}}{I_m} = \frac{R_m}{R_{sh}}.$$

That is, the current divides between the instrument and the shunt inversely as their resistances.

Example.—Assume that an instrument has a resistance of 4 ohms, the shunt a resistance of 0.0005 ohm, and that the line current is 90 amp. What is the value of the instrument current?

As the current in the line differs from the shunt current by a very small amount, the two may be assumed equal. Then,

$$\frac{90}{I_m} = \frac{4}{0.0005}$$

$$I_m = 0.0113 \text{ amp.} \quad \textit{Ans.}$$

For accuracy, the current must always divide between the instrument and the shunt in a fixed ratio. Therefore, the resistance of the shunt and the resistance of the instrument must not change at all, or both must change in the same ratio. Therefore, the leads with which the instrument is calibrated should always be used to connect the shunt to the instrument. The lugs and binding-post contacts should be kept clean from oxide and dirt.

An ammeter with an external shunt may be made to have a large number of scales or ranges. Assume, in the example just given, that the instrument gives full-scale deflection when the instrument current is 0.0125 amp. The potential drop across the instrument terminals is $0.0125 \times 4 = 0.050$ volt, or 50 millivolts. Dividing this voltage by the shunt resistance, the shunt current is

$$I = \frac{0.05}{0.0005} = 100 \text{ amp.}$$

The instrument then deflects full scale with 100 amp. in the line, practically.

If a shunt having a resistance of 0.005 ohm be substituted, the 50 millivolts drop across the shunt may be obtained with 10 amp. $(10 \times 0.005 = 0.050)$. Therefore, a 10-scale ammeter results. By the choice of suitable shunts, the same instrument

may be made to give full-scale deflection with 1 amp., and with 5,000 amp.

In the smaller sizes of instruments up to 50 amp., and where only one scale is desired, the shunt is usually placed within the instrument. For ranges between 50 and 100 amp., the use of an internal or an external shunt is optional. Above 100 amp., it is usual to have the shunt external to the instrument because of its size and its heating loss.

An ammeter can usually be distinguished from a voltmeter by the fact that its binding posts are heavy and are of bare metal, except in the case of an instrument having an external shunt. The posts of millivoltmeters and voltmeters are of much lighter construction and the metal posts are covered with hard rubber, mostly for insulation purposes.

Any instrument when connected in a circuit should disturb the circuit conditions as little as possible. An ammeter shunt, since it is connected in series with the line, should have as low a resistance as is practicable, so that, when it is connected, very little additional resistance is introduced into the circuit. To protect ammeters from heavy currents, etc., provision may be made for *short-circuiting* them when readings are not being taken.

110. Voltmeters.—The construction of a voltmeter does not differ materially from that of an ammeter, in so far as the movement and magnet are concerned (see Fig. 113). The moving coil of the voltmeter is usually wound with more turns and of finer wire than that of the ammeter and so has a higher resistance. The principal difference, however, lies in the manner of connecting the instrument to the circuit. As a voltmeter is connected directly across the line to measure the voltage, it is desirable that the voltmeter take as little current as is practicable. Because of its comparatively low resistance, the moving coil of the voltmeter cannot be connected directly across the line, as it would ordinarily take an excessive current and might be burnt out. Therefore it is necessary to connect a high resistance in series with the moving coil, as shown in Fig. 118. By Ohm's law the current through the instrument is proportional to the voltage, so that the instrument scale can be graduated in volts. The resistance required is easily determined. Assume that an instrument gives full-scale deflection with 0.01 amp. in

the moving coil, and that the coil resistance is 20 ohms. If it is desired that the instrument indicate 150 volts, full scale, the total resistance of the instrument circuit must be

$$R = \frac{V}{I} = \frac{150}{0.01} = 15{,}000 \text{ ohms}.$$

As the instrument has a resistance of 20 ohms, this means that 14,980 ohms additional are necessary.

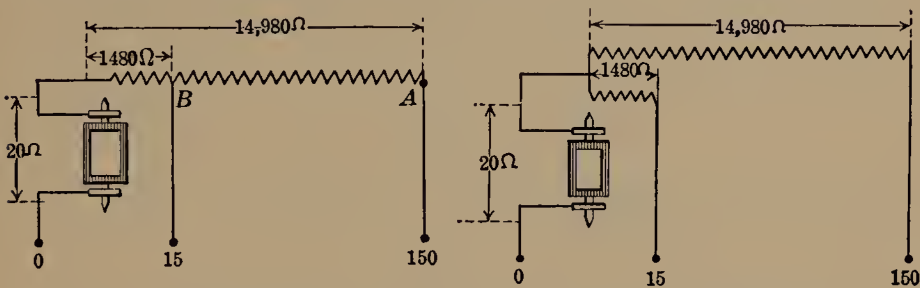


FIG. 118.—Methods of connecting resistance in a voltmeter.

If it be desired that this same instrument also have a full-scale deflection with 15 volts, the resistance of 14,980 ohms may be tapped so that the resistance OB (Fig. 118 (a)) $= \frac{15}{0.01} = 1{,}500$ ohms, and this tap can be brought to a binding post. Another method of securing the same result is shown in Fig. 118 (b). A separate resistance of $1{,}500 - 20 = 1{,}480$ ohms is connected from a binding post to the junction of the 14,980 ohms resistance and the moving coil. This last method is advantageous, as it permits independent adjustment of each resistance; injury or repair in one resistance does not affect the other.

111. Multipliers or Extension Coils.—The range of a voltmeter having its resistance incorporated within the instrument may be increased by the use of external resistance connected in series with the instrument.

Example.—A 150-scale voltmeter has a resistance of 17,000 ohms. What external resistance should be connected in series with it so that its range is (a) 300 volts? (b) 600 volts?

(a) The deflection of the voltmeter depends on the current flowing through its moving coil. Hence, in order to maintain the same current through the instrument at 300 volts as flows at 150 volts, the resistance of the circuit must be doubled.

Therefore, $17{,}000 \times 2 = 34{,}000$ ohms are necessary.

As the instrument already has 17,000 ohms, the added resistance will be

$34{,}000 - 17{,}000 = 17{,}000$ ohms. *Ans.*

(*b*) The total resistance must now be

$$^{600}\!/_{150} \times 17{,}000 = 68{,}000 \text{ ohms.}$$

As 17,000 ohms is already within the instrument, $68{,}000 - 17{,}000 =$ 51,000 ohms must be added external to the instrument. *Ans.*

External resistances used in this manner are called *multipliers*, or sometimes *extension coils*. Their multiplying power M is given by:

$$M = \frac{R_x + R_m}{R_m} \tag{46}$$

where R_x is the resistance of the multiplier and R_m the resistance of the instrument.

Example.—In the above problem (*b*) the multiplying power of the multiplier is as follows:

$$M = \frac{51{,}000 + 17{,}000}{17{,}000} = 4.$$

112. Hot-wire Instruments.—In the instruments heretofore considered, the indication depends on the electromagnetic action of the current. The hot-wire type of instrument depends for its indications upon the heating action of the current. A diagram of this instrument is shown in Fig. 119. AB is a fine wire of platinum-silver through which the current passes. At C, a wire CF is attached to AB. At E, on CF, a silk fiber EH is attached. This passes around the pulley W, and is held in tension by the spring H. When a current flows through AB, the heat expands the wire AB, reducing the tension in the wire CF, and allowing the spring H to pull the silk fiber to the left. This fiber, acting on the pulley W, moves the pointer P over the scale.

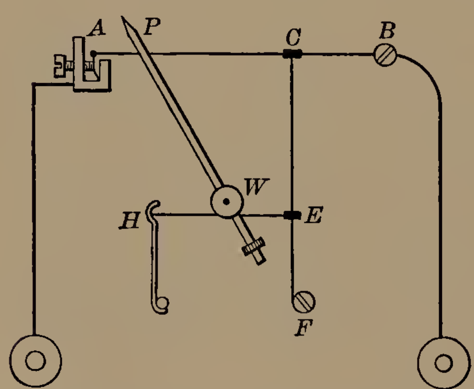


FIG. 119.—Principle of Hartmann and Braun hot-wire instruments.

When used as an ammeter, a shunt is necessary, unless the current is very small. When used as a voltmeter, a high resistance is connected in series with the wire AB

This type of instrument is very sluggish in its behavior and only reaches its ultimate deflection after the lapse of considerable time. An advantage of the hot-wire type of instrument is that it can be used for alternating as well as for direct currents. It is particularly useful for the measurement of high-frequency alternating currents, as its indications are independent of the frequency, provided a shunt is not used. As such instruments do not hold their calibration for very long periods, they should be calibrated at the time of using.

ELECTRICAL MEASUREMENTS

Measurement of Resistance

113. Voltmeter-ammeter Method.—The resistance of any portion of an electric circuit which does not contain a source of electromotive force is, by Ohm's law,

$$R = \frac{V}{I}$$

where V is the voltage across that portion of the circuit and I is the steady current flowing in that portion of the circuit. The

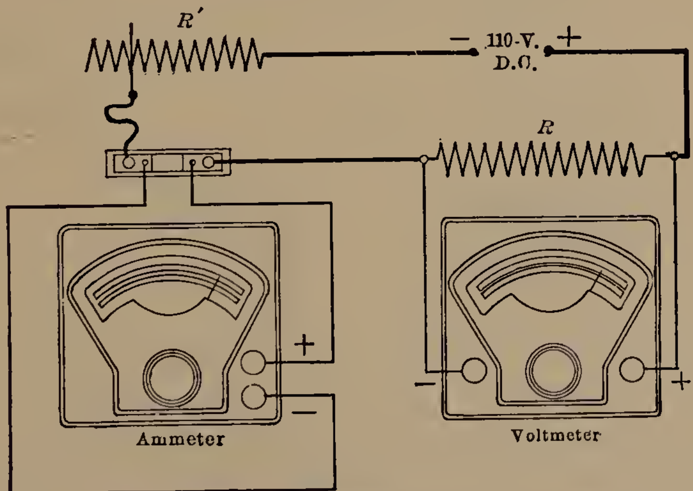


FIG. 120.—Voltmeter-ammeter method of measuring resistance.

voltage V may be measured with a voltmeter, the current I with an ammeter, and the resistance R may be computed.

Let it be required to determine the resistance R in the circuit shown in Fig. 120. The source of power is the 110-volt supply. The resistance R is comparatively small in value and, if connected directly across 110 volts, would take an excessive current. Therefore, it is necessary to insert a resistance R' in series with R to limit the current. The voltmeter, however, must be connected directly across R, as it is desired to know the resistance of this portion of the circuit only.

Example.—The voltmeter (Fig. 120) reads 19.0 volts when the ammeter reads 24.0 amp. What is the value of the resistance R?

$$R = \frac{19.0}{24.0} = 0.792 \text{ ohm.} \quad \textit{Ans.}$$

As a matter of interest, let it be required to determine the resistance ot R'. The voltmeter terminals are transferred from across R to across R- (a higher-range voltmeter will probably be necessary). Under these conditions the voltmeter reads 91.0 volts and the ammeter still reads 24.0 amp. Therefore:

$$R' = \frac{91.0}{24.0} = 3.79 \text{ ohms.} \quad \textit{Ans.}$$

It is sometimes desired to measure resistances of such low value that, if a voltmeter were connected directly across their termi-

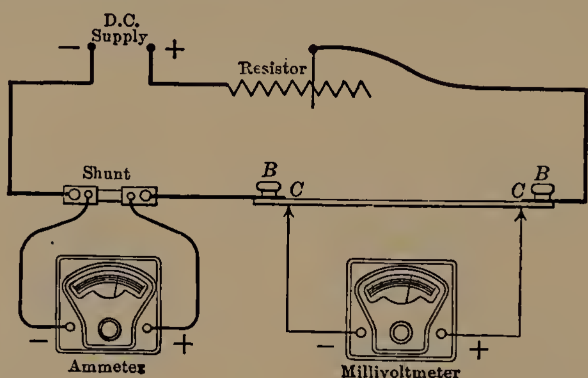


FIG. 121.—Measuring the resistance of a metal rod.

nals, the contact resistance, which may be comparatively large, would introduce considerable error and might even exceed in magnitude the resistance which it is desired to measure. To eliminate this error due to contact resistance, the voltmeter terminals are connected well inside the terminals BB (Fig. 121) through which the current is led to the specimen whose resis-

tance is being measured. As the voltmeter takes only a very small current, small sharp-pointed contacts CC may be used. Since the resistance of the voltmeter is comparatively high, it is only necessary that the contact resistances at CC be negligible compared to the resistance of the instrument. This condition is easily met. As these contacts are small and sharp, the points of contact on the specimen can be determined very accurately.

Example.—When the ammeter (Fig. 121) reads 50 amp., the millivoltmeter indicates 40 millivolts. The contacts CC are 23 in. apart. What is the resistance per inch length of the rod?

The resistance for 23 in. is

$$R = \frac{0.040}{50} = 0.00080 \text{ ohm.}$$

The resistance per inch

$$R = \frac{0.00080}{23} = 0.0000348 \text{ ohm.} \quad \textit{Ans.}$$

114. The Voltmeter Method.—It is possible to measure a resistance by means of a voltmeter alone, provided the resistance to be measured is comparable with that of the voltmeter. In

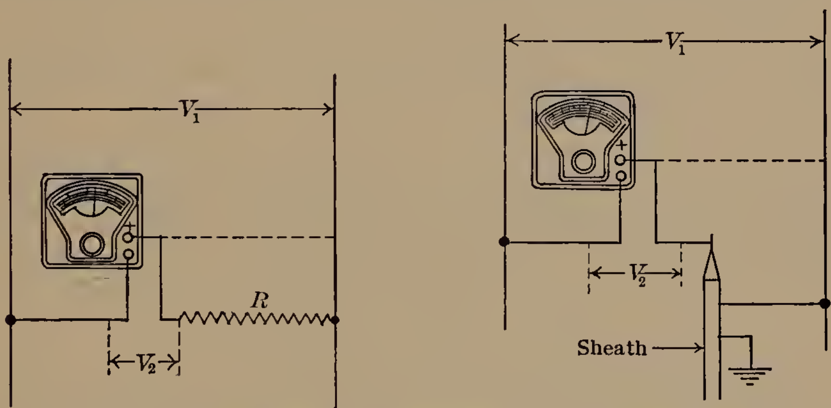


FIG. 122.—Measurement of resistance by the voltmeter method.

Fig. 122 (a), let it be required to measure the resistance R. The voltmeter is first connected across the source of supply and a reading V_1 taken. It is then transferred so that the resistance R is in series with it across the source of supply and the voltmeter reading is again taken. Let this reading be V_2.

As V_1 is the total circuit voltage and V_2 is the voltage across the instrument, the voltage across the unknown resistance R

is obviously $V_1 - V_2$. When the voltmeter is in series with R, the same current i must flow through each, so the voltages are as follows:

$$V_2 = iR_v \tag{1}$$

$$V_1 - V_2 = iR \tag{2}$$

where R_v is the resistance of the voltmeter.

Dividing (2) by (1) and solving for R,

$$R = R_v \frac{V_1 - V_2}{V_2}. \tag{47}$$

This method of measuring resistance is particularly useful in determining insulation resistance of dynamo windings, cables, etc. As such resistances are very high, they are usually expressed in megohms (1 megohm = 1,000,000 ohms). It will be seen from equation (47) that the greater the value of R_v, the greater the resistance that can be measured by this method. For this reason, special 150-scale voltmeters, having resistances of 100,000 ohms (one-tenth of a megohm) are available. These give a sensitivity about six times as great as can be obtained with the ordinary 150-scale voltmeter.

Figure 122 (*b*) shows the application of this method to the measurement of the insulation resistance of a cable.

Example.—When a 100,000-ohm voltmeter is connected across a direct-current line it reads 125.0 volts. One terminal of the voltmeter is then connected to the core of a lead-covered cable and the sheath of the cable is connected to the other side of the line (Fig. 122 (*b*)). The voltmeter now reads 8.0 volts. What is the insulation resistance of the cable?

$$x = 0.1 \frac{125.0 - 8.0}{8.0} = 1.56 \text{ megohms.} \quad \textit{Ans.}$$

115. The Wheatstone Bridge.—In distinction to the foregoing methods of measuring resistance, the Wheatstone-bridge method is one in which the unknown resistance is balanced against other known resistances. The bridge in its simplest form is shown in Fig. 123. Three known resistances, M, N, P, and the unknown resistance X are connected to form a diamond. Current from a battery B feeds the two opposite corners, o and c, of the diamond. Across the other two corners, a and b, is connected a galvanometer.

To make a measurement, the two arms M and N are each set at some fixed value of resistance, usually 1, 10, 100, 1,000 ohms, etc. The arm P is then adjusted until the galvanometer does not deflect. If the galvanometer does not deflect, no current flows through it and therefore the two points a and b must be at the *same potential.* Also, the currents $I_1 = I_3$ and $I_2 = I_4$, as no current passes through the galvanometer.

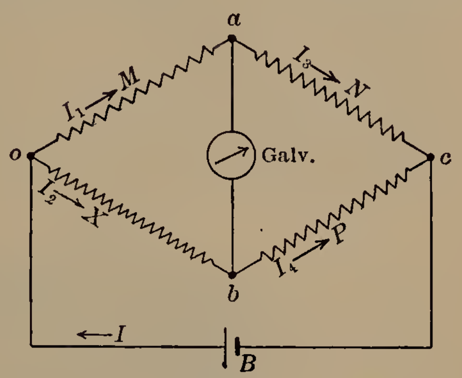


FIG. 123.—Elementary Wheatstone bridge.

If the points a and b are at the same potential, the voltage-drop oa = the voltage-drop ob, and

$$I_1M = I_2X. \qquad (1)$$

Also, the voltage-drop ac = the drop bc and

$$I_3N = I_4P.$$

And, since $\quad I_1 = I_3$ and $I_2 = I_4$,

$$I_1N = I_2P. \qquad (2)$$

Dividing (1) by (2),

$$\frac{I_1M}{I_1N} = \frac{I_2X}{I_2P} \text{ or } \frac{M}{N} = \frac{X}{P}$$

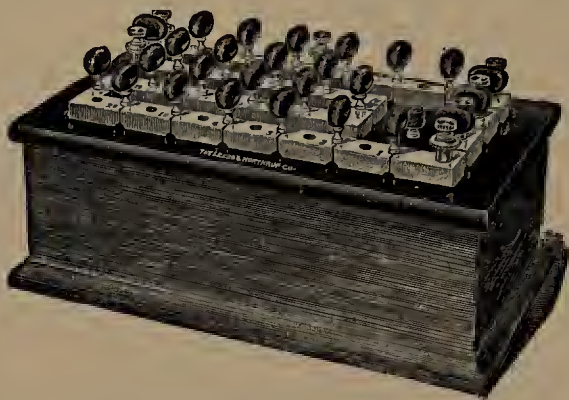
$$X = \frac{M}{N}P \qquad (48)$$

which is the equation of the Wheatstone bridge. M and N are called the ratio arms and P the balance or rheostat arm. Obviously, the battery and the galvanometer may be interchanged without affecting the relation given in equation (48).

The many types of Wheatstone bridge found in practice do not differ in principle from that shown in Fig. 123. The differences lie in the positions of the arms M, N, and P on the bridge, as well as in the manner in which the coils in these arms are cut in and out of circuit.

A diagram of a common plug-type of bridge is shown in Fig. 124. M consists of three resistances of 1,000, 100, and 1 ohms respec-

tively, and N consists of three of 10, 100, and 1,000 ohms respectively. P consists of a number of resistances ranging from 5,000 ohms to 1 ohm and of such values that, with the proper combinations, P may be made equal to any whole number between 1 and 11,110 ohms. Between the outer ends of N and P are two infinity plugs (∞) and a 10,000-ohm coil. The infinity plugs mean that the bridge can be open-circuited at these points



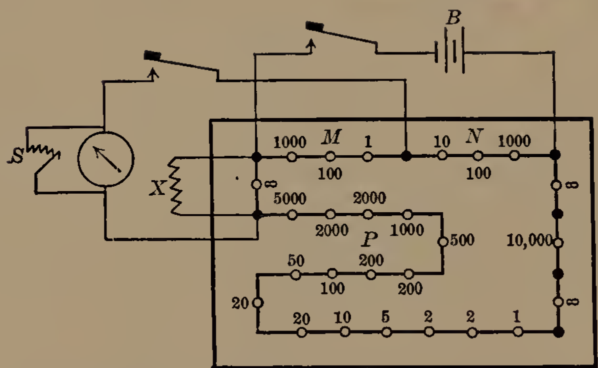


FIG. 124.—Connections of a typical bridge box.

and by their position the 10,000-ohm coil may be made a part of N or a part of P. The unknown resistance X may be connected across any one of the infinite resistances, as is found advisable. Between M and P is another infinite resistance, across which the unknown resistance may also be connected, the infinite plug being removed.

In this type of bridge, the resistance coils are connected across gaps cut in heavy brass or composition bars. When it is desired to insert a resistance, the plug is removed, and when it is desired

to remove a resistance, it is short-circuited by the plug. These plugs have hard-rubber tops and are tapered. As the principal source of error in this type of bridge lies in the contact resistance of the plugs, they should be made to fit tightly when used. This is accomplished by exerting a slight pressure and simultaneously twisting them, thus giving a wiping contact. As dirt and oxide are a frequent source of error, the plugs should be kept clean.

In using the bridge, much time may be saved if a systematic procedure is followed in obtaining a balance. Assume that it is desired to measure a certain unknown resistance. Connect the bridge as shown in Fig. 124, placing keys in the battery and galvanometer circuits and a shunt around the galvanometer to protect it from deflecting violently when the bridge is considerably out of balance. Make the ratio arms M and N each 1,000 ohms, a 1 to 1 ratio. With the galvanometer well shunted and all the plugs in P (Res. = 0), depress first the battery and then the galvanometer key. The galvanometer is observed to deflect to the left. Now remove the 5,000-ohm plug in P and the galvanometer deflects to the right. From these observations, two facts are determined. First, the unknown resistance is less than 5,000 ohms; second, the galvanometer deflects to the left if the resistance in P is too small, and deflects to the right if the resistance in P is too large. By inserting the 5,000-ohm plug and removing the 1,000-ohm plug, the galvanometer still deflects to the right, indicating that 1,000 ohms in P is too large. This is repeated with 500 ohms, 200 ohms, etc. By proceeding in this manner, it is found that the galvanometer does not reverse until a 2-ohm plug is removed. This means that the unknown resistance lies between 2 and 5 ohms. By removing the two 2-ohm plugs and then a 1 and a 2, the unknown resistance is narrowed down to between 2 and 3 ohms. To obtain a more precise value, the ratio arms must be changed. M is now made 1 ohm and 2,000 ohms are unplugged in P. By successive trials, at the same time reducing the shunt S, a balance is obtained at 2,761 ohms in P. Then:

$$X = \frac{M}{N} P = \frac{1}{1{,}000} 2{,}761 = 2.761 \text{ ohms.}$$

In obtaining a balance, the battery key should always be depressed before the galvanometer key, so that the current in the bridge has time to reach a constant value. Otherwise the electromotive force of self-induction may introduce an error.

A more convenient arrangement of the resistance units of the rheostat arm P is shown in Fig. 125. The resistances are arranged in groups of equal resistances, one group consisting of ten 1-ohm coils, the next of ten 10-ohm coils, the next of ten 100-ohm coils, etc. Each group is called a *decade*. Only one plug per decade is necessary. This arrangement has the advantage that the plugs are always in service, so are not so likely to be

mislaid or to become dirty; there is less probability of error in reading; it is a simple matter to see that the few plugs used are fitting tightly, and a balance can be quickly obtained. It is obvious that nine coils per decade

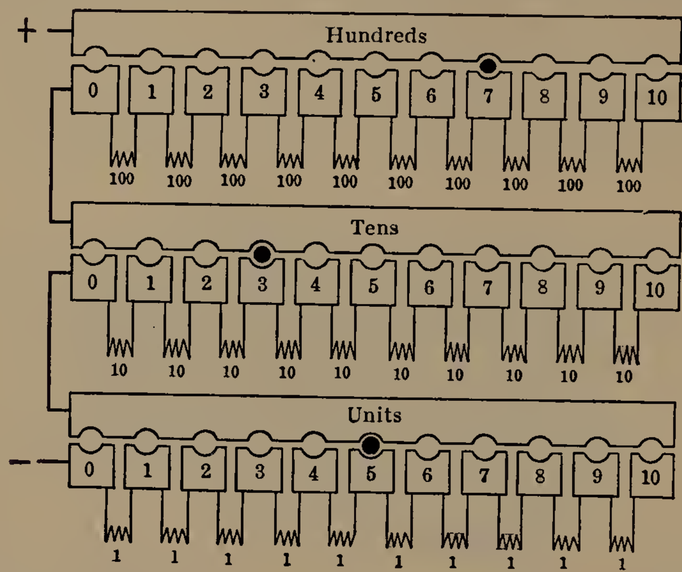


Fig. 125.—Arrangement of rheostat arm resistances in a decade bridge.

are sufficient for obtaining any desired resistance, although ten coils per decade are often used.

The decade principle has been extended to an even more convenient type of bridge, the dial bridge. Instead of using plugs, a dial arm similar to the

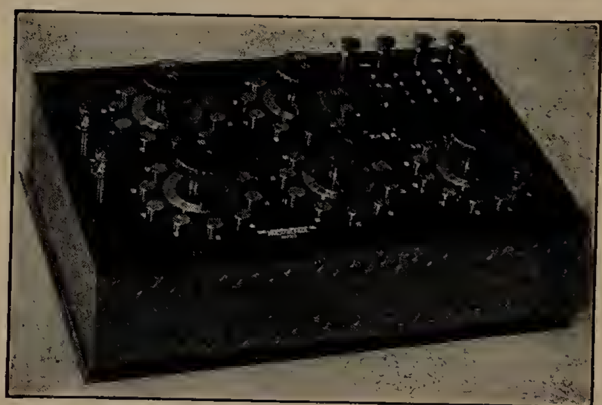


Fig. 126.—Leeds & Northrup dial bridge.

type used in rheostats is employed to select the required resistances. Because of its ease of manipulation, this type has come into extensive use. Care should be taken to keep the dials and contacts free from dirt and oxide. Figure 126 shows a dial bridge of the Leeds & Northrup type.

116. The Slide-wire Bridge.—The slide-wire bridge is a simplified Wheatstone bridge, in which the balance is obtained by means of a slider which moves over a German-silver or manganin resistance-wire. A typical slide-wire bridge is shown in Fig. 127. The resistance-wire AB, 100 cm. long, is stretched tightly between two heavy copper blocks CD, 100 cm. apart. A meter scale is placed along this wire. A contact key K' is movable along the scale and when the key K' is pressed, a knife edge makes contact with the wire. The rest of the bridge consists of a heavy copper bar E, a known resistance R, and the unknown resistance X. R is connected between D and E, and X is connected between C and E, although the positions of R and X are interchangeable.

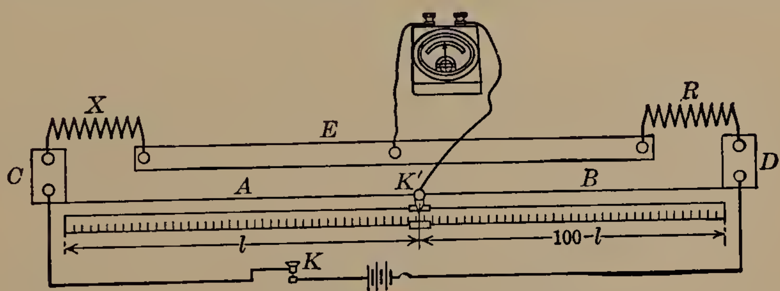


FIG. 127.—Slide-wire bridge.

The galvanometer is connected between the key K' and E and the battery terminals are connected to C and D. A balance is obtained by moving K' along the wire until the galvanometer shows no deflection.

Let l be the distance in centimeters from one end of the scale to K' when a balance is obtained. Then $100 - l$ is the distance from K' to the other end of the scale. Let r be the resistance per unit length of the wire. Then the resistance of l is lr and that of the remainder of the wire is $(100 - l)r$.

By the law of the Wheatstone bridge,

$$\frac{X}{lr} = \frac{R}{(100 - l)r}. \qquad \text{(I)}$$

r cancels and (I) becomes

$$X = R\frac{l}{(100 - l)}. \qquad (49)$$

(49) may also be written

$$\frac{X}{R} = \frac{l}{(100 - l)}. \qquad (50)$$

That is, when a balance is obtained, the slide wire is divided into two parts which are to each other as X is to R.

The slide wire is not as accurate as the coil bridge, because the slide wire may not be uniform; the solder at the points of contact at C and D makes the length of the wire uncertain; the slide wire cannot be read as accurately as the resistance units of a bridge can be adjusted.

Example.—Assume that R (Fig. 127) equals 10 ohms and that a balance is obtained at 74.6 cm. from the left-hand end of the scale. Find the unknown resistance X.

From equation (49)

$$X = 10\frac{74.6}{100 - 74.6} = 74.6\frac{10}{25.4} = 29.37 \text{ ohms. } \quad \textit{Ans.}$$

CABLE TESTING

117. The Murray Loop.—The slide-wire bridge offers a very convenient method of locating grounds in cables and wires. Figure 128 shows a cable AB which has become grounded at the

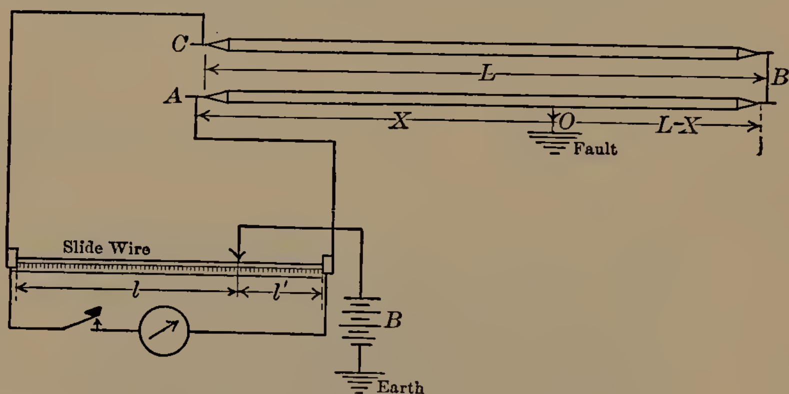


FIG. 128.—Murray-loop test.

point O, owing to a defect in the insulation. CB is the return conductor and is similar to AB, except that it has no ground or "fault." The two conductors are looped together at B, the far end of the two conductors, which may be at some power station, telephone exchange, etc.

The slide wire is then connected to the home ends of the cable as shown. It will be noted that the battery and the galvanometer are not in the positions shown in Fig. 127, but have been interchanged. This is done in order that earth currents shall not

disturb the galvanometer readings. Also, if the ground has a high resistance, the electromotive force of the battery B may be increased until sufficient current to operate the bridge is sent through the ground resistance. The resistance of the fault to ground does not produce any error in the measurement so long as the conductor is not broken. If the conductor is broken, with both ends lying on the ground, the resistance of the conductor is increased and a false location of the fault may result.

In Fig. 128, the distance X to the fault may be found as follows:

$$\frac{X}{l'} = \frac{L + (L - X)}{l} \tag{51}$$

where L is the length of the cable.

The slide wire is split into two sections which are to each other as the two lengths of cable on each side of the fault.

Solving (51) for X,

$$X = \frac{2Ll'}{l + l'}. \tag{52}$$

This assumes that the resistance per foot of both conductors is the same and is uniform. The jumper tying the cable ends together at B should make good connection, as contact resistance at this point may introduce an appreciable error. A ratio and rheostat arm of a bridge box may obviously be used instead of the slide wire.

Example.—A cable 2,000 ft. long consists of two conductors. One conductor is grounded at some point between stations. A Murray-loop test, with a 100-cm. slide-wire bridge, is connected as in Fig. 128 to locate the fault. A balance is obtained at 85 cm. How far from the station is the ground?

From equation (52)

$$L = 2{,}000 \quad l' = 15 \quad l = 85$$

$X = \dfrac{4{,}000 \times 15}{100} = 600$ ft. from the station at which the measurement is made.

118. Insulation Testing.—In practice it is necessary to measure the resistance of the insulation of cables, both at the factory and after the cable is installed. A low value of insulation resistance may indicate that the insulation is of an inferior grade. A low insulation resistance after installation may indicate

improper handling or faulty installation. The voltmeter method described in Par. 114 is applicable in many cases, but where the insulation resistance is high, even a high-resistance voltmeter is not sufficiently sensitive.

To make the measurement, a sensitive galvanometer is used. A source of potential, of moderately high voltage, from 100 to 500 volts, is also necessary. Such potential may be secured from direct-current mains, although dry cells, silver-chloride cells, and test-tube batteries connected in series are more satisfactory. A simple diagram of connections is shown in Fig. 129.

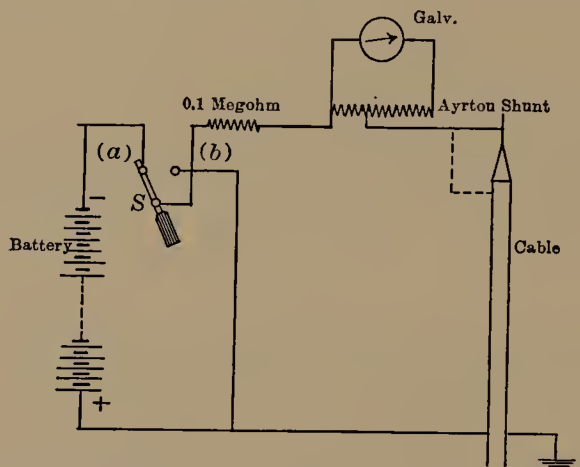


FIG. 129.—Measurement of the insulation resistance of a cable.

The method is one of substitution. A known resistance, usually 0.1 megohm (100,000 ohms), is first connected in the circuit and the galvanometer deflection noted. The unknown resistance X is then substituted and the galvanometer reading again noted. As the currents in the two cases are inversely proportional to the circuit resistances, the unknown resistance can be determined, the galvanometer deflections being used rather than actual values of current. Let D_1 be the deflection with the 0.1 megohm and D_2 be the deflection with the unknown resistance.

$$\frac{X}{0.1} = \frac{D_1}{D_2}$$

$$X = 0.1\,\frac{D_1}{D_2}. \tag{53}$$

Under ordinary circumstances it would not be possible to obtain accurate results under these conditions alone, because the

unknown resistance may be in the hundreds of megohms and the known resistance is but 0.1 megohm. This would make the deflection D_2 so many times smaller than D_1 that it would not be readable.

This difficulty is overcome by the use of the Ayrton shunt, described in Par. 108, page 119. When the 0.1 megohm only is in circuit, the galvanometer sensitivity ordinarily is such that it would deflect off the scale unless the galvanometer were shunted.

Therefore, the shunt is adjusted to some low value, as 0.0001. Call this setting of the shunt S_1 and the galvanometer deflection D_1. The multiplying power of the shunt is $M_1 = \frac{1}{S_1}$ (see page 119). The cable is now introduced in the circuit and the shunt adjusted until a reasonable deflection is obtained. Call this deflection D_2 and the value of the shunt S_2. Its multiplying power is now $M_2 = \frac{1}{S_2}$.

The ratio of the currents in the circuit in the two cases is

$$\frac{I_1}{I_2} = \frac{M_1 D_1}{M_2 D_2}.$$

Therefore the unknown resistance, from (53), is

$$X = 0.1 \frac{I_1}{I_2} = 0.1 \frac{M_1 D_1}{M_2 D_2}. \tag{54}$$

The galvanometer merely acts as an ammeter to measure the current leaking through the insulation. If the battery voltage and the galvanometer reading in terms of amperes were known, the insulation resistance could be computed directly. By the substitution of the 0.1 megohm, the insulation resistance may be determined directly from the galvanometer readings.

In practice, instead of substituting the cable for the 0.1 megohm, the cable is first short-circuited by the wire shown dotted (Fig. 129), and the constant determined. This wire is then removed, placing the cable in circuit. The 0.1 megohm is left permanently in circuit to protect the galvanometer in case of accidental short-circuit of the cable. The 0.1 megohm is usually not appreciable compared to the insulation resistance of the cable, so that ordinarily no correction is necessary for it.

A switch or key S is usually provided. When in position (a), the circuit is closed through the cable. When thrown over to (b), the cable, which is charged electrostatically, discharges through the galvanometer.

When the switch (a) is first closed, there is a rush of current which charges the cable electrostatically (see page 193). Due to dielectric absorption, it takes time to charge the cable. Therefore, the charging current flows for some time, decreasing continuously. As it is often inconvenient to wait for the galvanometer to reach a steady deflection, it has been agreed arbitrarily to take the deflection at the end of one minute as the value to be used in determining insulation resistance.

When the switch S is thrown to (b), the electrostatic charge in the cable rushes out through the galvanometer in the reverse direction. Due to absorption, it requires considerable time for the cable to become totally discharged.

In making insulation-resistance measurements, precautions must be taken to insulate thoroughly the apparatus itself. Hard-rubber posts should be used for supports and, wherever possible, the leads should be carried through the air rather than be allowed to rest on the ground. The insulation resistance varies enormously with temperature, so the temperature at which the measurements are made should be carefully determined and stated.

Example.—A cable is tested for insulation resistance, the connections given in Fig. 129 being used. When the cable is short-circuited, the galvanometer deflection, with the shunt set at 0.0001, is 24 cm., the resistance of the circuit being 0.1 megohm. When the short-circuit is removed from the cable, a deflection of 17 cm. is obtained after the cable has been electrified for 1 min. and with the shunt set at 1.0. The cable is 1,800 ft. long.

(a) What is its insulation resistance? (b) What is the insulation resistance of a mile length of the cable?

$$M_1 = \frac{1}{0.0001} = 10{,}000.$$

$$M_2 = \frac{1}{1.0} = 1.$$

$$D_1 = 24 \text{ cm. and } D_2 = 17 \text{ cm.}$$

From equation (54)

$$X = 0.1\,\frac{10{,}000 \times 24}{1 \times 17} = 1{,}410 \text{ megohms.} \quad \textit{Ans.}$$

(b) The insulation resistance of a mile length will be *less* than that of the 1,800-ft. length, because the amount of leakage current is directly proportional to the length of the cable. Therefore the resistance of this leakage path is inversely proportional to the length of cable. The cross-sectional area of the leakage path for the mile length is greater than it is for the 1,800-ft. length. Therefore the insulation resistance of a mile length

$$R = 1{,}410\frac{1{,}800}{5{,}280} = 481 \text{ megohms.} \quad \textit{Ans.}$$

POTENTIOMETERS

119. The Potentiometer.—The potentiometer is an instrument for making accurate measurements of voltage. Its standardization depends primarily on the Weston standard cell (see page 94). The principle is as follows:

Assume in Fig. 130 (a) that a standard cell S has an electromotive force of exactly 1 volt. Let a storage cell Ba supply current to a wire AB through a rheostat R. Let the wire AB be divided into 15 divisions each of 1 ohm resistance, making the total resistance of AB equal to 15 ohms. The standard cell is connected with its negative terminal to the negative terminal of the storage cell and its positive terminal is connected to the tenth 1-ohm coil C through a key and galvanometer. If 0.1 ampere flows through the wire AB, the voltage-drop through each division of AB will be 0.1 volt and the voltage-drop across AC will be 1.0 volt. If the key be depressed, no current will flow through the galvanometer, as the standard-cell electromotive force is equal and opposite to this 1-volt drop. If, however, the current in AB is not exactly 0.1 amp., current will flow through the standard-cell circuit, due to the voltage-drop AC being either greater or less than 1 volt. If the current is less than 0.1 amp., the galvanometer deflects in one direction, and if it is greater than 0.1 amp., the galvanometer deflects in the reverse direction. Obviously, it is possible to adjust the current in AB to such a value that the galvanometer deflection is zero. Under these conditions, the current in AB is exactly 0.1 amp. and the potential-drop across each resistance in AB is 0.1 volt. Therefore, AB may be marked in volts as shown.

Let it be required to measure some unknown electromotive force E whose value is known to be less than 1.5 volts. Its negative terminal is connected to the end A of the wire AB

(Fig. 130 (*b*)). The positive terminal of the electromotive force is connected through the galvanometer and key to a movable contact *b*. It is assumed that the current in *AB* has been adjusted to exactly 0.1 amp. Contact *b* is moved along *AB* until

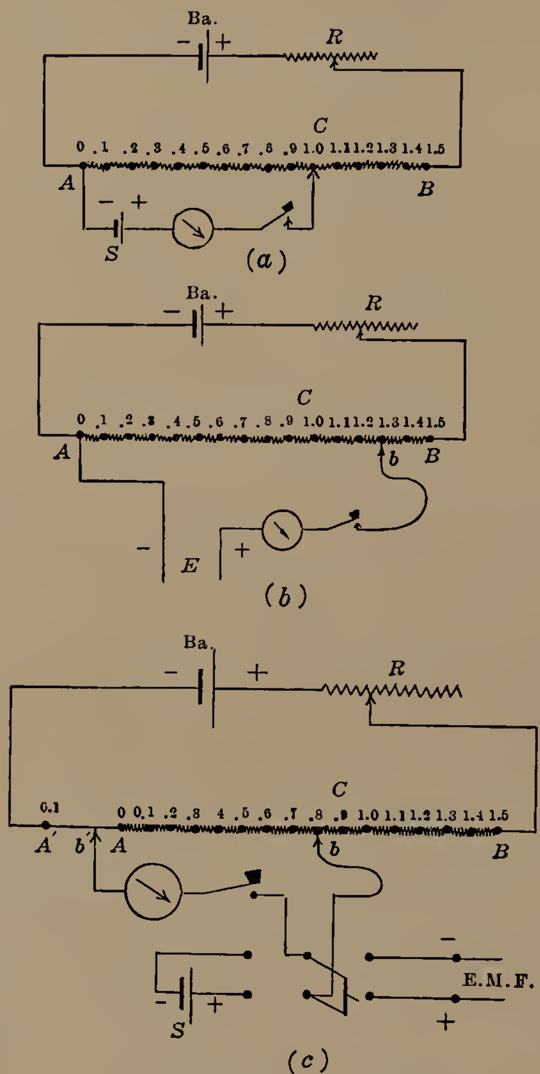


Fig. 130.—Simple potentiometer.

the galvanometer deflection is zero. This means that the electromotive force *E* is just balanced against an equal drop in the wire *AB*. As *AB* is calibrated in volts, the value of *E* may be read directly on *AB*. This method of measuring voltage is the

Poggendorff method and is the fundamental principle of the potentiometer.

The transfer from the standard cell to the electromotive force to be measured may be accomplished by means of a double-pole, double-throw (D.-P. D.-T.) switch, such as is shown in Fig. 130 (*c*). The standard-cell electromotive force actually is slightly greater than 1 volt. In Fig. 130 (*c*) an extra slide wire $A'A$, having a resistance of 1.0 ohm, is added to the potentiometer wire AB. This makes the fractional parts of 0.1 volt readily obtainable. To make the standard-cell adjustment, the contacts b and b' are set to values corresponding to the electromotive force of the standard cell, and the potentiometer is then balanced with the switch to the left. The switch is then thrown to the right and the potentiometer is balanced against the unknown electromotive force by moving the contacts bb'.

120. The Leeds & Northrup Student's Potentiometer.—Figure 131 shows a Leeds & Northrup student's potentiometer,

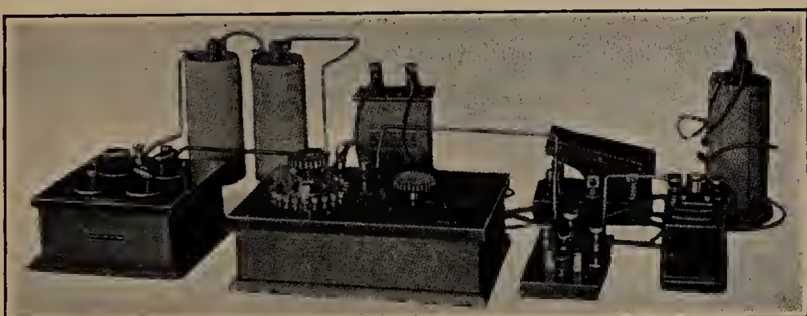
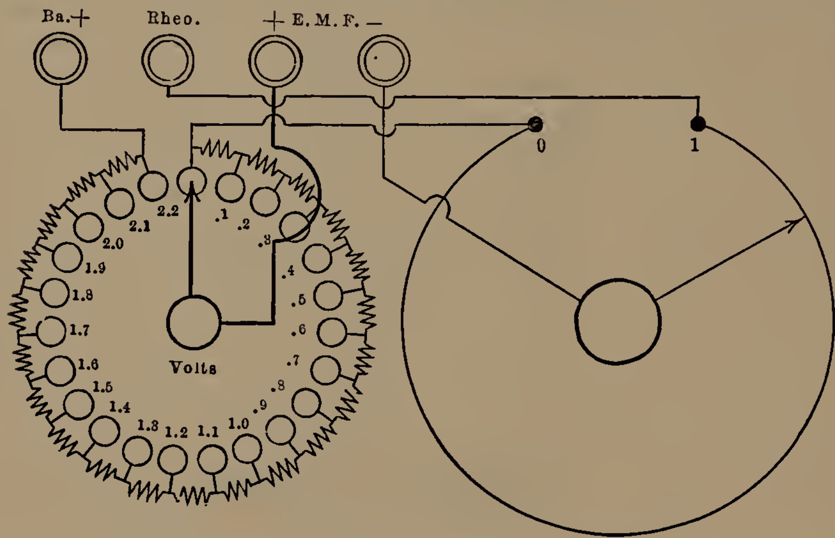


FIG. 131.—Student's potentiometer and accessories set up for voltage measurements. Student's potentiometer in center of foreground.

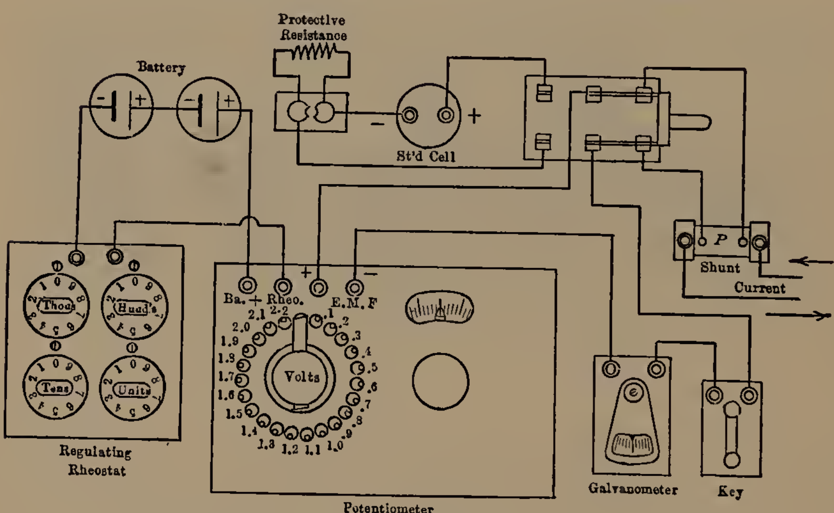
with its accessories, set up for voltage measurement. Although this type of potentiometer is not so conveniently arranged and manipulated as the commercial types,[1] yet, because of its simplicity, the principle of its operation is more readily understood than are the more complicated types. The range and resistance of this potentiometer are slightly different from the one shown in Fig. 130, but the principle of operation is the same. The interior wiring diagram is shown in Fig. 132 (*a*) and the complete connections for measuring current in Fig. 132 (*b*). The poten-

[1] See DAWES, "A Course in Electrical Engineering," Vol. I, p. 153.

tiometer wire consists of 22 100-ohm resistances mounted between contactor points, and a 100-ohm circular slide wire connected in series. Since the voltage-drop between adjacent



(*a*) Interior wiring diagram of student's potentiometer.



(*b*) Wiring diagram of potentiometer and its accessories.

FIG. 132.

contactor points is 0.1 volt when the instrument is in adjustment, the working current is $\frac{0.1}{100}$ or 0.001 amp. (1 milliampere). Since the resistance of the slide wire is 100 ohms, it has a total range of 0.1 volt, and fractional parts of 0.1 volt

may be read on the dial (Fig. 132 (*b*)). The range of the instrument is from 0 to 2.3 volts.

The working current from the positive battery terminal is led in at the *Ba* + terminal. It then flows through the 22 100-ohm resistance coils and the slide wire. It passes out at the *Rheo* terminal, flows through the regulating rheostat to the negative terminal of the battery. The two *E.M.F.* terminals are connected to the two dial-switches. The left-hand contactor gives a total range of 2.2 volts in 0.1-volt divisions and the slide wire gives the fractional parts of 0.1 volt.

The accessories consist of two dry cells in series for supplying the working current, a regulating rheostat, a standard cell, and a galvanometer and key. A double-pole, double-throw (D.-P. D.-T.) switch connects the *E.M.F.* terminals to either the standard cell or the electromotive force to be measured. A protective resistance may be connected in series with the standard cell to prevent it from delivering too much current when the potentiometer is out of balance.

To use the potentiometer, the two dials are set to read the electromotive force of the standard cell, and the switch is then thrown to the standard-cell side. The regulating rheostat is next adjusted until the galvanometer shows no deflection. The switch is then thrown to the *E.M.F.* side and the unknown electromotive force is balanced against a corresponding voltage-drop in the slide wire. The unknown electromotive force is read directly on the two dials. For example, if the dial switch is on 1.6 volts and the slide wire is at 0.0612 volt, the electromotive force is 1.6612 volts. The working current should be checked frequently by throwing the switch to the standard-cell position and rebalancing. This potentiometer of itself cannot measure voltages in excess of 2.3 volts.

121. Voltage Measurements with the Potentiometer.—The maximum range of a potentiometer is ordinarily from 1.6 to 2.3 volts. For the measurement of potentials in excess of these values, a *volt-box* is necessary. A volt-box is merely a very high resistance from which suitable taps are brought. This is illustrated by the resistance *AD* (Fig. 133). Assume *AD* to have a resistance of 10,000 ohms and *AB* a resistance of 100 ohms. *If no current leaves the wire at B*, the voltage drop across *AB* will be $\frac{100}{10{,}000}$

$= \frac{1}{100}$ of that across AD. If leads be carried from AB to the potentiometer, the potentiometer will measure $\frac{1}{100}$ of the voltage across AD, since no current is taken from B, the potentiometer principle being an opposition method. Therefore, when a voltmeter V is being calibrated, it should be connected in parallel with AD. If the voltmeter reads 113.8 volts and the potentiometer reads 1.142 volts, the true line voltage across the voltmeter will be $1.142 \times 100 = 114.2$ volts. Therefore, the correction to the voltmeter is +0.4 volt.

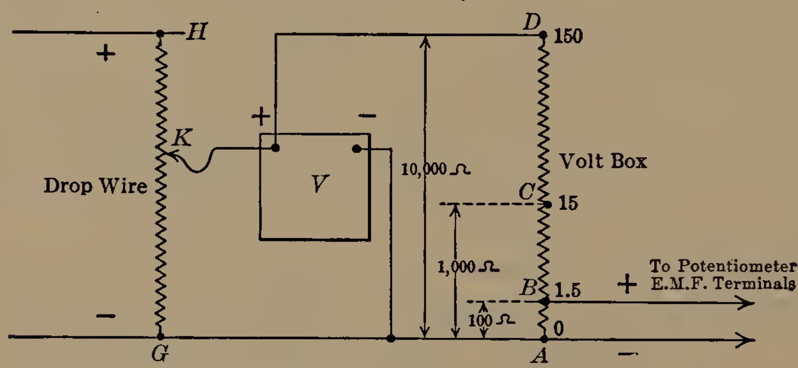


FIG. 133.—Volt-box and drop-wire connections.

In a similar manner, voltages from 1.5 to 15 volts are connected across AC, the multiplying factor in this case being 10.

The Drop-wire.—GH is a resistance connected directly across the line. One voltmeter terminal and one terminal of the volt-box are connected to the end G of this wire. The other terminal of the voltmeter and the remaining terminal of the volt-box are connected to a movable contact K. By sliding K along GH, any desired voltage up to that across the drop-wire, may be obtained. When used in this manner, GH is called a *drop-wire* or a *potentiometer wire.* It is not necessary to the operation of the volt-box, and is merely a convenient means for adjusting the voltage.

122. Measurement of Current with the Potentiometer.—A potentiometer is designed primarily to measure *voltage.* It may also be used to measure current by applying Ohm's law. Let an unknown current I flow through a known resistance R. If E, the voltage-drop across R, be measured, the current I is

immediately determined, since, for this part of the circuit, both the voltage and the resistance are known. Therefore

$$I = \frac{E}{R}.$$

The method of making the measurement is shown in Fig. 134. In order to determine the errors of the ammeter, it is necessary

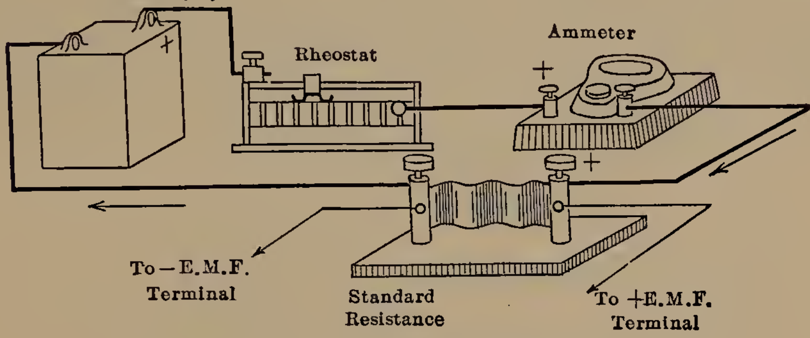


FIG. 134.—Connections for calibrating an ammeter.

to know the exact current passing through the instrument. The ammeter is connected in series with a battery, the standard resistance, and a rheostat for controlling the current. Standard resistances are provided with four terminals as a rule, two heavy ones for current and two smaller binding posts for potential (see Fig. 135 and also Fig. 121, page 128). The two potential binding posts are connected to the potentiometer, the proper polarity being observed. The voltage across the standard resistance is then measured by means of the potentiometer.

0.01 ohm.

FIG. 135.—Standard resistance.

Standard resistances are usually adjusted to decimal values, such as 10, 1, 0.1, 0.01, etc. ohms. They are ordinarily rated to carry a current that will give 1.0-volt drop. Thus, the 1 ohm can carry 1 amp., the 0.001 ohm, 1,000 amp., etc. To keep the resistances cool, they are often immersed in oil.

Knowing that the potentiometer is limited to 2.3 volts, it is a simple matter to select the proper standard resistance.

An instrument having a range of 100 amp. would require $\frac{2.3}{100} = 0.023$ ohm, approximately; 0.01 ohm would be used. Likewise, a 15-scale instrument would require $\frac{2.3}{15} = 0.15$ ohm, approximately. A 0.1 ohm resistance would be used.

When instruments are calibrated, they should be checked at ten or fifteen different points throughout the scale. The corrections are plotted as ordinates with the corresponding scale readings as abscissas. (The instrument readings are plotted as

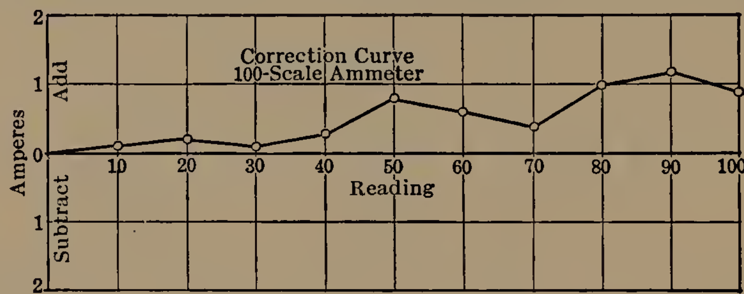


FIG. 136.—Ammeter calibration curve.

abscissas.) As an instrument scale is subject to scale errors, etc., it is customary to connect successive points of the plot, by straight lines, as shown in Fig. 136. For example, in Fig. 136, the correct current when the instrument reads 50 amp. is 50 + 0.8 = 50.8 amp.

MEASUREMENT OF POWER

123. The Voltmeter-ammeter Method.—Since power is equal to the product of volts and amperes (see page 61), the power taken by a circuit or a device may ordinarily be measured by an ammeter in series and a voltmeter in parallel with the circuit. The power is the product of the two readings. By using calibrated instruments, the power may be measured to one-half of 1 per cent. With commercial uncalibrated instruments, the precision is less, usually from 1 to 2 per cent.

124. The Wattmeter.—The wattmeter measures power directly. It consists of fixed coils *FF* and a pivoted coil *M*, free to turn within the magnetic field produced by coils *FF*, as shown in Fig. 137. The coils *FF* are wound with comparatively few turns of

wire and are capable of carrying the entire current of the circuit. The moving coil M is wound with very fine wire and the current is led into it through two control springs in the same manner that current is led into the coil of a Weston instrument. The fixed coil is connected in series with the load in the same manner as an ammeter is connected. The moving coil is connected across the line in series with a high resistance R in the same manner as a voltmeter coil is ordinarily connected.

The field of the coils FF is proportional to the current and the current in the coil M is proportional to the voltage. Therefore,

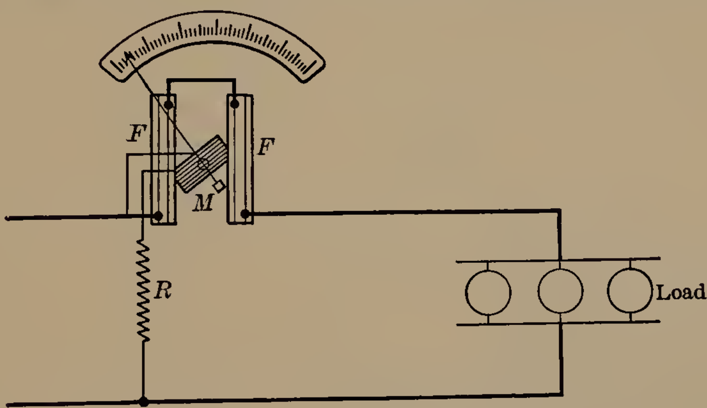


FIG. 137.—The indicating wattmeter.

the turning moment is proportional to the power of the circuit and also depends on the angular position of M with respect to FF, which is taken into consideration when the scale is marked.

Owing to the high degree of accuracy obtainable by the use of the voltmeter and ammeter, the wattmeter is seldom used for direct-current measurements. As it is affected by stray fields, reversed readings should be taken, that is, both the current and voltage should be reversed and the average of the two readings used. The wattmeter is used more extensively for alternating current than for direct current. A more complete description, together with its uses, is found in Part II.

MEASUREMENT OF ENERGY

125. The Watthour Meter.—The watthour meter is a device for measuring *energy* (see page 63). As energy is the product of power and time, the watthour meter must take into considera-

tion both of these factors. As power is usually sold on an energy basis, many dollars may depend upon the accuracy of such a meter. Therefore, a proper understanding of its mechanism and the method of adjustment is very often essential.

In principle, the watthour meter is a small motor whose instantaneous speed is proportional to the *power* passing through it, and whose total revolutions in a given time are proportional to the total *energy* or watt-hours delivered during that time.

Referring to Fig. 138, the line is connected to two terminals on the left-hand side of the meter. The upper terminal is connected to two coils *FF* in series. These coils are wound with

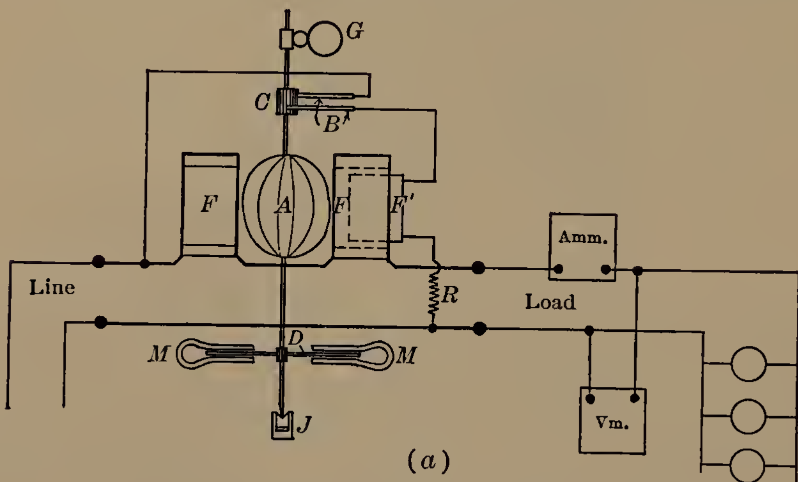


FIG. 138.—Connections of the watthour meter.

wire sufficiently heavy to carry the maximum current taken by the load and are so connected that their magnetic fields act in conjunction. The armature rotates in the magnetic field so produced.

The other line wire runs straight through the meter to the load. A shunt circuit is tapped to the upper line on the left-hand side. It runs first to the armature, through the silver brushes *B*, which rest on the small commutator *C*. From the brushes the line passes through coil *F'*, and through a resistance *R* to the lower line wire. This resistance *R* is omitted in certain types of meters.

As the load current passes through *FF*, and there is no iron in circuit, the magnetic field produced by these coils is proportional to the *load current*. As the armature, in series with

resistance, is connected directly across the line, the current in the meter armature is proportional to the *line voltage.* Neglecting the small voltage-drop in *FF*, the torque acting on the armature must be proportional to the product of the load current and the load voltage or, in other words, proportional to the power passing through the meter to the load.

It can be proved that, if the meter is to register correctly, there must be a retarding torque acting on the moving element which is proportional to its speed of rotation. To meet this condition, an aluminum disc *D* is pressed on the motor shaft. This disc rotates between the poles of two permanent magnets *MM*. In cutting the field produced by these magnets, eddy currents are set up in the disc, retarding its motion. As the strength of these currents is proportional to the velocity of the disc and they are acting in conjunction with a magnetic field of constant strength, their retarding effect is proportional to the speed of rotation.

Fig. 139.—Direct-current watthour meter with cover removed.

Friction cannot be entirely eliminated in the rotating element. Near the rated load of the meter the effect of the friction torque is practically negligible, but at light loads the friction torque, which is nearly a constant at all loads, is a much greater proportion of the load torque. As the ordinary meter may operate at light loads during a considerable portion of the time, it is desirable that the error due to friction be eliminated. This is accomplished by means of coil *F′* connected in series with the armature. *F′* is so connected that its field acts in the same direction as that due to coils *FF*. Therefore, it assists the armature *A* to rotate. Being connected in the shunt circuit, it is acting continuously. The coil is movable and its position can be so adjusted that the friction error is just compensated.

To reduce friction and wear, the rotating element of the meter is made as light as possible. The element rests on a jewel bearing *J*, which is a sapphire in the smaller sizes and a diamond in the

heavier types of meter. The jewel is supported on a spring. A hardened steel pivot rests in the jewel. In time, the pivot becomes dulled and the jewel roughened, which increases friction and causes the meter to run more slowly unless F' is readjusted. The moving element turns the clockwork of the meter dials through a worm and the gears G.

Figure 139 shows the interior of a Thomson watthour meter.

126. Adjustment of the Watthour Meter.—Even if the initial adjustment be accurate, the registration of a watthour meter may, in time, become incorrect. This is due to many causes, such as pitting of the commutator, roughening of the jewel, wear on the pivot, change in the strength of the retarding magnets, etc. As the cost of energy to consumers is largely based on the registration of such meters, it is important that they be kept in adjustment, as a small error in the larger sizes may ultimately mean a difference of many dollars one way or the other.

To adjust the meter, it may be loaded as shown in Fig. 138. The power taken by the load is measured by a calibrated voltmeter and ammeter. The revolutions of the disc D are counted over a period of time which is measured with a stop watch. The relation between watt-hours and the revolutions of the *disc*, in most meters, is as follows:

$$\frac{W \times t}{3{,}600} = K \times N \qquad (55)$$

where W is in watts, t is in seconds, K is the meter "constant," and N is the revolutions of the disc in time t.

When the meter is tested, the voltmeter and ammeter are read intermittently while the revolutions of the disc are being counted. A run of about a minute gives good results.

Let the average watts determined from the corrected voltmeter and ammeter readings be W_1.

The average watts, as indicated by the meter during the same period, are, from (55),

$$W = \frac{K \times N \times 3{,}600}{t}. \qquad (56)$$

The per cent. accuracy of the meter is

$$\frac{100\ W}{W_1}.$$

Example.—In the test of a 10-amp. watthour meter, having a constant of 0.4, the disc makes 40 revolutions in 53.6 sec. The average volts and amperes during this period are 116 volts and 9.4 amp. What is the per cent. accuracy of the meter at this load?

Average standard watts $W_1 = 116 \times 9.4 = 1{,}090.$

Average meter watts, from (56),

$$W = \frac{0.4 \times 40 \times 3{,}600}{53.6} = 1{,}074$$

$$\text{Per cent. accuracy} = \frac{100 \times 1{,}074}{1{,}090} = 98.5. \quad \textit{Ans.}$$

This means that the meter is 1.5 per cent. slow and should be speeded up slightly. With calibrated indicating instruments and careful adjustment, a meter may easily be brought within 0.5 per cent. of accurate registration.

There are two adjustments to be made. Near full load, if the meter is running slow, the magnets are moved nearer the

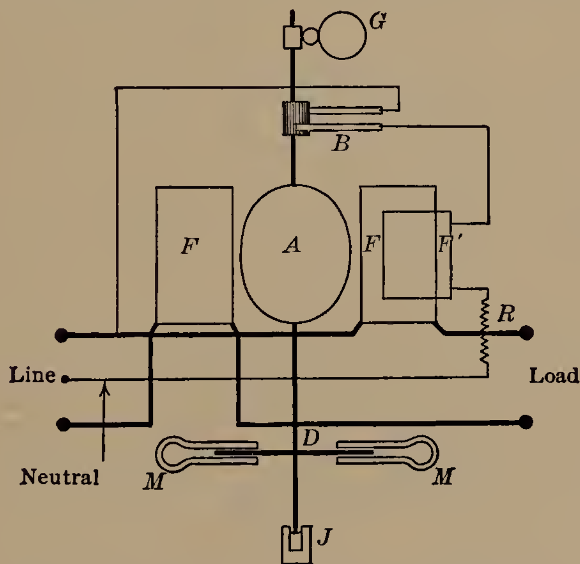


FIG. 140.—Diagram of a 3-wire watthour meter.

center of the disc where the effect of the retarding currents is reduced and, if the meter is running fast, the magnets are moved farther from the center. If the meter has been correctly adjusted near full load and is found to be in error near light load, the error is obviously due to friction. The light load adjustment (made at from 5 to 10 per cent. rated load) is effected by moving the friction-compensating coil F'. If the meter is slow, the coil F' is

moved in nearer the armature and, if the meter is fast, it is pulled out farther from the armature. This adjustment of F' may affect the full-load adjustment slightly so that the meter should be rechecked at full load and then again at light load.

127. The Three-wire Watthour Meter.—The three-wire meter is designed to register energy on a three-wire system. It does not differ materially from the meter shown in Fig. 138, except that the two coils *FF* are connected in opposite sides of the line, as shown in Fig. 140. The armature circuit may be connected to the neutral as shown, or it may be connected across the outer wires. If this latter connection is used, the neutral connection to the meter is omitted. In the former case the meter does not register accurately unless the voltages between the two outer lines and neutral are equal, although the error is usually small.

CHAPTER VIII

THE MAGNETIC CIRCUIT

The general nature of magnetic phenomena was considered in Chaps. I and II, though the quantitative relations existing among the magnetic flux, the ampere-turns, and the magnetic circuit itself were not given. As with the electric circuit, it is possible to determine the values of magnetic quantities in a circuit, provided sufficient data are available.

128. The Magnetic Circuit.—The magnetic circuit resembles the electric circuit in many ways. For example, magnetic flux is equal to the magnetomotive force divided by the magnetic resistance (reluctance); drop in magnetic potential is equal to the magnetic flux multiplied by the reluctance, etc. Also, Kirchhoff's two laws are applicable to the magnetic circuit as well as to the electric circuit.

On the other hand, magnetic calculations and measurements cannot be made with the same high degree of precision with which electric calculations can be made. Electric currents are usually confined to definite paths whose geometry is accurately known. Magnetic flux cannot be confined to definite paths, since there is no known insulator for magnetic flux. Air itself is a fairly good magnetic conductor.

Magnetic paths are ordinarily short and have large cross-sections as compared with their lengths. Their geometry is usually not simple. Again, the reluctance of iron and steel varies over wide limits depending both on its previous magnetic condition and also on the flux density. The reluctance of a given piece of steel may increase from 50 to 100 times with increase of flux density.

All the foregoing factors make it extremely difficult to calculate magnetic quantities with a high degree of precision.

MAGNETIC UNITS

129. Ampere-turns (*IN*).—The ampere-turns acting on a circuit are given by the product of the turns linked with the circuit and the amperes flowing through these turns. For example, 10 amp. flowing through 150 turns give 1,500 amp.-turns. The same result is produced by 15 amp. flowing through 100 turns. If any ampere-turns act in opposition, they must be subtracted.

Magnetomotive Force (m.m.f. also F).—Magnetomotive force tends to drive the flux through the circuit and corresponds to electromotive force in the electric circuit. It is directly proportional to the ampere-turns of the circuit and differs from the value of the ampere-turns by the constant factor $0.4\pi = 1.257$. That is, $F = 0.4\pi IN = 1.257IN$.

The magnetomotive force of a circuit is measured by the work done in carrying a unit north pole through the entire circuit.

The unit of magnetomotive force is the *gilbert*. The gilberts acting on a circuit are obtained by multiplying the ampere-turns by 0.4π or 1.257.

Reluctance ($\mathcal{R}$).—Reluctance is resistance to the passage of magnetic flux and corresponds to resistance in the electric circuit. The unit of reluctance is that of a centimeter-cube of air. This unit is called the *oersted*.

Permeance ($\mathcal{P}$).—The permeance of a circuit is the reciprocal of the reluctance $\left(\mathcal{P} = \frac{1}{\mathcal{R}}\right)$ and may be defined as that property of the circuit which permits the passage of magnetic flux or of lines of induction. Permeance corresponds to conductance in the electric circuit.

Permeability (μ).—The permeability of a material is the ratio of the flux or of the number of lines of induction existing in that material to the flux or the number of lines of induction which would exist if that material were replaced by air, the m.m.f. acting on this part of the circuit remaining unchanged. The permeability of air is taken as unity and with the exception of iron, steel, nickel, liquid oxygen, and certain iron oxides, practically all materials may be considered as having a permeability of unity. The permeability of commercial iron and steel ranges from 50 and even lower to about 2,000. In special investiga-

tions, vacuum-treated iron has attained a permeability of 5,000 and even greater.

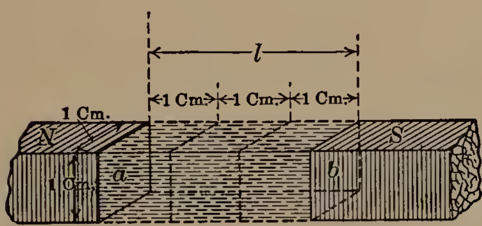
Flux (ϕ).—The magnetic flux is equal to the total number of lines of induction existing in the circuit and corresponds to current in the electric circuit. The unit of flux is the *maxwell*, but "line of induction" or simply "line" is more often used.

Flux Density (B).—The flux density is the number of maxwells or of lines of induction per unit area, the area being taken at right angles to the direction of the flux. The unit of flux density in the c.g.s. system is one line per square centimeter and is called the *gauss*. Flux density is usually expressed in "lines per square centimeter" or "lines per square inch."

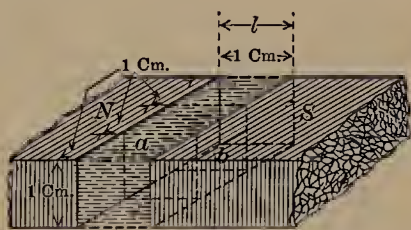
$$B = \frac{\phi}{A}$$

where A is the area and ϕ the flux through and normal to this area.

130. Reluctance of the Magnetic Circuit.—The unit reluctance is defined as that of a centimeter-cube of air. If a portion of a magnetic circuit between pole-faces a and b (Fig. 141 (a))[1] con-



(a) Path whose reluctance is 3 oersteds.



(b) Path whose reluctance is ⅓ oersted.

Fig. 141.—Reluctance of simple magnetic paths.

sists of a path in air having a length of 3 cm. and a cross-section of 1 sq. cm., as shown in the figure, this path is equivalent to 3 cm.-cubes placed in series. As the total flux must pass through each cube in turn, it is evident that the total reluctance is 3 units (oersteds). The reluctance is proportional to the length of the flux path.

On the other hand, if the path has a length of 1 cm. and a cross-section of 3 sq. cm., as shown in Fig. 141 (b), the reluctance

[1] The actual flux path between pole-faces would not be as shown in Fig. 141 (a), but the flux would "fringe," as shown in Figs. 13 (b) and 14, page 13.

of the path through which the flux passes is one-third that of one cube alone, or ⅓ oersted. The reluctance is inversely proportional to the cross-section of the path.

Moreover, if these paths were in iron, having a permeability μ, the flux would be μ times its value in air, provided the same magnetomotive force were maintained between the two pole-faces. This means a lower reluctance. *The reluctance of any portion of a magnetic circuit is proportional to its length, inversely proportional to its cross-section and inversely proportional to the permeability of the material.* The constant of proportionality is unity, since the reluctance of a path in air *one* centimeter long and *one* square centimeter cross-section is *one* oersted. Hence,

$$\mathcal{R}_1 = \frac{l_1}{A_1\mu_1} \tag{57}$$

where l_1 is the length in centimeters of that part of the circuit under consideration; A_1 is the uniform cross-section in square centimeters of that portion of the circuit; and μ_1 is the permeability of that portion of the circuit.

Example.—A portion of a magnetic circuit consists of a cylindrical steel casting 3.0 in. (7.62 cm.) diameter, 8.0 in. (20.3 cm.) long, whose permeability is 1,300 at the flux density at which it is to operate. Find the reluctance and the permeance of this portion of the circuit.

The cross-section,

$$A = \frac{\pi(7.62)^2}{4} = 45.6 \text{ sq. cm.}$$

The reluctance,

$$\mathcal{R} = \frac{20.3}{45.6 \times 1{,}300} = 0.000342 \text{ oersted.}$$

The permeance,

$$\mathcal{P} = \frac{1}{0.000342} = 2{,}920 \text{ units.}$$

If a magnetic circuit consists of several parts in series (Fig. 142), the total reluctance is:

$$\begin{aligned} \mathcal{R} &= \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4 \qquad (58) \\ &= \frac{l_1}{A_1\mu_1} + \frac{l_2}{A_2\mu_2} + \frac{l_3}{A_3\mu_3} + \frac{l_4}{A_4\mu_4}. \end{aligned}$$

Permeances in parallel are added together to find the total permeance, just as conductances in parallel are added together to find the total conductance.

The total permeance

$$\mathcal{P} = \mathcal{P}_1 + \mathcal{P}_2 + \mathcal{P}_3 + \mathcal{P}_4 + \ . \ . \ . \qquad (59)$$

Reluctances in parallel are combined in the same manner as resistances in parallel.

$$\frac{1}{\mathcal{R}} = \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3} + \frac{1}{\mathcal{R}} + \ . \ . \ . \qquad (60)$$

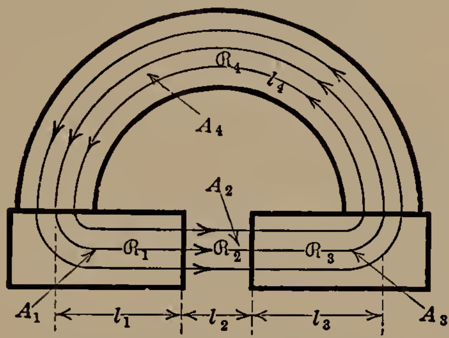


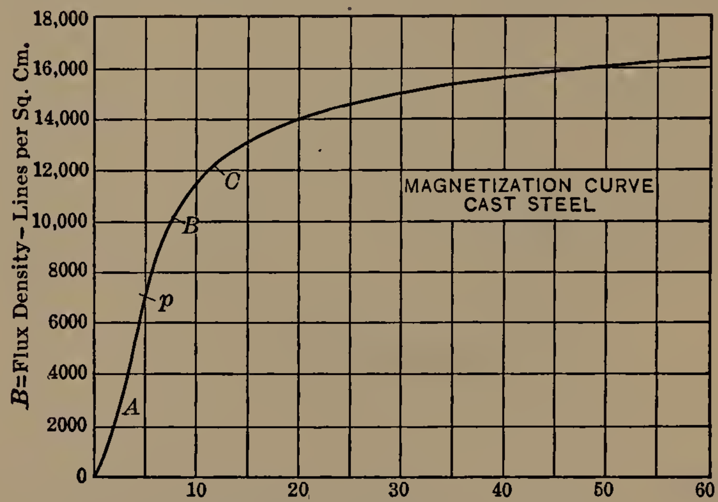
FIG. 142.—Reluctances in series.

131. Permeability of Iron and Steel.—The permeability of iron and steel depends on the quality of the material, the flux density, and the previous magnetic history.

If the permeability of iron and steel were constant, a graph plotted with flux density B as ordinates and magnetomotive force per unit length H as abscissas would be a straight line, since the flux density would always be proportional to the magnetomotive force per unit length (see equation (61), page 161). As the permeability varies over wide ranges, the relation of the flux density B to the magnetomotive force per unit length H is not a straight line, but a curve (Fig. 143) whose equation is not simple. A curve which shows this relation of flux density to magnetomotive force per unit length is called a *magnetization curve.* Figure 143 gives the magnetization curve for one grade of cast steel. Magnetomotive force gradient in gilberts per centimeter H is plotted as abscissa and the corresponding flux density in gausses B is plotted as ordinate. The gilberts per centimeter are equal to $0.4\pi nI$, where nI is equal to the ampere-turns per centimeter length of the material.

From the origin to point A, the curve concaves slightly upwards. From A to B the curve approaches a straight line. Beyond B the flux density increases much less rapidly for a given increase

in magnetomotive force per unit length and the steel approaches saturation. The point C, where the bend in the curve is very *pronounced*, is the "knee of the curve." Beyond C the flux can be increased but slightly, even with a great increase in the



H = Gilberts per centimeter = $0.4\ \pi nI$.

FIG. 143.—Magnetization curve for cast steel.

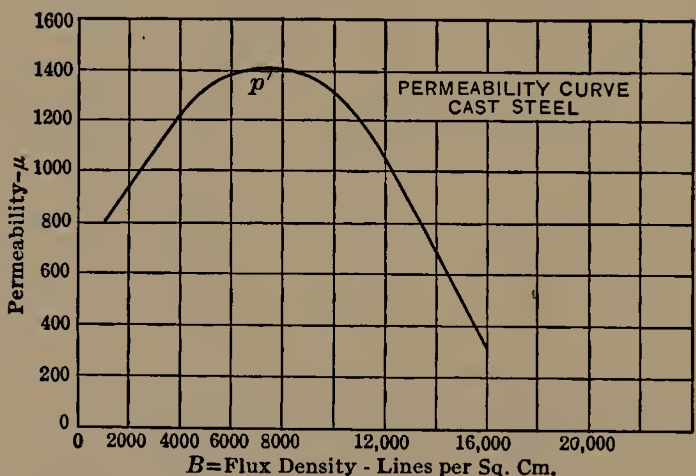


FIG. 144.—Permeability curve for cast steel.

magnetomotive force. The steel is then said to be *saturated.* The type of curve shown in Fig. 143 is called the *normal* saturation or induction curve.

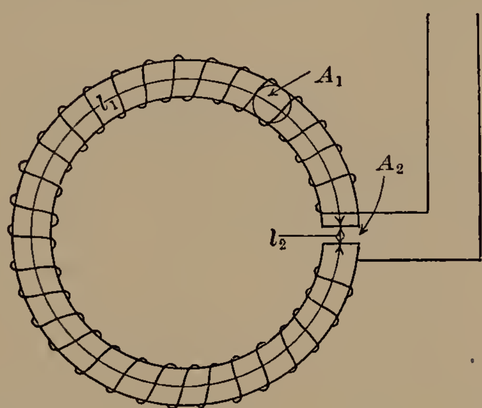
Figure 144 shows the permeability curve for this same steel. Each ordinate is obtained by dividing B by H for each point of

the curve in Fig. 143. It will be noted that the permeability varies over a wide range. It begins at a comparatively low value, increases to a maximum at the point p, and then decreases to about one-fifth its maximum value.

132. Law of the Magnetic Circuit.—The relation among flux, magnetomotive force, and reluctance, for the magnetic circuit, is identical with the relation among current, electromotive force, and resistance for the electric circuit.

$$\phi = \frac{F}{\mathcal{R}} \tag{61}$$

The flux is directly proportional to the magnetomotive force and inversely proportional to the reluctance of the circuit.



l_1 = 18.0 in. (45.7 cm.), $A_1 = A_2$ = 0.20 sq. in. (1.29 sq. cm.), l_2 = 3/16 in. (0.476 cm.)

FIG. 145.—Ring-type electromagnet.

If the magnetic circuit consists of several distinct parts having reluctances $\mathcal{R}_1$, $\mathcal{R}_2$, etc. in series, and magnetomotive forces F_1, F_2, etc., then, from equations (58) and (61),

$$\phi = \frac{F_1 + F_2 + F_3 + \cdots}{\dfrac{l_1}{A_1\mu_1} + \dfrac{l_2}{A_2\mu_2} + \dfrac{l_3}{A_3\mu_3} + \cdots} = \frac{0.4\pi(I_1N_1 + I_2N_2 + I_3N_3 + \cdots)}{\dfrac{l_1}{A_1\mu_1} + \dfrac{l_2}{A_2\mu_2} + \dfrac{l_3}{A_3\mu_3} + \cdots} \tag{62}$$

Example.—The ring magnet (Fig. 145), is wound with 250 turns of wire, through which a current of 1.5 amp. flows. Assume the permeability of

the iron to be 800. Neglecting fringing, determine the flux in the ring, and also the flux density.

$$F = 0.4\pi \times 1.5 \times 250 = 471 \text{ gilberts}$$
$$l_1 = 18 \text{ in.} = 18 \times 2.54 = 45.7 \text{ cm.}$$
$$l_2 = 3/16 \text{ in.} = 3/16 \times 2.54 = 0.476 \text{ cm.}$$
$$A_1 = A_2 = 0.2 \text{ sq. in.} = 0.2 \times 2.54 \times 2.54 = 1.29 \text{ sq. cm.}$$

From equation (62),

$$\phi = \frac{471}{\frac{45.7}{1.29 \times 800} + \frac{0.476}{1.29 \times 1.0}} = \frac{471}{0.0443 + 0.369} = 1{,}140 \text{ lines (maxwells).}$$

Ans.

The flux density

$$B = \frac{1{,}140}{1.29} = 884 \text{ lines per square centimeter (gausses)}$$

or 5,700 lines per square inch. *Ans.*

133. Magnetization Curves for Standard Iron and Steel.—Typical magnetization curves for commercial iron and steel are

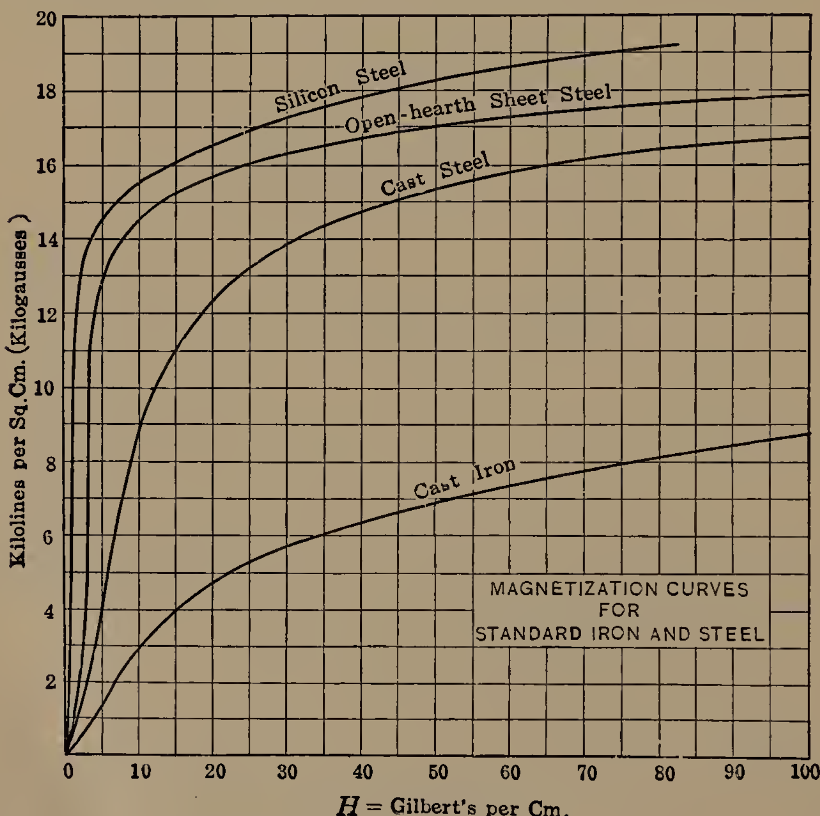


Fig. 146.—Typical magnetization curves.

shown in Fig. 146. It will be noted that silicon steel has the best magnetic properties, although open-hearth sheet steel is

only slightly inferior. The eddy-current and hysteresis losses per unit volume in the silicon steel are so much less than in the open-hearth sheet steel that silicon steel is used almost exclusively for transformer cores (see Part II), where the iron is subjected to rapid reversals of flux direction. On the other hand, open-hearth sheet steel is used for dynamo cores (see page 224), because it is cheaper than silicon steel and the iron losses are of less importance than in transformers. Cast iron has less than half the permeability of cast steel, and can be used only where weight is not important.

The values of B (Fig. 146) are found by multiplying the ordinates by 1,000.

The permeability for any given flux density is found by dividing the value of B at that point by the corresponding value of H, as described on page 160.

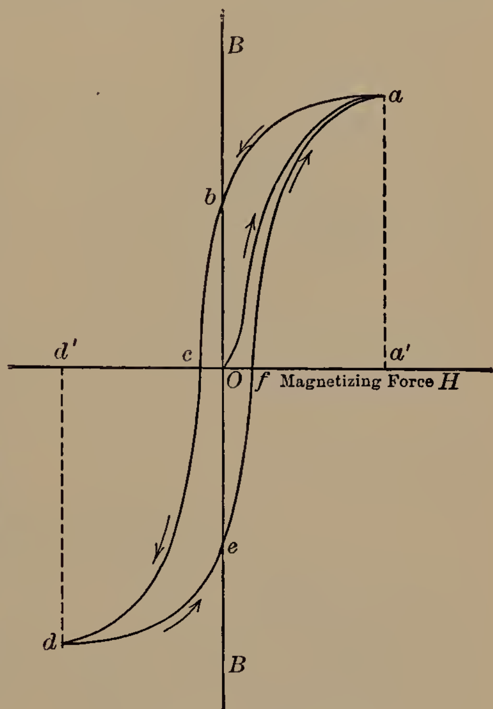


FIG. 147.—Hysteresis loop.

134. Hysteresis.—If the magnetomotive force acting on an iron sample forming a closed magnetic circuit begins at zero and increases, the relation between magnetomotive force gradient and flux density will be similar to that shown by curve Oa (Fig. 147). This curve is called the *normal saturation* or magnetization curve and has already been discussed.

If the magnetomotive force now decreases, the flux will *not* decrease along the line aO, but will decrease less rapidly along ab. When point b is reached, the magnetomotive force is zero, but the magnetic induction has not reached zero. The flux density Ob is called the *remanence*. Before the flux density can be reduced to zero, the magnetizing force must be reversed in direction. That is, it requires a negative magnetizing force Oc to reduce the flux density to zero. The magnetizing force Oc is called the *coercive force*.

If now the magnetizing force be increased in the negative direction to d', where $Od' = Oa'$, the flux density will be carried to a negative maximum $d'd$. The negative maximum flux density $d'd$ is equal to $a'a$. If the magnetizing force is now increased toward zero, the curve will pass through point e when the magnetizing force is again zero and the negative remanence $Oe = Ob$. A positive coercive force $Of = Oc$ is necessary to bring the flux density again to zero. When the magnetizing force again becomes Oa' the flux density will return to its original maximum value at a, closing the loop.

In Fig. 147, it will be noted that the magnetization *lags* behind the magnetizing force. Starting with any degree of magnetization, if the magnetizing force is increased and then decreased, or if the magnetizing force is decreased and then increased, it will be found that the values of magnetization which are associated with increasing values of the magnetizing force are different from the values which are associated with corresponding decreasing values of the magnetizing force. That is, with iron, the magnetization *lags* behind the magnetizing force. For example, at point b the magnetizing force is zero and the flux does not become zero until later at point c. This tendency to lag is called *hysteresis*, which means "to lag."

The cycle of magnetization shown in Fig. 147 is called a *hysteresis loop*.

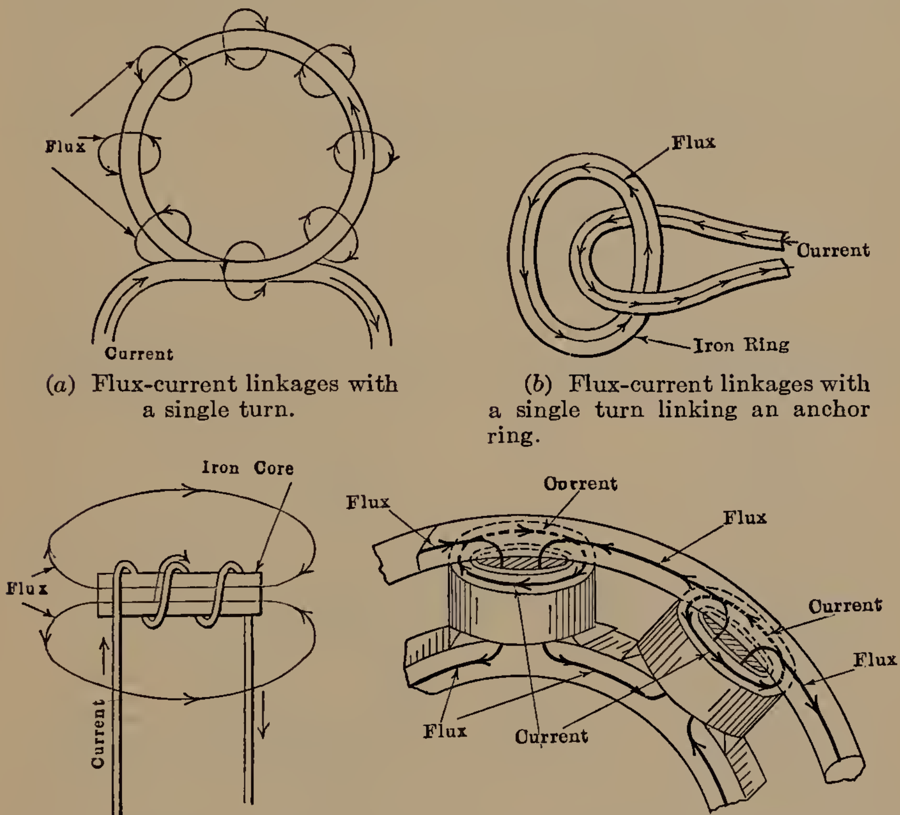
135. Hysteresis Loss.—The lag of magnetization behind the magnetizing force is attributed to molecular friction (see Weber's theory, page 3). Therefore, it must require an expenditure of energy to carry the iron through a cycle. The expenditure of energy per cycle is proportional to the area of the hysteresis loop. The energy involved is called *hysteresis loss*. In silicon steel, this loss is less than in ordinary steel, which is one reason for using silicon steel for transformer cores.

INDUCTANCE

136. Magnetic Linkages.—If a current flows in a conductor, a magnetic flux is set up about the conductor. *This magnetic flux completely encircles the conductor and the current in the conductor completely encircles the flux* (see Chap. II, pages 17 to 23). Some familiar examples of this are given in Fig. 148, where the

currents and related fluxes are shown. As a current and the resulting flux always completely encircle each other, they are said to link with each other. This is shown particularly well in Fig. 148 (*b*), where a conductor carrying a current is linked with an anchor ring.

The product of the turns of conductor and the number of lines of flux linking these turns is called the *linkages* of the circuit.



(*a*) Flux-current linkages with a single turn.

(*b*) Flux-current linkages with a single turn linking an anchor ring.

(*c*) Flux-current linkages with an iron core solenoid.

(*d*) Flux-current linkages in the magnetic circuit of a dynamo.

FIG. 148.—Typical examples of flux-current linkages.

Example.—There are two exciting coils, each of 700 turns, on a certain electromagnet. When the current is 8.0 amp. the flux in the magnetic circuit is 3,000,000 maxwells. What are the magnetic linkages?

$$1{,}400 \times 3{,}000{,}000 = 4.2 \times 10^9 \text{ ampere-maxwell linkages. } Ans.$$

The number of these *linkages per unit current* in a circuit is called the self-inductance of the circuit and is represented by the symbol L, implying linkages. The unit of self-inductance is the

henry, which is defined as "*the induction in a circuit when the electromotive force induced in this circuit is one international volt, while the inducing current varies at the rate of one ampere per second.*"

Self-inductance from definition:

$$L = \frac{N\phi}{I \times 10^8} \tag{63}$$

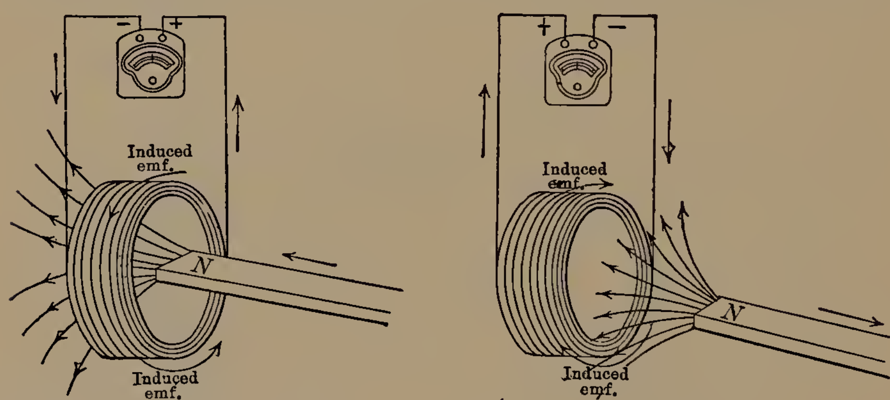
where L is the self-inductance in henrys, ϕ is the flux in maxwells, and I is the current in amperes.

Note.—It is necessary to divide by 10^8 because 10^8 c.g.s. magnetic lines or maxwells are equal to one line or weber in the practical system of volts, amperes, etc.

Example.—What is the self-inductance of the above circuit?

$$L = \frac{4.2 \times 10^9}{8.0 \times 10^8} = 5.25 \text{ henrys.} \quad \textit{Ans.}$$

137. Induced Electromotive Force.—If the terminals of an insulated coil (Fig. 149 (*a*)), be connected to a galvanometer, and a



(*a*) North pole inserted in coil. (*b*) North pole withdrawn.

Fig. 149.—Induced electromotive force.

magnetic field be set up through this coil, either by thrusting a bar magnet into the coil or by some other means, the galvanometer will be observed to deflect momentarily and then to return to its zero. This shows that an electromotive force has been temporarily induced in the coil. When the flux through the coil has ceased to change, this electromotive force also ceases. If investigation be made, it will be found that the direction of this induced electromotive force is that shown in the figure and that

this direction is such that if the electromotive force be allowed to produce a current, this current will tend to push the bar magnet *out* of the coil, or, what is equivalent, will oppose its entering the coil. A study of Fig. 149 (a) shows that the induced current flows in a counter-clockwise direction. Hence it produces a magnetomotive force acting outwards through the center of the coil, and therefore opposes the entrance of the N-pole of the bar magnet.

If the magnet be withdrawn from the coil (Fig. 149 (b)), the galvanometer will be observed to deflect again, momentarily as before, but the deflection is opposite to its direction in the first case. The direction of the induced electromotive force is now such that if it causes current to flow, this current will tend to prevent the magnet from being withdrawn from the coil. That is, in Fig. 149 (b), the current would flow in a clockwise direction and produce a magnetomotive force, hence a flux, whose direction is inward. Since a north pole tends to move in the direction of the lines of force, this magnetomotive force will oppose the withdrawal of the bar magnet. The electromotive force in each case is transient and ceases when the *change* of flux through the coil ceases.

If careful measurements be made, the value of this electromotive force will be found to depend on: (1) the number of turns in the coil, (2) the rate at which the flux linked with the coil changes.

The average electromotive force in volts is given by

$$e = -\frac{N\phi\,10^{-8}}{t} \qquad (64)$$

where N is the number of turns in the coil, ϕ is the total *change* of flux linked with the coil, and t is the time in seconds required to produce this change of flux. 10^{-8} reduces the flux ϕ to practical units, so that e is given in volts. The minus sign indicates that the induced electromotive force acts in *opposition* to the effect which produces it.

$\dfrac{\phi}{t}$ is the average rate of change of flux, so that the induced electromotive force may be said to be proportional to the *number of turns* and the *rate of change of flux* linked with these turns.

Example.—When an armature coil of a direct-current generator is directly under a north pole, 2,400,000 maxwells link the coil. When the coil has moved to such a position that its center line is midway between poles, the

flux linking it is zero. The coil has eight turns and the time required for the coil to move half the distance between poles is $\frac{1}{180}$ sec. What is the average electromotive force induced in the coil? (See Fig. 181, page 206.)

Applying equation (64),

$$e = -\frac{8 \times 2{,}400{,}000 \times 10^{-8}}{\frac{1}{180}}$$

$$= -8 \times (2.4 \times 10^{6}) \times (1.8 \times 10^{2}) \times 10^{-8} = -34.6 \text{ volts. } \textit{Ans.}$$

The fact that the currents produced by induction *oppose* the motion producing them should be carefully noted, for this principle is manifest in practically all types of electric machinery. This principle was first formulated by Lenz, in a form known as Lenz's law, which says:

"*In all cases of electromagnetic induction, the induced currents have such a direction that their reaction tends to stop the motion which produces them.*"

This law is also based upon the law of the conservation of energy. That is, the induced currents, which represent energy, are produced at the expense of the mechanical energy required to push the magnet into the coil against their opposition, or the energy required to withdraw the magnet against the opposition of the induced currents, which try to prevent this withdrawal.

138. Electromotive Force of Self-induction.—If a coil be connected to a battery and a switch *S* closed (Fig. 150), current

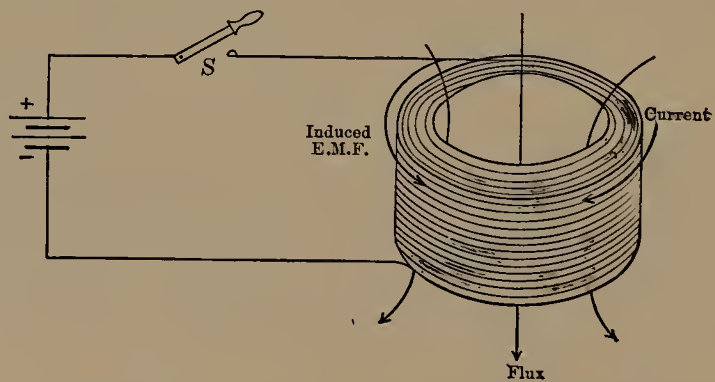


FIG. 150.—Relation of electromotive force of self-induction to current.

begins to flow in the coil. This current produces a flux linking the coil. As this flux increases, it must induce an electromotive force in the coil, the magnitude of which depends on the number of turns in the coil and the rate at which the flux increases. By Lenz's law, and also from a consideration of Fig. 149 (*a*), the

electromotive force thus induced must have such a direction as to oppose the increase in the flux linking the coil. That is, in Fig. 150, the current, by the corkscrew rule, produces a flux acting downwards, which is practically equivalent to thrusting the N-pole of a bar magnet downwards through the coil. The direction of the induced electromotive force must be such that, if it pro-

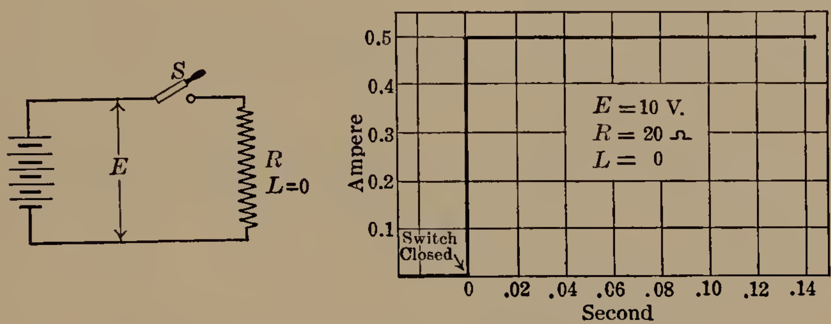


FIG. 151.—Rise of current in a non-inductive circuit.

duced a current, this current would oppose the increase of flux. Hence the induced electromotive force must flow in a counter-clockwise direction. This induced electromotive force, therefore, must *oppose* any increase of current. Hence the current cannot reach its maximum value at once, but is retarded in its rise by the opposing electromotive force of self-induction.

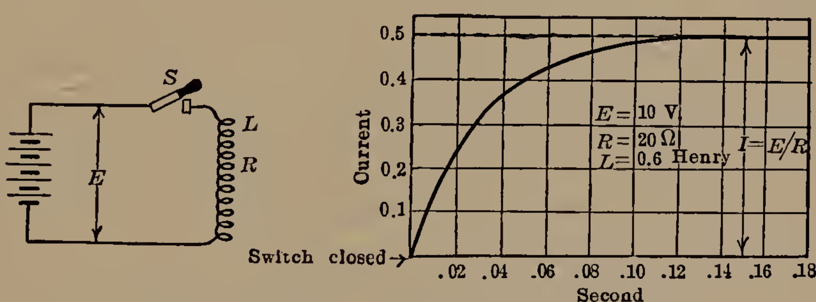


FIG. 152.—Rise of current in an inductive circuit.

In Fig. 151 is shown graphically the rise of current in a circuit containing resistance only, the impressed voltage being 10 volts and the resistance 20 ohms. When the switch S is closed, the current at once reaches its maximum or Ohm's law value of 0.5 amp.

With the inductive circuit, however, the current gradually approaches its Ohm's law value. To be exact, it takes an infinite time for the current to reach its Ohm's law value, although in a comparatively short time it reaches substantially this value. Figure 152 shows the rise of current in a circuit in which the impressed voltage is 10 volts, the resistance 20 ohms, and the self-inductance 0.6 henry.

It will be noted that at the instant the switch is closed, the current begins to rise at a comparatively high rate. Its rate of increase, however, diminishes with time, until the rate of increase becomes practically zero, and the current equals its Ohm's law value.[1] This curve should be compared with Fig. 151, in which the circuit has the same impressed voltage and the same resistance but has no inductance.

This delayed rise of current in a circuit due to self-inductance should be carefully kept in mind, since it accounts for some of the time lag observed in relays, trip coils, etc. The effect of inductance is also one of the controlling factors in the initial current rush on short-circuit.

If an inductive circuit, carrying current, be short-circuited, the current does not cease immediately, as it does in a non-inductive circuit under similar conditions, but continues to flow and does not become zero until an appreciable time after the instant of short-circuit. This is due to the electromotive force of self-induction. The flux linking the coil is due to the current, and, when the current decreases, this flux also decreases. In decreasing, the flux induces an electromotive force in the coil. In the same way that the current due to the induced electromotive force tends to prevent the flux being withdrawn in Fig. 149 (*b*), so now the electromotive force of self-induction tends to *prevent* the decrease of the current.

A curve showing the decrease of current with time is given in Fig. 153. The circuit has the same constants as the circuit shown in Fig. 152. It is usually advisable to fuse the battery so that it

[1] The equation of this curve is

$$i = \frac{E}{R}\left(1 - \epsilon^{-\frac{Rt}{L}}\right)$$

where i is the current at a time t sec. after the closing of the switch. ϵ is the Napierian logarithmic base = 2.7183.

will not be injured, since short-circuiting the inductive circuit also short-circuits the battery, as is shown in Fig. 153.

It thus appears that the effect of inductance is always to oppose any change in circuit conditions. If the current tends to increase, inductance *opposes* it; if it tends to decrease, inductance tends to *oppose* this decrease. Inductance corresponds to inertia in mechanics. A body having inertia opposes any force tending to set it in motion when the body is at rest, and if the body is in motion, inertia opposes any force tending to bring the body to rest.

After having established the current in the circuit of Fig. 152, if the switch S be opened, a noticeable arc will appear at the switch blades. This arc will be much greater in magnitude than

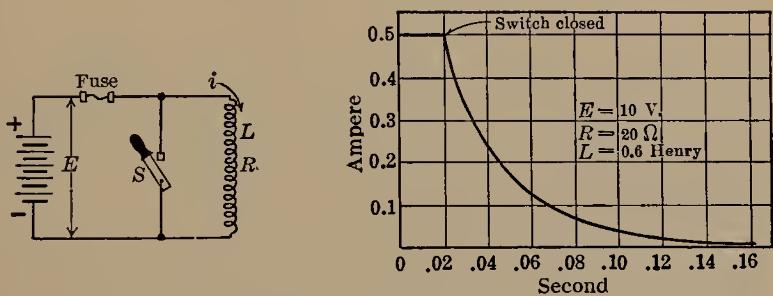


Fig. 153.—Decay of current in an inductive circuit.

that formed at the contacts of the switch in the circuit of Fig. 151, with resistance only in the circuit, although the current and circuit voltage are the same in each case. This arc is due to the electromotive force of self-induction, which in some circuits may have such a value as to cause severe arcing at the switch contacts. In fact, this voltage has been known to reach such values in alternator fields as to puncture their insulation when the field circuit is suddenly opened. To protect the field from puncture, a field discharge switch is ordinarily used (see Part II).

A voltmeter should never be allowed to remain connected across the field circuit of a dynamo while the switch is opened. The electromotive force of self-induction resulting from the large rate of change of flux linking the large number of field-turns may be many times the normal field voltage. Voltmeters have been ruined by neglecting this precaution.

139. Calculation of the Electromotive Force of Self-induction.—From equation (64) page 167, the electromotive force induced in a coil, due to a change in the flux linking the coil, is

$$e = -N\frac{\phi}{t}10^{-8}$$

where N is the number of turns, and $\frac{\phi}{t}$ the average rate at which the flux changes.

Remembering that

$$L = \frac{N\phi}{I}10^{-8} \text{ or } N\phi 10^{-8} = LI \text{ (equation (63), page 166),}$$

and also that the electromotive force of self-induction opposes the change in current, its value may be written

$$e = -\frac{N\phi 10^{-8}}{t} = -L\frac{I}{t} \qquad (65)$$

The electromotive force of self-induction is proportional to the product of the self-inductance and the rate of change of current with respect to time. The minus sign indicates that this electromotive force opposes the change of current.

From the definition, page 166, and also from equation (65), it is obvious that a current changing at the rate of one ampere per second in an inductance of one henry induces an electromotive force of one volt.

Example.—The field circuit of a generator has an inductance of 6 henrys. If the field current of 12 amp. is interrupted in 0.05 sec., what is the average induced electromotive force in the field winding?

$$e = -6\frac{12}{0.05} = -1{,}440 \text{ volts. } \textit{Ans.}$$

140. Energy of the Magnetic Field.—To establish a magnetic field requires the expenditure of energy. This is obvious from the fact that the current, in building up, flows against a counter electromotive force. To maintain a *constant field* does *not* require an expenditure of energy even in electromagnets. The energy lost in the exciting coils of electromagnets is accounted for as heat in the copper and is not concerned with the energy of the magnetic field itself.

That is, if the exciting coils be removed entirely from the iron cores of the magnet and rewound so that they are practically

non-inductive, and produce no field, therefore, the energy loss for a given constant direct current will be the same as when the coil is associated with the magnetic field.

The energy of the magnetic field is stored or potential energy and is similar to the energy of a suspended weight (Fig. 154). Work is performed in raising the weight to its position, *but no expenditure of energy is required to maintain the weight in this position.* The energy of the weight due to its position is Wh ft.-lb., where W is the weight in pounds and h the height in feet through which the weight has been raised. This energy is available and can be utilized in many ways.

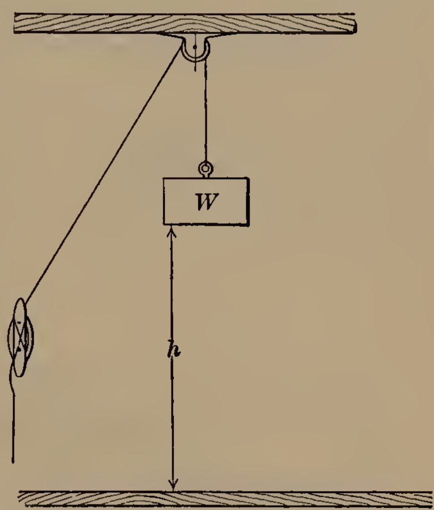


Fig. 154.—Energy of a suspended weight.

In the same way the energy stored in the magnetic field is available and may make itself manifest in many ways, as, for example, the arc at the switch contacts. In an alternating current circuit this energy may all be returned to the circuit.

The energy of the field in joules, or watt-seconds, is

$$W = \frac{1}{2} LI^2 \tag{66}$$

where L is the circuit inductance in henrys and I the current in amperes.

Example.—Find the energy stored in the generator field in the problem of Par. 139. What is the average power during the time that the circuit is being interrupted?

$$W = \frac{1}{2} \times 6 \times 12^2 = 432 \text{ watt-sec.} \quad \textit{Ans.}$$

$$P = \frac{432}{0.05} = 8{,}640 \text{ watts} = 8.64 \text{ kw.} \quad \textit{Ans.}$$

Equation (66) shows that the energy of the magnetic field is proportional to the *square of the current* when the inductance is fixed. Therefore, if the current can be reduced by a suitable resistance to one-half its initial value before opening a highly

inductive circuit, the energy of the arc at the switch contacts can be reduced to one-fourth of its initial value. This fact should be remembered when opening the field circuit of a dynamo.

A very common use of the electromotive force of self-induction occurs in the so-called spark coil used for gas lighting. This coil consists of a considerable number of turns of wire wound on a laminated iron core. The core is usually made of iron wires, as shown in Fig. 155. This coil is connected between the bell-ringing battery *B* and the grounded gas pipe. The other terminal of the battery is connected directly to the insulated contact on the gas burner. When the two contacts on the burner meet, the circuit is closed, and a magnetic field is established in

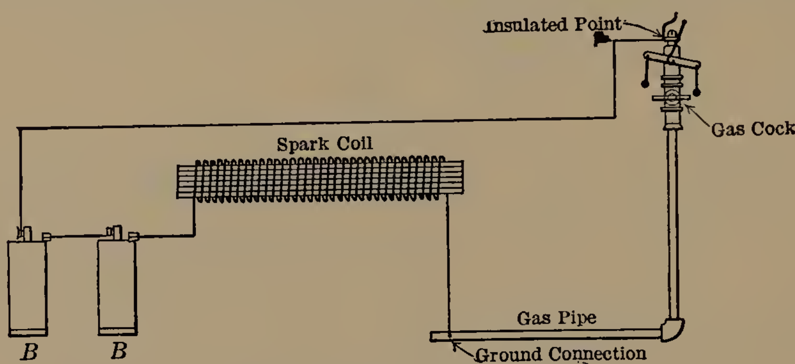


Fig. 155.—Electric gas ignition.

the laminated core of the spark coil. As the two contacts of the burner separate, they snap apart and the circuit is broken suddenly. Consequently, a high electromotive force of self-induction is produced in the spark coil. This causes a hot arc at the contacts, which ignites the gas, the gas being turned on simultaneously with the closing of the contact points and by the same mechanism.

The spark coil may be considered as having a magnetic field which is built up as the two contacts at the gas jet wipe each other. Energy is thus stored slowly in the magnetic field. When this energy is released suddenly by the contacts snapping open, considerable power is developed, resulting in a hot spark at the contact points.

This same principle is also utilized in the "make and break" system of gas-engine ignition. The spark is drawn out at the proper time by opening a contact within the cylinder.

141. Mutual Inductance.—In Fig. 156 are shown two coils, A and B. Coil A is connected to a battery through a switch S. Coil B is not connected to any source of voltage, but is connected to a galvanometer. Coil B is placed so that its axis is nearly coincident with that of A and the two coils are close together. When the switch S is closed, current flows in coil A, building up a field which links the coil. The position of B with respect to A

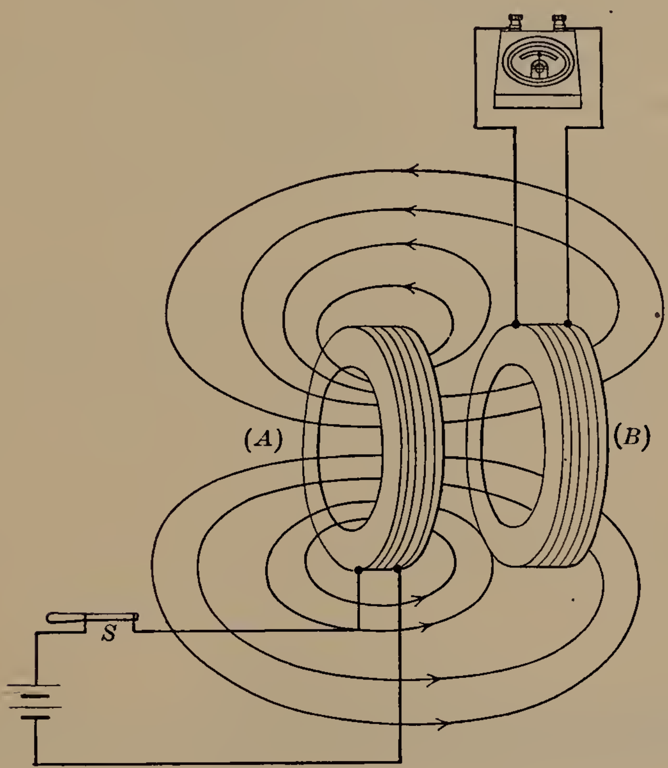


FIG. 156.—Mutual inductance between two coils.

results in a considerable part of the magnetic flux produced by A linking B. Therefore, if the current in A be interrupted by opening the switch S, or if it be altered in magnitude, a *change of flux simultaneously* occurs in B, inducing an electromotive force in B. This electromotive force is detected by the galvanometer connected across the terminals of B. Upon closing the switch S the galvanometer will deflect momentarily, and upon opening the switch S the deflection will reverse, showing that the induced electromotive force on opening the circuit is opposite in direction

to the induced electromotive force on closing the circuit. Because coil B is in such a relation to A that an electromotive force is induced in B, due to a change of current in A, these two coils are said to possess *mutual inductance.* The induced electromotive force is an *electromotive force of mutual induction.*

Even though coils A and B be brought close together, all the flux, ϕ_1, produced by coil A does not link coil B. Only a certain proportion, K, of ϕ_1 links B, K being less than unity. K is often called the *coefficient of coupling* of the circuits A and B.

If the current in coil A, when changing at the rate of one ampere per second, causes one volt to be induced in coil B, the coils have a mutual inductance of *one henry.* If the two coils have a mutual inductance of one henry, the current in coil B, when changing at the rate of one ampere per second, also causes one volt to be induced in coil A.

From the foregoing, it is obvious that the electromotive force induced in coil B, due to a rate of change of current $\frac{I_1}{t}$ in coil A, is

$$e_2 = -M\frac{I_1}{t}\text{ volts} \tag{67}$$

Likewise the electromotive force induced in coil A, due to a rate of change of current $\frac{I_2}{t}$ in coil B is

$$e_1 = -M\frac{I_2}{t}\text{ volts,} \tag{68}$$

where M is the mutual inductance in henrys, and t is the time in seconds.

Example.—The mutual inductance of coils A and B (Fig. 156), is 0.3 henry. A current of 2 amp. in coil A is interrupted in 0.04 sec. What is the average electromotive force induced in coil B?

$$e_2 = -0.3\frac{2}{0.04} = -15\text{ volts.}\quad \textit{Ans.}$$

Likewise, if a current of 2 amp. in coil B be interrupted in 0.04 sec., -15 volts will be induced in coil A. The minus sign signifies that current, flowing as a result of this induced electromotive force, opposes the flux (see Lenz's law, page 168).

The mutual inductance of two circuits may be materially increased by linking the circuits with an iron core. Thus, if two coils, similar to those shown in Fig. 156, be placed upon an iron core (Fig. 157), the coefficient of coupling, K, may be made

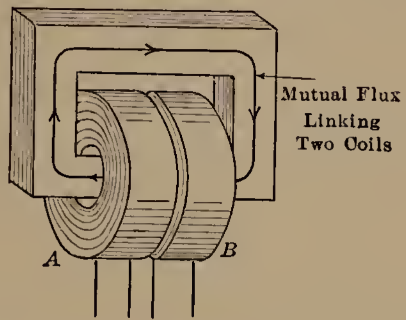


Fig. 157.—Effect of iron core on mutual inductance.

very nearly unity. That is, practically all the flux linking coil A also links coil B. (Fig. 157 shows a transformer.)

142. The Induction Coil.—A very common example of mutual inductance occurs in the induction coil (Fig. 158). A primary winding P, of comparatively coarse wire and few turns, is wound

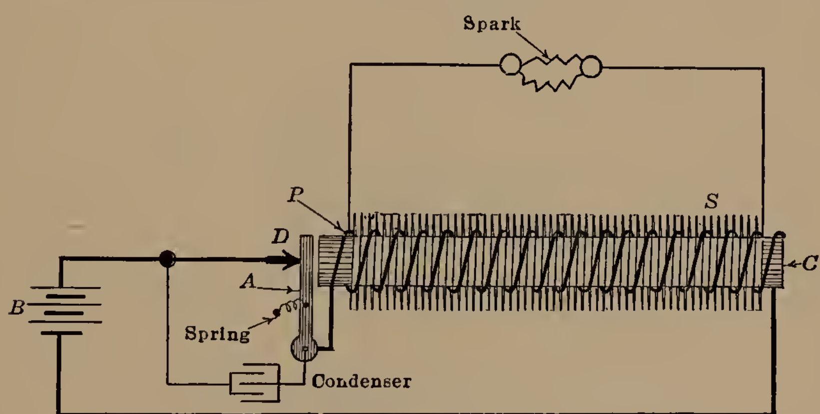


Fig. 158.—Induction coil.

on a laminated iron core C. This winding is connected to a battery B. The primary current is interrupted by passing through the contact D, against which the iron armature A is held by a spring. When the core C is magnetized by the primary current, the armature A is drawn toward it and away from D, opening the circuit and causing the flux in the core to drop prac-

tically to zero. The spring then pulls the armature A against the contact D again, and the cycle is repeated. By this process the flux in the core C is continually being established and then destroyed. A condenser is shunted across the contacts in order to reduce sparking when the circuit is opened.

On the same core is placed a secondary winding S, consisting of many turns of fine wire. This winding is thoroughly insulated from the primary winding, but as it is wound on the same core as P, the two coils have a high value of mutual inductance. Because of the change of flux in the core, due to the interruptions of the primary current, a high alternating electromotive force is induced in the secondary winding. This induced electromotive force may be considered as due to the mutual inductance existing between the primary and secondary coils. The induction coil has many practical applications. Its wide use in automobile and gas-engine ignition systems is important.

143. Battery Ignition Systems.—A very common example of the combined use of mutual inductance and the energy stored in the magnetic field is given in battery ignition systems which are used to such a large extent with internal combustion engines.

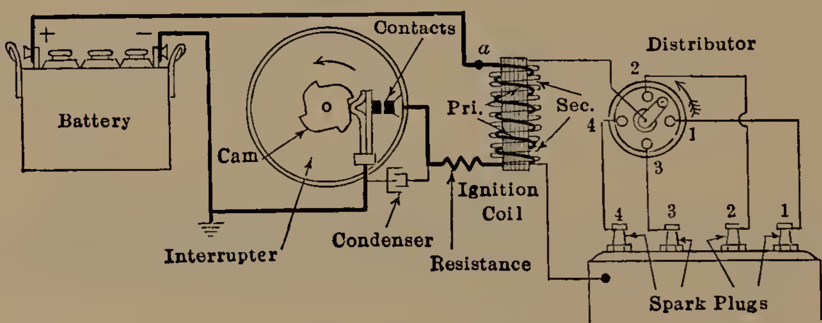


Fig. 159.—Typical battery ignition system.

The wiring diagram of a typical system is shown in Fig. 159. The ignition coil itself does not differ materially from the induction coil. It consists of an iron core made of either laminations or wires, a primary winding of comparatively few turns, and a secondary winding of a comparatively large number of turns. The primary and secondary windings are insulated from each other, although each circuit is grounded at one point. The primary of the ignition coil, however, is not interrupted by a

vibrator, as it is with the induction coil, but by a set of tungsten contacts, actuated by a cam driven by the engine crankshaft through gears. The cam ordinarily has the same number of projections as the engine has cylinders and runs at half the engine speed, since in a four-cycle engine each cylinder fires only once in two revolutions of the crankshaft. One of the contacts is mounted on a spring, which tends to hold the contacts open. As the cam rotates, the projections close the contacts and hence the primary circuit. Because of the inductance of the primary, the current builds up slowly (see page 169, Fig. 152). Therefore, the induced secondary electromotive force is not sufficient to cause a spark to jump at the spark plugs. The cam allows the contacts to remain closed a sufficient time to permit the primary current to attain practically its Ohm's law value. When the cam projection passes the follower, the spring causes the contacts to open quickly, thus suddenly interrupting the primary current. The magnetic flux in the core of the ignition coil suddenly collapses and induces a high voltage in the secondary. This high voltage causes a spark to jump across the gap in the spark plug. The secondary ground terminal is usually connected to the primary terminal of the ignition coil at a. The induced electromotive force of the primary and the condenser charge then contribute to the energy of the spark.

The distributor is usually made of molded insulation, such as bakelite, to give it good insulating and mechanical properties. A number of equally spaced metal contacts are set in the insulation, there being one contact for each cylinder. A rotating arm, which is well insulated, is connected to the secondary of the ignition coil. This arm passes over the contacts and conducts the current to the proper spark plug. The order of the contacts is such that the cylinders are fired in the proper sequence. In a four-cylinder engine the arrangement of the cranks on the crankshaft ordinarily is such that the cylinders must fire either in the sequence 1-3-4-2, or 1-2-4-3. These sequences also minimize vibration. The engine shown in Fig. 159 fires in the sequence 1-2-4-3. The distributor is ordinarily incorporated with the interrupter.

A condenser is shunted across the primary contacts in order to reduce sparking when they open. A resistance of high-tem-

perature-coefficient material is connected in series with the primary of the ignition coil in order to limit the current, particularly if the ignition switch is inadvertently left closed when the engine is not running.

The operation of this ignition system may also be considered from the energy point of view. When the primary contacts are closed, a magnetic field is built up in the core of the ignition coil. When the primary circuit is opened suddenly, this energy is suddenly released. A considerable portion of the energy appears at the spark plug. Naturally, some of the energy is lost at the interrupter contacts and energy is stored in the condenser, which is later dissipated when the contacts reclose.

It is obvious that with this system but one spark is obtained each time a cylinder is fired. With the induction coil, a number of sparks are obtained whenever the primary and interrupter circuits are supplied with voltage. This is an advantage when starting an engine, particularly if it is cold. On the other hand, the induction coil is not well adapted to high-speed automobile and airplane engines, because of the comparative sluggishness in the action of the vibrator. (Also see page 300.)

CHAPTER IX

ELECTROSTATICS: CAPACITANCE

Thus far, the electric current, or electricity in motion, has been considered. Electricity in motion is called *dynamic* electricity. Electricity may be stationary or at rest. Under these conditions electricity is called *static* electricity. There is no difference in the nature of static and dynamic electricity. Static electricity usually appears different because of its extremely high potential and small quantity.

144. Electricity Produced by Friction.—If a glass rod or a rod of hard rubber, amber, or other resinous material be rubbed with a dry woolen cloth, its surface will acquire a charge of electricity.

If the rod be held near a suspended pith, cork, or even an insulated metal ball, such as *a* (Fig. 160), the ball at first will deflect toward the rod and take some such position as *b* (Fig. 160). If the ball be allowed to touch the rod, it will remain in contact for a short time and then will be repelled to some such position as *c*.

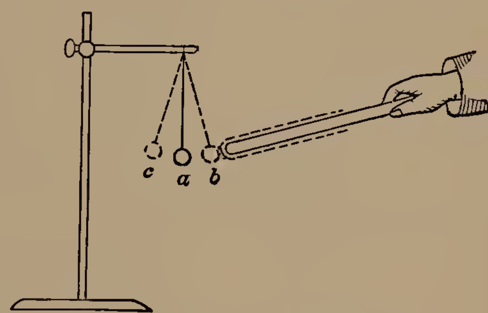
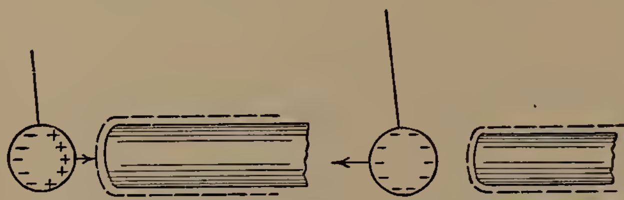


Fig. 160.—Suspended ball attracted and repelled by electrostatic charge.

This experiment indicates the presence of electricity on the rod. The reasons for the foregoing phenomena are: Rubbing the rod with the woolen cloth usually produces negative electricity on the rod and positive electricity on the cloth. Being an insulator, the rod allows electricity to leak away only very slowly. When the negatively charged rod is held near the pith ball, a positive charge is induced on the side of the pith ball nearer the rod (Fig. 161). The total charge on the ball was zero before the rod was brought near it and since the ball is well insulated, its *net* charge cannot have changed appreciably. Hence, there must be equal

quantities of negative and positive electricity on the ball. The negative electricity is repelled by the negative charge on the rod, and moves to those parts of the ball which are farthest removed from the inducing charge on the rod.

Unlike charges attract and like charges repel each other. The negative charge on the rod attracts the positive charge and



(*a*) Ball attracted by induced positive charge. (*b*) Ball repelled by negative charge given it by conduction.

FIG. 161.

repels the negative charge on the ball. The positive charge being nearer the inducing charge, attraction prevails over repulsion and the ball moves toward the rod.

If the ball be allowed to touch the rod, the induced positive charge on the ball is neutralized by the negative charge on the rod and the ball acquires further negative charge from the rod by conduction. The negative charge which the ball thus acquires is repelled by the negative charge on the rod; and the ball takes position *c* (Fig. 160).

145. The Electrophorus.—Figure 162 shows a flat metal dish *a*, filled with some resinous material *b*, such as a mixture of resin and wax, or hard rubber may be used. If the resinous material be rubbed with either a woolen cloth or a cat-skin, it will acquire a negative charge by friction. If a metal disc *c* be laid on the resinous material, it will touch it at only a very few points. The negative charge on the resinous material will induce a positive charge on the under side of the disc and a negative charge must obviously come into existence on the upper side of the disc. If the upper side of the disc be touched or grounded, the *free* negative charge flows to ground, leaving only the *bound* positive

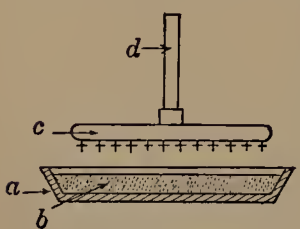


FIG. 162.—The electrophorus.

charge on the disc (Fig. 162). If the disc be raised by the glass handle d, a strong positive charge will be found on it. After discharging, the disc may again be laid on the resinous material, its top surface grounded, and again on being raised it will be found to have a strong positive charge. This operation may be repeated a number of times before the negative charge on the resinous material leaks away.

The positive charge on the disc represents energy. This energy is obviously not acquired at the expense of the negative charge on the surface of the resinous material, since the decrease in the charge on this material is due wholly to leakage. The energy is obtained in virtue of the work done in raising the disc against the force of attraction between the inducing negative charge and the induced positive charge.

A principle similar to this is involved in the operation of influence machines which generate static electricity (see page 184).

146. The Gold-leaf Electroscope.—An electroscope is a sensitive instrument for detecting small electric charges. It consists of a wide-mouthed glass jar (Fig. 163), closed with a stopper of insulating material in which is fitted a varnished glass tube. A stiff wire or rod passes through the tube. At the top of the rod there is either a metal ball or disc, and at the bottom of the rod two pieces of gold leaf are suspended. When a charge is brought into the vicinity of the electroscope (Fig. 163), a charge of opposite sign is induced on the metal ball and a charge of the same sign as the inducing charge appears on the two pieces of gold leaf (Fig. 161 (a)). Since the two pieces of gold leaf now have charges of like sign, they repel each other.

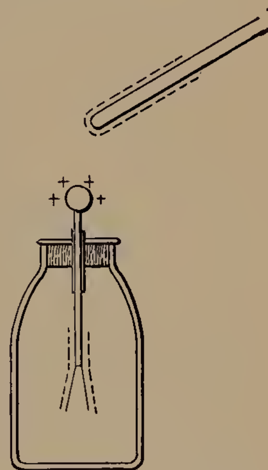


FIG. 163.—Gold-leaf electroscope.

As an example, a negatively charged glass rod is shown as brought into the vicinity of the electroscope (Fig. 163). A positive charge is induced on the knob and a negative charge appears on the two pieces of gold leaf.

The positive charge is a *bound charge*. That is, the negative inducing charge prevents its leaving the knob, even if a conduct-

ing path to ground be provided. The negative charge is not so held and is a *free charge.* If a conducting path be provided, it will flow to ground.

If, therefore, the knob be touched, or grounded with the negatively charged rod still in position, the negative charge on the gold leaves flows to ground, as will be shown by the pieces of gold leaf coming together. If now the inducing charge be removed, by withdrawing the rod, the bound positive charge distributes itself over the rod, gold leaves, etc., and the pieces of gold leaf repel each other again.

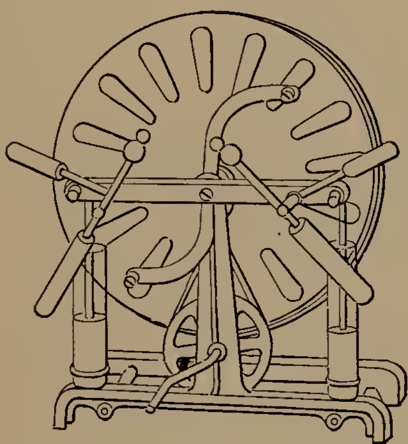
The polarity of a charge may be determined by means of the electroscope. Assume that the electroscope be charged negatively throughout by *touching* the knob with a rod of hard rubber which has been rubbed with flannel or silk. If the unknown charge be brought into the vicinity, it will induce on the knob a charge of opposite polarity to itself, and on the gold leaves a charge of the same polarity as that of the unknown charge. Therefore, if the unknown charge is negative, the gold leaves will spread further apart; if it is positive, they will come closer together.

147. Influence Machines.—There are various types of machines for generating so-called "static" electricity. Some depend on friction for the generation of electricity, but this type is not satisfactory. The machines which give the best results and which are in common use depend on electrostatic induction or influence for their operation. The principle is fundamentally that of the electrophorus. Small conducting strips called carriers, usually of tin foil, are brought consecutively under the influence of a charged body at a time when the charge on the carriers is zero. Simultaneously, each carrier is touched by a conductor which allows the "free" charge (see page 182) to escape. The "bound" charge is then carried to a collecting brush. With the electrophorus a conducting plate is brought near the charged resinous material and grounded to permit the free charge to escape. The bound charge is then carried away.

A common type of influence machine is the Wimshurst machine (Fig. 164 (*a*)). Two circular glass or ebonite plates are mounted on a common shaft with small clearance between them. The two plates rotate at the same speed, but in opposite directions.

A number of carriers, usually of tin foil, are mounted on the outer surface of each plate. The machine requires a pair of neutralizing brushes and a pair of collecting combs for each plate.

The details of operation of the Wimshurst machine are illustrated in Fig. 164 (*b*), in which, for clearness, the two flat circular plates are represented by two concentric glass or ebonite cylinders. The back plate is represented by the outer cylinder, shown as rotating in a counter-clockwise direction, and the front plate is represented by the inner cylinder, shown as rotating in a clock-



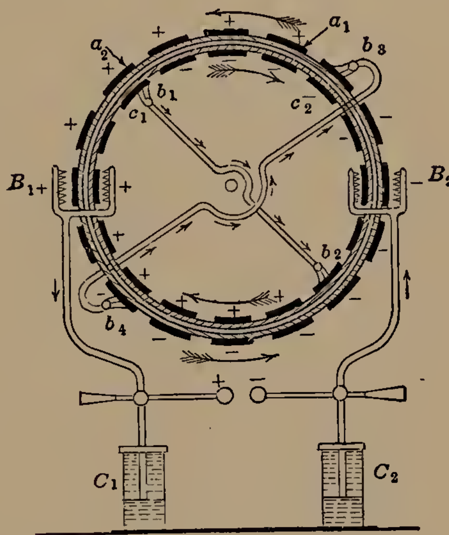


FIG. 164.—The Wimshurst influence machine.

wise direction. The tin-foil sectors in the actual machine are represented by black segments. The neutralizing brushes are shown as b_1, b_2 and b_3, b_4. The sharp-pointed collecting combs B_1 and B_2 are also shown.

The action of the machine is as follows: Assume that some segment a_1 on the back plate acquires in some manner a positive charge, which may be ever so small. This charge is carried from right to left. When it reaches position a_2 directly opposite the neutralizing brush b_1, a segment c_1 on the front plate comes directly under its influence when being touched by brush b_1. This results in a negative charge, which is a *bound* charge, being induced on this front segment c_1, and a *free* positive charge also

appearing on c_1 (see Pars. 145 and 151). The positive charge, which is a free charge, flows away through the brush b_1. This leaves only the bound negative charge on this segment c_1.

The segment c_1 now carries this negative charge from left to right. When it reaches position c_2, a segment on the back plate comes directly under its influence when being touched by brush b_3. This results in a positive bound charge appearing on this back segment, the free negative charge escaping through neutralizing brush b_3. In this manner the charges on the segments of the back plate moving from right to left are increased. They, in turn, increase the charges on the segments of the front plate. As this action is cumulative, the machine will build up its charge.

The positive charge on a_2 will move directly under the sharp points of the positive collecting brush B_1, where, in virtue of brush discharge (see Fig. 170, page 192), it is taken off. When this segment a_2 reaches the neutralizing brush b_4, it comes under the influence of a positive charge. Hence it obtains a bound negative charge. Its free positive charge neutralizes the free negative charge coming from brush b_3.

Throughout the machine the foregoing reactions are occurring. These reactions are all of such character as to cause an increase in the generated energy.

The electrical energy generated by such a machine is of extremely high voltage. The current must necessarily be very small, since its value is determined by the rate at which the carriers can bring the charges to the collecting brushes. The capacity of the machine may be increased by increasing the number of pairs of plates.

A sphere gap, shown in Fig. 164, (*a*) and (*b*), is usually connected across the terminals of such a machine. A series of small, thin, and rapidly occurring sparks ordinarily jump the gap. By using two Leyden jars $C_1 C_2$ (see page 196), whose outer coatings are connected together, large, vigorous sparks occurring at less frequent intervals are obtained.

The energy which this machine generates is obtained from the work done in separating positive and negative charges.

This is the same principle as in the electrophorus, where the free charge is allowed to escape and the energy is obtained in

virtue of the work done in separating the positive induced charge and negative inducing charge.

148. Electrostatic Field.—The preceding experiments show that a condition of stress exists in the medium surrounding an electrostatic charge. If a charged body or a neutral body be placed in this region, a force will be found to act on the body. As with the magnetic field, this condition of stress may be represented by lines of force, the number of lines per square centimeter, taken normal to their direction, representing the force in dynes exerted on a unit charge.

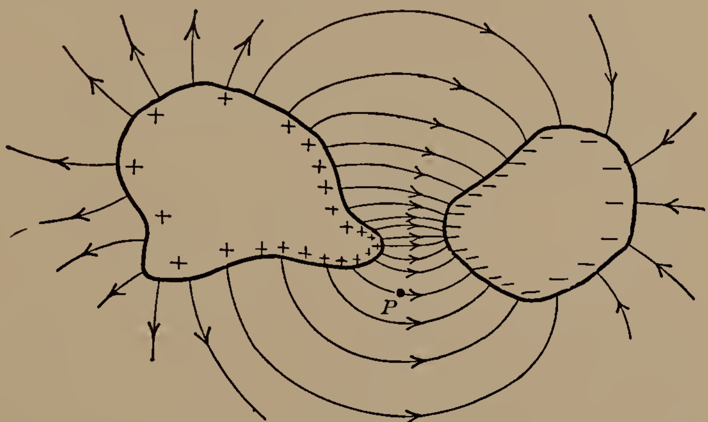


FIG. 165.—Electrostatic field between charged conducting bodies.

These lines have the following properties: They originate on a positive charge and terminate on a negative charge; they must always leave and enter a conducting surface normal to the surface, as otherwise there would be a component of force or stress tangential to the surface, and a resulting flow of current; they can never intersect, since two separate electrostatic forces cannot exist simultaneously at a point in space; they cannot exist within a conducting body, since there can be no force without a flow of current in such a body; a positive charge, P, free to move (Fig. 165), if placed at the positive end of an electrostatic line will move along the line until it reaches the negative end of the line. Figure 165 shows a typical electrostatic field between two irregular, charged, conducting bodies.

The properties of these lines are almost identical with those of magnetic lines of force.

Electrostatic induction (or influence) may be explained by means of these lines. Figure 166 shows a positively charged rod and a pith ball. Some of the lines leaving the positively charged rod terminate at the negative charge on the pith ball (Fig. 161 (*a*), page 182). Likewise, lines leave the free positive charge on the pith ball and go out into space. The lines between the charged rod and the pith ball in contracting tend to pull the

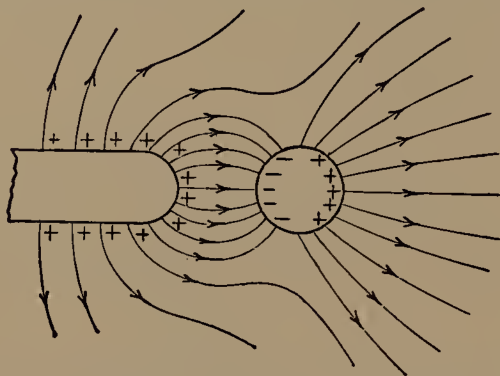


FIG. 166.—Electrostatic induction explained by electrostatic lines.

pith ball nearer the rod, whereas the lines leaving the pith ball tend to pull it away from the rod. The former predominate (compare Fig. 166 with Fig. 11, page 11).

149. Unit Electrostatic Charge.—*Unit*[1] *electrostatic charge is that quantity which, when placed at a distance of one centimeter in air from a similar and equal quantity, repels it with a force of one dyne.*

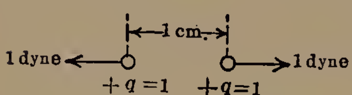


FIG. 167.—Unit electrostatic charge.

This is illustrated in Fig. 167, which shows two unit positive charges, $+q$, 1 cm. apart, and repelling each other with a force of 1 dyne.

150. Force between Charged Bodies (Coulomb's Law).—By means of a torsion balance, Coulomb, after a series of experiments conducted between the years 1785 to 1789, proved that *the force between two small charged bodies in air is proportional to the product of the charges and inversely proportional to the square of the distance between them.* If the charges q_1 and q_2 are given in terms

[1] The commonly accepted name for the unit of quantity in the electrostatic system is *statcoulomb*.

of unit electrostatic charges, and the distance r between them is given in centimeters, the force in dynes is

$$F = \frac{q_1 q_2}{r^2}. \tag{69}$$

If the charges are in a medium having a dielectric constant κ (see page 195), the force is given by

$$F = \frac{q_1 q_2}{\kappa r^2}. \tag{70}$$

For air and vacuum, κ is equal to unity. For other dielectrics, κ is greater than unity. Hence, with fixed *distance* and *charges*, the force between charged bodies is decreased if the medium is other than air or vacuum.

If the charges are not concentrated at points and the distance between them is comparatively small, (69) and (70) do not apply.

A charge placed on a sphere which is remote from all other electrified bodies distributes itself uniformly over the *surface* of the sphere.

The electrostatic lines of force or flux must emanate radially and uniformly from such a charged sphere, that is, as if they came from its center. It follows that the effect of a charged sphere at all points in space external to the sphere is the same as if the entire charge on the sphere were concentrated at its center.

Example.—Two equal spheres A and B, each of which has a radius of 0.5 cm., are spaced 30 cm. between centers in air. If sphere A is given a charge of 100 positive electrostatic units (statcoulombs) and sphere B a charge of 150 negative units (statcoulombs), find: (*a*) the force (acting on a unit charge) just outside the surface of each sphere, neglecting the effect of the other sphere; (*b*) the force acting between the two spheres.

(*a*) The force acting just outside the surface of sphere A, from equation (69),

$$F_A = \frac{100 \times 1}{(0.5)^2} = 400 \text{ dynes, radially outwards.} \quad \textit{Ans.}$$

And at the surface of sphere B

$$F_B = \frac{150 \times 1}{(0.5)^2} = 600 \text{ dynes, radially inwards.} \quad \textit{Ans.}$$

(*b*) The force acting between the two spheres, from equation (69),

$$F = \frac{100 \times 150}{(30)^2} = 16.67 \text{ dynes.} \quad \textit{Ans.}$$

151. Examples of Electrostatic Induction.—The following experiments illustrate the electrostatic principles which have just been stated.

If the terminals of an electrostatic induction machine be connected to two equal ellipsoids, which are conducting and are insulated (Fig. 168) the ellipsoid connected to the positive ter-

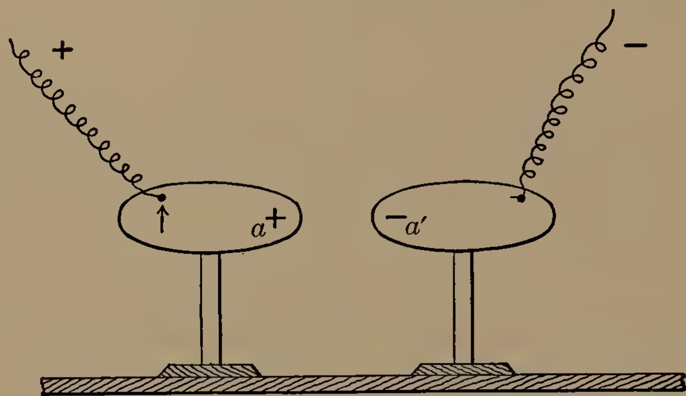


FIG. 168.—Electrostatic charges on insulated ellipsoids.

minal will be charged with positive electricity and that connected to the negative terminal will be charged with an equal amount of negative electricity. The charges will distribute themselves over the entire surface of the ellipsoids, but the density of the charge will be greatest on the ends of the ellipsoids which are adjacent, due to the fact that the positive and negative charges attract each other.

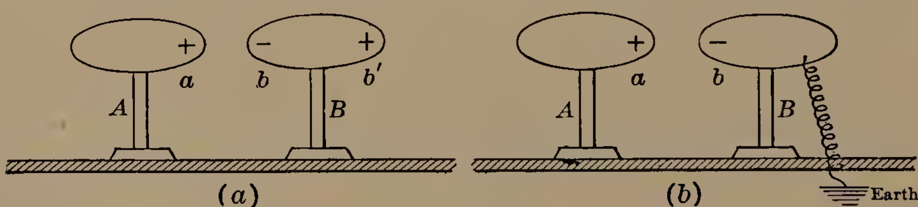


FIG. 169.—Electrostatic induction.

If the two wires from the electrostatic machine be disconnected, the two charges will not be sensibly affected. In time they will leak away through the insulating supports.

If a positively charged ellipsoid A (Fig. 169 (a)) be brought near another insulated ellipsoid B, which initially has no charge, a minus charge will be found on the end of B nearest A. As B did not hold any charge initially, and it is assumed to be perfectly

insulated, no electricity can have left B and none can have reached it from external sources, so that the net charge on B must still be zero. Therefore, a positive charge b' equal to b must also appear on B at the end farthest from A. Since these two charges are of opposite sign, the net charge on B is still zero. The minus charge b is as near as possible to the positive inducing charge a, whereas the positive charge b' is as far away as possible from the positive charge a, due to the fact that unlike charges attract each other and like charges repel each other.

Charges a and b are bound charges, and charge b' is a free charge. This may be proved by connecting B to ground (see Fig. 169 (b)). The charge b' will escape to ground, whereas the two charges a and b remain bound by their mutual attraction. Charge b' will seek a position as far away from a as possible. If a were a negative charge, b would be a positive charge.

152. Fundamental Laws of Electrostatics.—The preceding discussions and experiments are all illustrative of the following fundamental laws of electrostatics.

For every positive charge in the Universe there must exist an equal negative charge.

Charges of unlike sign attract each other and charges of like sign repel each other.

A positive charge will induce a negative charge on a body near it.

A negative charge will induce a positive charge on a body near it.

An electrostatic line originates at a positive charge and terminates at a negative charge.

Electrostatic lines so distribute themselves that the electrostatic reluctance of the system is a minimum.

153. Dielectrics.—If electrostatic phenomena are being considered, the medium between two conductors is called a *dielectric.* This is in distinction to the properties of the same medium as an insulator, which relates to electrical *conduction.* The dielectric properties of a medium relate to the number of electrostatic lines which it permits to pass through it with a given potential difference and also to the number of electrostatic lines per unit area taken normal to their direction which it can permit without being ruptured. On the other hand, the insulating properties of a medium relate to the current which it conducts with a given potential difference.

For example, air is not a particularly good dielectric. It does not permit the passage of electrostatic lines as readily as most media and it ruptures when the potential gradient (see Par. 154) reaches 75,000 volts per inch. On the other hand, it is one of the best insulators known.

154. Dielectric Strength.—There is one difference, however, between electrostatic lines, on the one hand, and either magnetic lines of induction or electric-current stream-lines on the other. No matter how much current flows in a conductor, the conductor is not injured mechanically, provided it be kept cool. Neither is a magnetic conductor injured, no matter how many magnetic lines exist in it. But there is a limit to the number of electrostatic lines which may exist in a medium. If the lines become too concentrated, the medium cannot withstand the stresses which result, and it is ruptured or "breaks down." This break-down may be followed by a dynamic arc, which increases the injury to the medium by burning.

The ability of a substance to resist electrostatic break-down is called its *dielectric strength*. This is expressed in volts per unit thickness when the substance is placed between flat electrodes having rounded corners. For example, the dielectric strength of air is approximately 3,000 volts per millimeter, or 75 volts per mil. Rubber and varnished cambric have a much greater dielectric strength than air, that of rubber being in the neighborhood of 16,000 volts per millimeter, or 400 volts per mil, and that of cambric being about twice as great as the value for rubber.

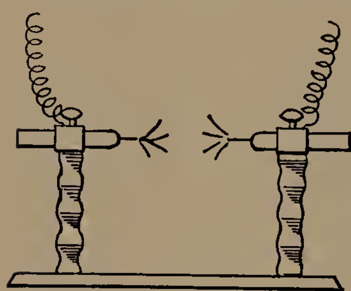


FIG. 170.—Brush or corona discharge from needle points.

The volts per unit thickness impressed across a dielectric is called the *voltage gradient*. For example, if 24,000 volts are impressed across 30 mils of dielectric between flat parallel electrodes, the gradient is $\frac{24,000}{30}$, or 800 volts per mil.

An example of air being ruptured under dielectric stress is seen by connecting a gap having needles or other sharp-pointed electrodes (Fig. 170) across the terminals of an influence machine

or high-voltage transformer. A bluish discharge occurs around the points, due to the air being ruptured by the concentration of electrostatic lines at the points. This bluish brush discharge is often called *corona*. It occasionally appears on high-voltage transmission lines (see Part II).

155. Capacitance.—A condenser consists of two conductors separated by a dielectric.

The ellipsoids of Figs. 168 and 169 form condensers.

Figure 171 shows two conducting plates connected to a battery, the plates being separated by a dielectric. There is also a single-pole, double-throw (S.-P. D.-T.) switch S and a galvanometer G in the circuit. If the switch S be closed to the left, the galvanometer will deflect momentarily, and then come back to zero. This indicates that, when the switch is closed, a quantity

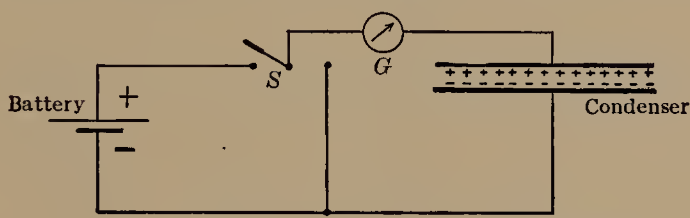


FIG. 171.—Charging and discharging a condenser.

of electricity passes through the galvanometer, but that the current ceases to flow almost immediately. This current flows for a time sufficient to charge the condenser. After the condenser has become fully charged, the current ceases, because the electromotive force of the condenser is equal and opposite to that of the battery. As this condenser electromotive force opposes the current entering the condenser, it may be considered as a back electromotive force. Any current which may flow after the condenser has become fully charged is a leakage current flowing through the insulation. If the switch S be opened for a short time, and then closed again, no deflection of the galvanometer will be noted unless there is leakage through the insulation.

This phenomenon of charging a condenser from a battery is not unlike the filling of a tank T from a reservoir R (Fig. 172). When the valve V is first opened, water will rush through the connecting pipe and will continue to flow at a diminishing rate until the level H, of the water in the tank T, is equal to the level

of the water in the reservoir. If the tank does not leak, no water flows through the pipe after the water levels have become equal. In the same way the condenser (Fig. 171) takes current until its potential is equal to that of the battery, after which current ceases to flow.

To prove that electricity has actually been stored in the condenser (Fig. 171) the switch S may be closed to the right. This short-circuits the condenser through the galvanometer. The galvanometer now deflects momentarily in a direction opposite to that on charge, showing that the current now flows *out* of the positive plate. The condenser now becomes discharged, as is shown by there being no longer any deflection of the galvanometer.

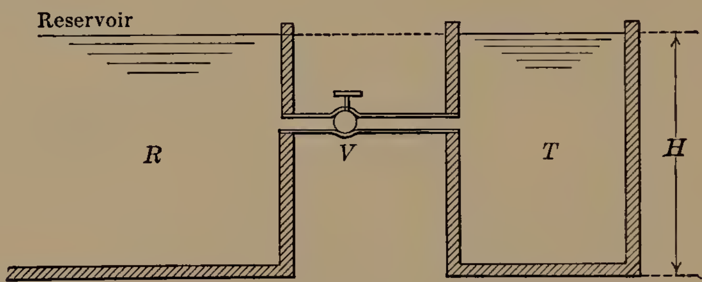


Fig. 172.—Reservoir and connected tank.

If the voltage of the battery (Fig. 171) be increased, the galvanometer deflection on charge and on discharge will increase also. This is due to the fact that the charge given to the condenser is proportional to the voltage across its terminals, just as the amount of water in the tank will be proportional to its height H (Fig. 172). The relation between the voltage and the charge in a condenser may be expressed by the equation

$$Q = CE. \qquad (71)$$

That is, the quantity of electricity in a condenser is equal to the voltage multiplied by a constant C. This constant C is called the *capacitance* of the condenser. The practical unit of capacitance is the *farad*. If C is in farads and E in volts, Q is in coulombs or ampere-seconds.

The farad is too large a unit for practical purposes, as a condenser having a capacitance of 1 farad would have a prohibitively large volume. The capacitance of the earth, as an isolated

sphere, is less than one-thousandth of a farad. The *microfarad*,[1] equal to one-millionth of a farad, is the unit of capacitance ordinarily used.

By transposition, equation (71) may be written as follows:

$$C = \frac{Q}{E} \tag{72}$$

That is, the capacitance of a condenser in *farads* is the ratio of its charge in *coulombs* to the potential difference between its terminals in *volts*.

Also

$$E = \frac{Q}{C} \tag{73}$$

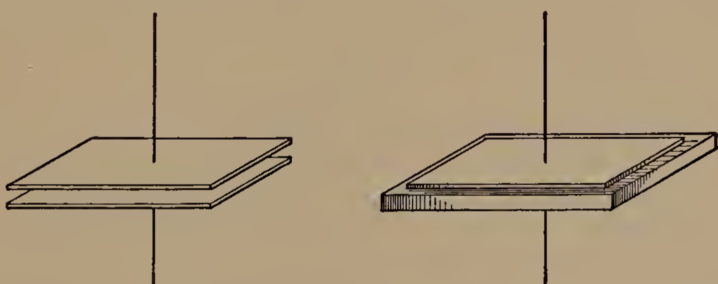
That is, the voltage across a condenser is the ratio of its charge in *coulombs* to its capacitance in *farads*.

Example.—A condenser has a capacitance of 200 μf. and is connected across 600-volt mains. If the current is maintained constant at 0.1 amp., how long must it flow before the condenser is fully charged?

The quantity in the condenser, when fully charged, is $Q = 0.000200 \times 600 = 0.12$ coulomb or ampere-second.

$$0.12 = 0.1t$$
$$t = 1.2 \text{ sec.} \quad \textit{Ans.}$$

156. Specific Inductive Capacity or Dielectric Constant.—A parallel-plate condenser (Fig. 173 (*a*)) with air as a dielectric, has



(*a*) (*b*)

FIG. 173.—Plate condenser having air and then glass as a dielectric.

a measured capacitance C_1. If a slab of glass or of hard rubber be inserted between the plates so as to fill the intervening space completely (Fig. 173 (*b*)) and the capacitance of the condenser again be measured, it will be found to be greater than its previous

[1] The recognized abbreviation for microfarad is μf.

value. Let this new value be C_2. The increase in capacitance must be due to the presence of the glass or hard rubber.

The ratio $\frac{C_2}{C_1} = \kappa$ is called the *specific inductive capacity*, or *dielectric constant* of the material between the condenser plates. The specific inductive capacity of air is assumed to be unity, just as the magnetic permeability of air is assumed to be unity.

In the table are given the specific inductive capacities of some of the more common dielectrics:[1]

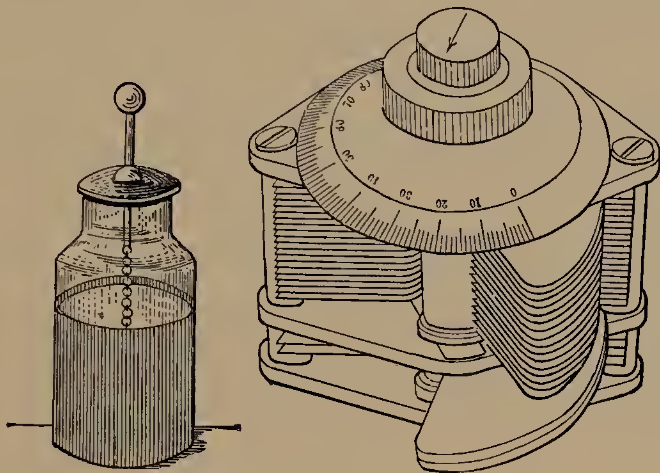
Bakelite	4.1 to 8.8	Paraffin	1.9 to 2.3
Glass	5.5 to 10	Rubber compounds	3 to 6
Ice	86.4	Hard rubber	1.5 to 3.5
Mica	2.5 to 5.5	Transformer oils	2.3 to 2.6
Paper	1.7 to 2.6		

157. Types of Condensers.—There are several types of condensers. A simple and common type is the Leyden jar (Fig. 174 (*a*)). The base and lower portion of a glass jar are covered both inside and outside with a thin, metallic coating, such as tin foil. Contact with the inner coating is made through a loose chain, the lower end of which touches the inner coating and the upper end of which is fastened to a metallic rod. The rod passes through the cover of the jar, usually of wood, and is often terminated at the top by a round knob. The inner and outer coatings of the jar constitute the condenser plates, while the glass of the jar is the dielectric. Such condensers have capacitances of the order of 0.003 μf.

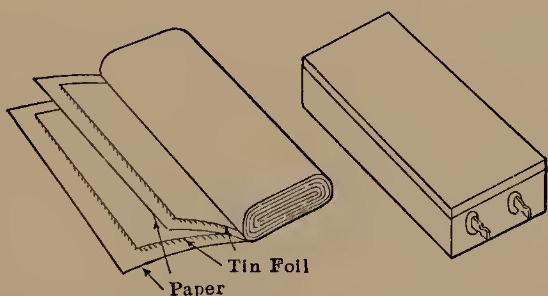
A common type of variable air condenser is shown in Fig. 174 (*b*). It consists of a set of equally spaced, semicircular fixed plates which are connected together, and a similar set of equally spaced, semicircular movable plates which are fastened rigidly together, but mounted as a unit on an axis free to rotate. The mounting is such that the movable plates move into the spaces between the fixed plates. The fixed set of plates and the movable set of plates are insulated from each other. The capacitance is varied by turning the movable set of plates so that more or less of their plate area is between the fixed plates. Even with the movable plates entirely outside the spaces between the fixed

[1] For more complete data, see "Standard Handbook," Fifth Edition, Sec. 4, Par. 238, *et seq.*

plates, the capacitance is not zero. With the exception of the insulating supports, this type of condenser has air as the dielectric. At radio frequencies, losses in dielectrics other than air are excessive. The capacitance of such condensers is nearly proportional to the number of plates. The capacitance of a condenser of 11 plates is of the order of 0.00025 μf. A vernier on the top



(a) (b)



(c) (d)

Fig. 174.—Types of condensers. (a) Leyden jar; (b) variable radio air condenser; (c) telephone condenser (internal view); (d) telephone condenser (assembled).

usually serves to indicate the condenser setting rather than its capacitance.

The foregoing two types of condensers have very low capacitance. Telephone condensers, on the other hand, are made to have very large capacitance per unit volume. They are made by rolling together very tightly two long strips of very thin foil, alternating with two strips of thin paraffined paper (Fig. 174 (c)).

The width of the paper is slightly greater than that of the foil. The roll is then sealed in a tin box and the two terminals brought out through the top (Fig. 174 (*d*)). As much as 2 μf. may be obtained with a box 1 by 2¼ by 4½ in. (2.54 by 5.7 by 11.4 cm.). Such condensers will withstand voltages as high as 600 volts, direct-current, without puncturing.

Condensers are also made using flat layers of waxed paper and tin foil, and paper and tin foil immersed in oil or impregnating compound.

158. Equivalent Capacitance of Condensers in Parallel.—Let it be required to determine the capacitance, C, of three condensers in parallel, the condensers having capacitances C_1, C_2, C_3. This arrangement of condensers is shown in Fig. 175. Let the common voltage across the condensers be E and the total resulting charge Q. Obviously,

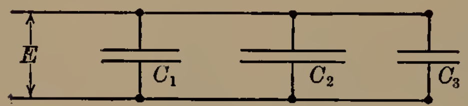


FIG. 175.—Capacitances in parallel.

$$Q = CE$$

and

$$Q_1 = C_1E, \; Q_2 = C_2E, \; Q_3 = C_3E. \qquad \text{(I)}$$

The total charge

$$Q = Q_1 + Q_2 + Q_3 = CE \qquad \text{(II)}$$

Substituting from (I) in (II),

$$CE = C_1E + C_2E + C_3E$$

$$CE = E(C_1 + C_2 + C_3)$$

$$\therefore C = C_1 + C_2 + C_3. \qquad (74)$$

That is, if condensers are connected in parallel, the resulting capacitance is the sum of the individual capacitances.

This is analogous to the grouping of conductances in parallel in the electric circuit.

Example.—Three condensers, having capacitances of 5, 10, and 12 μf., are connected in parallel across 600-volt mains. (*a*) What single condenser would replace the combination? (*b*) What is the charge on each condenser?

(*a*) $C = 5 + 10 + 12 = 27$ μf. *Ans.*

(*b*) $Q_1 = 5 \times 600 = 3{,}000$ microcoulombs

$Q_2 = 10 \times 600 = 6{,}000$ microcoulombs

$Q_3 = 12 \times 600 = 7{,}200$ microcoulombs. *Ans.*

Total charge = 16,200 microcoulombs = 27 × 600 microcoulombs (check).

159. Equivalent Capacitance of Condensers in Series.—In Fig. 176, three condensers, having capacitances of C_1, C_2, and C_3, are connected in series across the voltage E. It is desired to determine the capacitance of an equivalent single condenser. Let E_1, E_2, and E_3 be the potential differences across the condensers C_1, C_2, and C_3, respectively. After the voltage E is applied to the system, there will be $+Q$ units of charge on the positive plate of C_1 and, by the law of electrostatic induction, $-Q$ units must be induced on its negative plate.

Now consider the region a, which consists of the negative plate of C_1, the positive plate of C_2, and the connecting lead. This system is insulated from all external potentials, since it is assumed that the condensers have perfect insulation. Before the voltage was applied to the system of condensers, no charge existed in the region a. After the application of the voltage, the net charge in this region must still be zero, as no charge can flow through the insulation. Therefore, $+Q$ units must come into existence in order that the net charge in the region a may remain zero. $(+Q + (-Q)) = 0$. This charge of $+Q$ units will go to the plate of C_2, since it is repelled by the $+$ charge on C_1, just as the charge b' (Fig. 169 (a), page 190), took a position on the end of the ellipsoid as far as possible from the positive inducing charge a. The same reasoning holds for the region b, between C_2 and C_3. Therefore, each of the three condensers in series has the same charge Q. (This is analogous to resistances in series, each of which must carry the same current, if no leakage exists.)

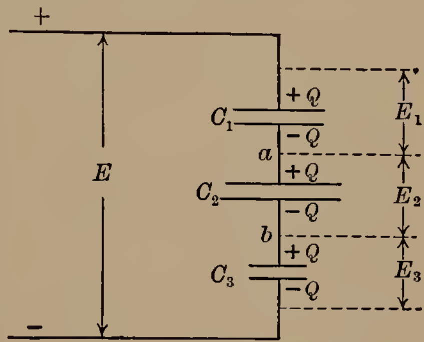


FIG. 176.—Capacitances in series.

Consider the voltages E_1, E_2, E_3.

$$E_1 = \frac{Q}{C_1},\ E_2 = \frac{Q}{C_2},\ E_3 = \frac{Q}{C_3} \text{ from equation (73), page 195.}$$

The sum of the three condenser voltages must equal the line voltage:

$$E_1 + E_2 + E_3 = E. \tag{II}$$

From (I) and (II)

$$E = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

Also $E = \frac{Q}{C}$, as by definition the equivalent condenser C must have a charge Q.

Substituting this value for E,

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}. \tag{75}$$

That is, the reciprocal of the equivalent capacitance of a number of condensers in series is equal to the sum of the reciprocals of the capacitances of the individual condensers.

Equation (75), giving the equivalent capacitance of condensers in series, is very similar to equation (10), page 39, giving the equivalent resistance of a number of resistances in parallel.

In assuming for condensers connected in series that, with direct current, the potential across each condenser is inversely proportional to its capacitance, the factor of leakage is absolutely neglected. If the condensers are even slightly leaky, however, a current flows through the series and eventually the potential distributes itself according to Ohm's law.

$$E_1 = IR_1,\ E_2 = IR_2, \text{ and } E_3 = IR_3$$

where I is the leakage current, and R_1, R_2, and R_3 are the ohmic resistances of the three condensers.

Example of condensers connected in series.

Consider that the three condensers of Par. 158, page 198, having capacitances of 5, 10, and 12 μf., are connected in series across 600-volt mains. Determine (*a*) the equivalent capacitance of the combination; (*b*) the charge on each condenser; (*c*) the potential across each condenser, assuming no leakage.

(*a*)
$$\frac{1}{C} = \frac{1}{5} + \frac{1}{10} + \frac{1}{12} = 0.383$$

$$C = \frac{1}{0.383} = 2.61\ \mu\text{f.} \quad \textit{Ans.}$$

(*b*) $Q = 2.61 \times 600 = 1{,}566$ microcoulombs, on each condenser. *Ans.*

From equation (73), page 195.

(c)
$$E_1 = \frac{1{,}566 \times 10^{-6}}{5 \times 10^{-6}} = 313 \text{ volts}$$
$$E_2 = \frac{1{,}566 \times 10^{-6}}{10 \times 10^{-6}} = 157 \text{ volts}$$
$$E_3 = \frac{1{,}566 \times 10^{-6}}{12 \times 10^{-6}} = 130 \text{ volts.} \quad \textit{Ans.}$$

160. Energy Stored in Condensers.—As a certain quantity of electricity is stored in a condenser and a difference of potential exists between the positive and negative plates, energy must be stored in the condenser. The existence of this energy is shown by the spark resulting from short-circuiting the condenser plates. The energy in joules or watt-seconds is

$$W = \frac{1}{2} QE \tag{76}$$

Since $Q = CE$ and $E = \frac{Q}{C}$, (76) may also be written

$$W = \frac{1}{2} CE^2 \tag{77}$$

$$W = \frac{1}{2} \frac{Q^2}{C} \tag{78}$$

The similarity in form of (77) to the equation for the energy stored in the magnetic field should be noted (see equation (66), page 173, Par. 140). The energy stored in the electrostatic field is proportional to the square of the *voltage*, whereas the energy stored in the electromagnetic field is proportional to the square of the *current*.

Example.—Determine the stored energy in each of the condensers in series of Par. 159 and the total stored energy.

Using equation (78),

$$W_1 = \frac{1}{2} \frac{(1{,}566 \times 10^{-6})^2}{5 \times 10^{-6}} = 0.2453 \text{ joule.} \quad \textit{Ans.}$$
$$W_2 = \frac{1}{2} \frac{(1{,}566 \times 10^{-6})^2}{10 \times 10^{-6}} = 0.1225 \text{ joule.} \quad \textit{Ans.}$$
$$W_3 = \frac{1}{2} \frac{(1{,}566 \times 10^{-6})^2}{12 \times 10^{-6}} = 0.1020 \text{ joule.} \quad \textit{Ans.}$$

The total energy $W = \frac{1}{2} (1{,}566 \times 10^{-6} \times 600) = 0.4698$ joule. *Ans.*

161. Measurement of Capacitance.—There are two common methods of measuring capacitance, the direct-current or ballistic method and the alternating-current or bridge method.

The direct-current method employs a galvanometer which is used ballistically. It can be shown that if the moving coil of the ordinary galvanometer have considerable inertia and be properly damped, its maximum throw, due to the impulse produced by the sudden passage of a current through the coil, is proportional to the total quantity of electricity passing through the galvanometer. This assumes that the entire charge passes

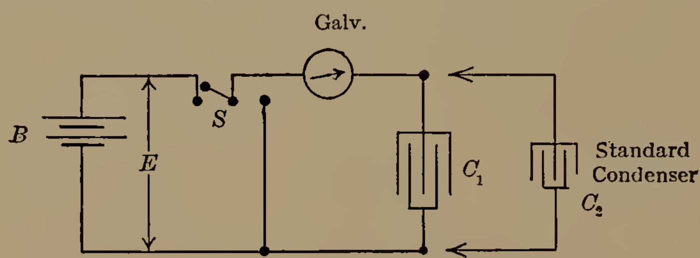


FIG. 177.—Ballistic method of measuring capacitance.

through the coil before the coil begins to move. Let D be the maximum galvanometer throw in centimeters. Then

$$Q = KD \tag{79}$$

where Q is the quantity and K the galvanometer constant.

To make the measurement, the apparatus is connected as shown in Fig. 177. A battery B supplies the current for the apparatus. The measurement may be made on either the charge or the discharge of the condenser, or check measurements may be made using both the charge and the discharge. If the condenser is at all leaky, the discharge method is preferable.

When the switch S is closed to the left, the condenser C_1 is charged through the galvanometer and the maximum throw of the galvanometer is read. The galvanometer should return immediately to zero. If it shows a steady deflection it indicates a leaky condenser. In a corresponding manner the ballistic throw of the galvanometer may be read on discharge by closing switch S to the right after charging. Let D_1 be the deflection of the galvanometer, either on charge or discharge, when C_1, the unknown capacitance, is connected.

A standard capacitance C_2 is now substituted for the unknown condenser. Let D_2 be the corresponding galvanometer deflection. Then

$$\frac{C_1}{C_2} = \frac{D_1}{D_2}$$

or

$$C_1 = C_2 \frac{D_1}{D_2} \tag{80}$$

$\frac{C_2}{D_2}$ is the galvanometer constant.

It is often desirable to use an Ayrton shunt in such measurements, as it gives the apparatus greater range. When such a shunt is used, proper correction must be made for its multiplying power (see page 119).

In the bridge method, two capacitances form adjacent arms of a Wheatstone bridge and two resistances form the other two arms, (Fig. 178). An alternating current having a frequency of about 1,000 cycles is desirable. The secondary of an induction coil may be used as the source of power and a telephone as a detector. Let C_x be the unknown capacitance and C_2 a standard capacitance, which may or may not be adjustable. R_1 and R_2 are two known resistances, one of which should be adjustable unless C_2 is so.

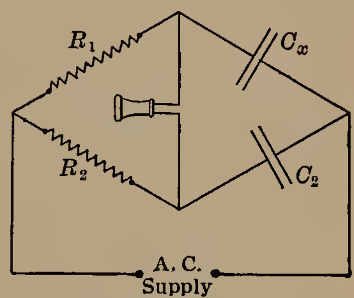


FIG. 178.—Bridge method of measuring capacitance.

Either C_2 or one of the resistances is adjusted until there is no sound in the telephone, showing that the bridge is in balance. Under these conditions

$$\frac{C_x}{C_2} = \frac{R_2}{R_1}$$

$$C_x = C_2 \frac{R_2}{R_1}. \tag{81}$$

Compare this equation with (48), page 131. Capacitances in the electrostatic circuit correspond to conductances in the electric circuit.

In the above measurements, it is assumed that there is little if any leakage through the condensers.

162. Cable Testing—Location of a Total Disconnection.—In Chap. VII, it was shown that a grounded fault in a cable could be located by suitable resistance measurements, as with the Murray-loop test. If a cable be totally disconnected and its broken ends remain insulated, these loop tests are impossible. The distance to the fault may now be determined by capacitance measurements. The connections are shown in Fig. 179. The

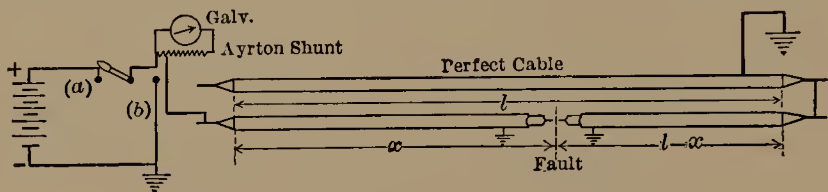


FIG. 179.—Locating an "open" in a cable.

capacitance C_1 of the length x to the fault is first measured by the ballistic method. If a *similar* perfect cable parallels the faulty cable, the two are looped at the far end and the capacitance C_2 of the combined length l of the perfect cable and the length $l - x$ of the faulty cable is measured. Let D_1 and D_2 be the respective galvanometer deflections. If the cables are uniform, the capacitances are proportional to the respective lengths.

Therefore

$$\frac{C_1}{C_2} = \frac{x}{2l - x} = \frac{D_1}{D_2}$$

$$x = l\frac{2D_1}{D_1 + D_2}. \qquad (82)$$

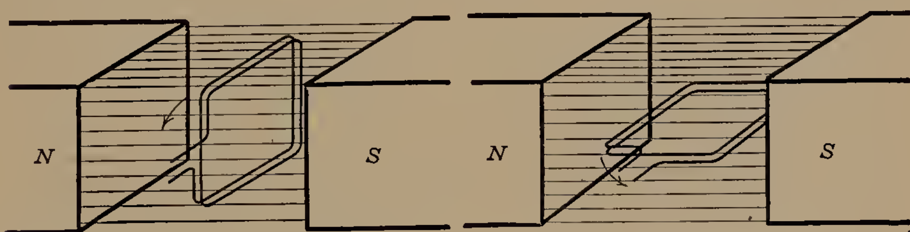
CHAPTER X

THE GENERATOR

A generator is a machine which converts mechanical energy into electrical energy. This is accomplished by means of an armature, carrying conductors upon its surface, acting in conjunction with a magnetic field. Electrical power results from the relative motion of the armature conductors and the magnetic field.

In the direct-current generator the field is usually stationary and the armature rotates. In most types of alternating-current generators the armature is stationary and the field rotates. In either case the rotating member is driven by mechanical power applied to its shaft.

163. Induced Electromotive Force in Generator.—It was shown in Chap. VIII that if the flux linking a coil is varied in any manner, an electromotive force is *induced* in the turns of the coil. The



(*a*) Maximum lines linking coil. (*b*) No lines linking coil.

Fig. 180.—Simple coil rotating in a magnetic field.

action of the generator is based on this principle. The flux linking the armature coils is varied by the relative motion of the armature and field.

In Fig. 180 a coil revolves in a uniform magnetic field produced by a north and a south pole. In (*a*) the coil is perpendicular to the magnetic field and in this position the maximum possible flux links the coil. Let this flux be ϕ.

If the coil be rotated counter-clockwise a quarter of a revolution, it will lie in the position shown in (*b*). As the plane of the coil

is parallel to the direction of the flux, no lines link the coil in this position. Therefore, in a quarter revolution the flux which links the coil has been decreased by ϕ lines. Since the flux linking the coil has changed in the quarter revolution, an induced electromotive force must result.

The change in flux linkages of a generator coil brought about by the change in relative position of armature coil and flux is

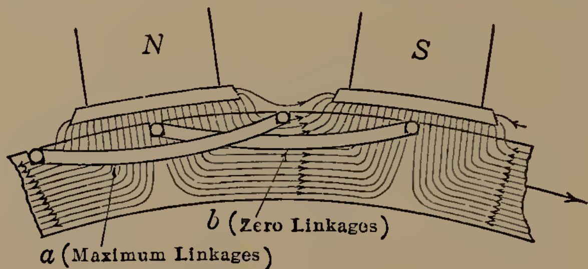


FIG. 181.—Change of coil linkages in a generator.

illustrated in Fig. 181 for a multipolar machine. In position a the entire flux from a north pole links the coil. In position b no flux *links* the coil, the flux merely entering and leaving the coil without linking it. When the coil moves to a position directly under the south pole, the entire flux to the south pole links it. The change in the flux linking the coil during the time required for the coil to move through one pole pitch is 2ϕ, where ϕ is the total flux entering the armature from the north pole. Therefore, an induced electromotive force must result.

164. Electromotive Force Induced in a Single Conductor. When an electromotive force is induced by the relative motion of a coil and a magnetic field, it is usually more convenient, for purposes of analysis, to consider the electromotive force as being due to the *cutting* of the magnetic flux by the individual conductors forming the coil-sides, rather than to the change in flux linkages with the coil itself. For example, in Fig. 182, the single-turn coil may be considered as composed of two active conductors ab and cd, connected by the inactive conductors ac and bd. (The conductor

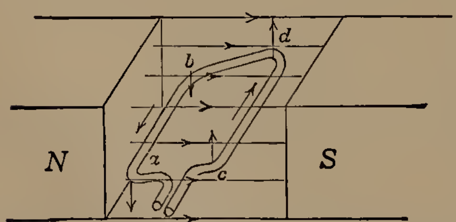


FIG. 182.—Electromotive force induced in individual coil-sides.

ac is closed through the external circuit.) No electromotive force is induced in the inactive conductors *ac* and *bd* because of their geometrical relation to the flux, as will be shown later. The coil in Fig. 182 is shown as rotating counter-clockwise. The direction of the induced electromotive force in conductor *ab* will be outwards and that in *cd* will be inwards by Fleming's right-hand rule (see Par. 165, page 208). Considering the electromotive force of the coil as a whole, the two electromotive forces in *ab* and *ad* are additive even though their space directions are opposite.

The method of considering the electromotive force as being due fundamentally to the cutting of the flux by the individual conductors gives the same numerical result as is obtained by considering the electromotive force as being due to change in flux linkages.

The electromotive force in volts generated by a single conductor which cuts a magnetic field is given by the fundamental relation

$$e = Blv10^{-8} \qquad (83)$$

where B, l, and v are mutually perpendicular.

B is the flux density of the field in gausses and is assumed uniform over the entire length l of the conductor; l is the length of the conductor in *centimeters;* and v is the velocity of the conductor in centimeters per second (Fig. 183).

That the electromotive force in any single conductor is not large is illustrated by the following example.

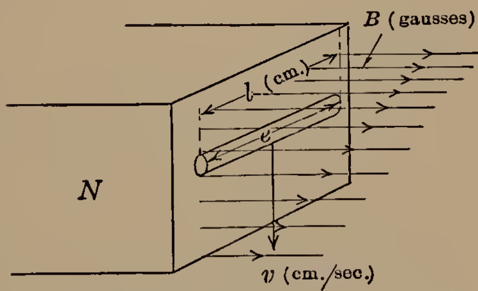


FIG. 183.—Electromotive force induced in a single conductor.

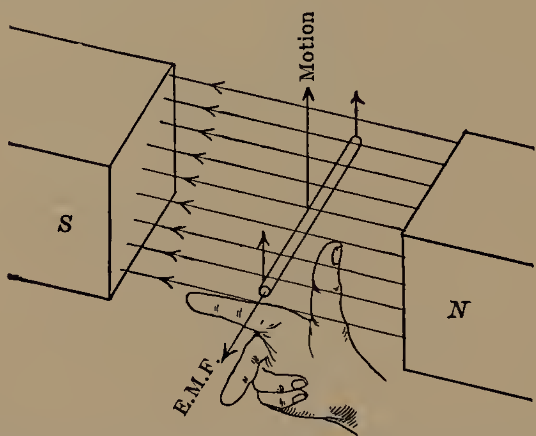
Example.—The average flux density over the pole-face of the ordinary generator is approximately 45,000 lines per square inch; hence $B = \frac{45,000}{(2.54)^2} =$ 7,000 gausses. The pole-faces cover only about 0.7 the peripheral surface of the armature. Hence the average density over the entire peripheral surface is $7,000 \times 0.7 = 4,900$ gausses. The peripheral speed of the ordinary armature is of the order of 100 ft. per second, or approximately 3,000 cm. per second. As a rule, the axial length of the armature of such a machine will not exceed 2 ft. Hence the length $l = 24 \times 2\ 54 = 61$ cm.

The induced electromotive force per single conductor is

$$e = 4,900 \times 61 \times 3,000 \times 10^{-8} = 9 \text{ volts}$$

It is necessary, therefore, to connect a number of such conductors in series in order to obtain the usual commercial voltages of 115, 230, and 600 volts.

165. Direction of Induced Electromotive Force. Fleming's Right-hand Rule.—A definite relation exists among the direction of the flux, the direction of motion of the conductor and the direction of the electromotive force induced in the conductor, just as a definite relation exists between the direction of current and of the flux which it produces.



Fore finger along lines of force. Thumb in direction of motion. Middle finger gives direction of induced emf.

Fig. 184.—Fleming's right-hand rule.

A convenient method for determining this relation is the *Fleming right-hand rule.* In this rule the fingers of the *right* hand are utilized as follows:

Set the forefinger, the thumb, and the middle finger of the right hand at right angles to one another (Fig. 184). If the *forefinger points along the lines of flux and the thumb in the direction of motion of the conductor, the middle finger will point in the direction of the induced electromotive force.* (The fact that the *forefinger* points along the lines of *flux* assists in memorizing this rule.)

This rule is illustrated by Fig. 184.

166. Electromotive Force Generated by the Rotation of a Coil. A coil of a single turn is shown in Fig. 185 (*a*). The coil rotates in a counter-clockwise direction at a uniform speed in a uniform magnetic field. As the coil assumes successive positions, the

electromotive force induced in it changes. When it is in position (1), the electromotive force generated is zero, for in this position neither active conductor is *cutting* magnetic lines, but is actually moving parallel to these lines. When the coil reaches position (2) (shown dotted), its conductors are cutting across the lines obliquely and the electromotive force has a value indicated at (2) in Fig. 185 (*b*). When the coil reaches position (3), the conductors are cutting the lines *perpendicularly*, and are therefore cutting at the maximum possible rate. Hence the electromotive force is a maximum when the coil is in this position. At position (4) the electromotive force is less, due to a lesser rate of cutting.

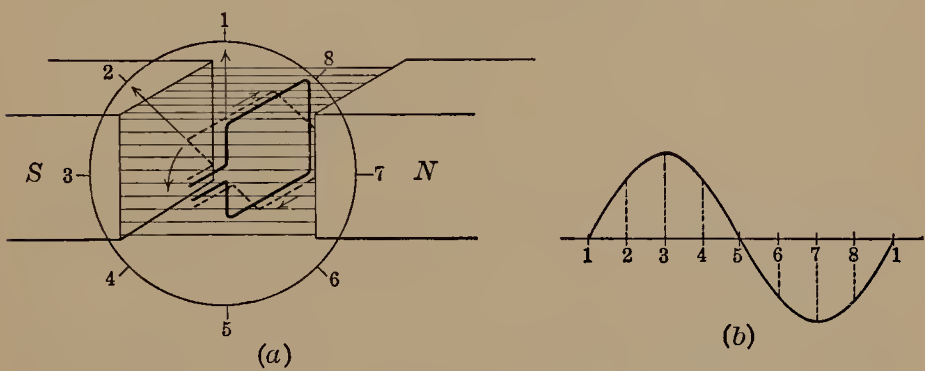


Fig. 185.—Emf. induced in a coil rotating at constant speed in a uniform magnetic field.

At position (5) no lines are being cut and, as in (1), there is no electromotive force. In position (6), the direction of the electromotive force in the conductors will have reversed, as each conductor is under a pole of opposite sign to that for positions (1) to (5). The electromotive force increases to a negative maximum at (7) and then decreases negatively until the coil again reaches position (1). After this the coil merely repeats the cycle.

This induced electromotive force is alternating and an electromotive force varying in the manner shown is called a *sine wave*[1] of electromotive force. This alternating electromotive force may be impressed on an external circuit by means of two *slip-rings* (Fig. 186). Each ring is continuous and insulated from the other ring and from the shaft. A metal or a carbon brush rests on

[1] See Part II, Chap. I, for a more detailed discussion.

each ring and conducts the current from the coil to the external circuit.

If a unidirectional or *direct current* is desired, that is, one whose direction of flow in the external circuit is always the same, such rings cannot be used. A direct current must always flow into the external circuit in the *same direction*. As the coil current must

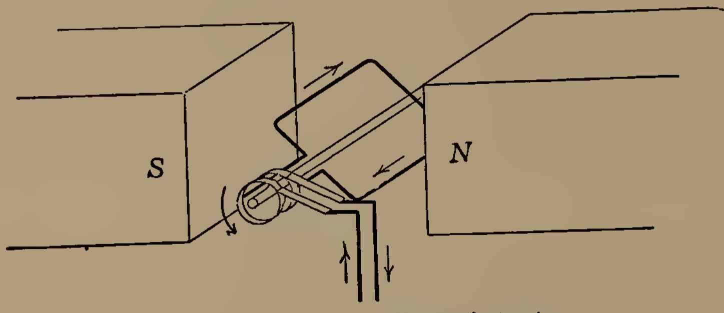


FIG. 186.—Current taken from rotating coil by means of slip-rings.

necessarily be alternating, since the electromotive force which produces it is alternating, as has just been shown, this current must be rectified before it is allowed to enter the external circuit. This rectification can be accomplished by using a split-ring, as shown in Fig. 187. Instead of using two rings, as in Fig. 186, one ring only is used. This is split by saw cuts at two points diametrically

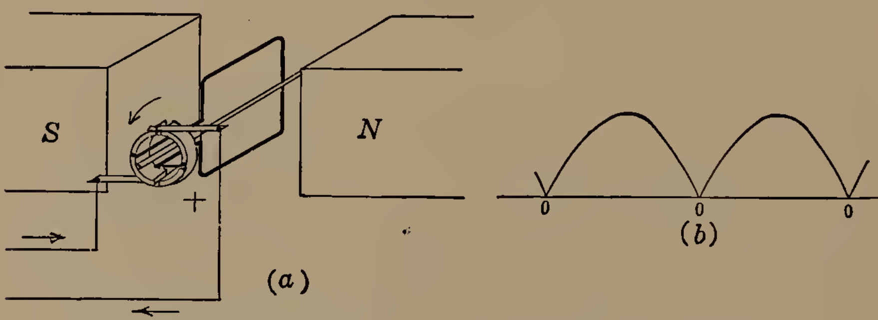


FIG. 187.—Rectifying effect of a split-ring or commutator.

opposite each other. Each of the two ends of the coil is connected to one of the sections or segments so produced.

A careful consideration of Fig. 187 shows that as the direction of the current in the coil reverses, its connections to the external circuit are simultaneously reversed. Therefore, the direction of flow of the current in the external circuit is not

changed. The brushes pass over the cuts in the ring when the coil is perpendicular to the magnetic field or when it is in the so-called neutral plane and is generating no voltage, as shown in Fig. 185. These neutral points are marked 0-0-0 in Fig. 187 (*b*).

By comparing Fig. 185 (*b*) with Fig. 187 (*b*), it will be seen that so far as the *external* circuit is concerned the negative half of the wave has been reversed, and so made positive.

The split ring shown in Fig. 187 is a simple *commutator*. The commutator causes the alternating current generated in the individual armature coils to flow to the external circuit as a unidirectional or direct current. In commercial machines there is a large number of segments or commutator bars.

A voltage with a zero value twice in each cycle, as shown in Fig. 187, could not be used commercially for direct-current service. Also a single-coil machine would have a small output for its size and weight. By employing a large number of coils and commutator segments, the electromotive force becomes practically steady and the output of the machine per unit weight becomes sufficiently large to make the machine commercially economical.

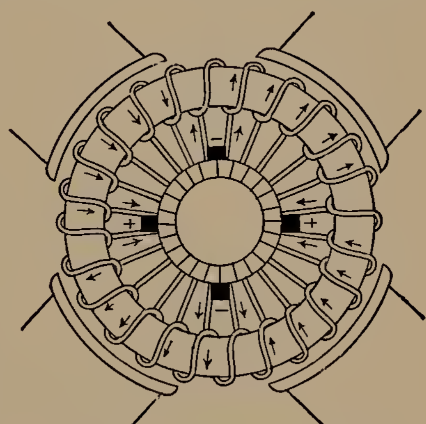


Fig. 188.—Gramme-ring winding.

167. Gramme-ring Winding. This type of winding in diagrammatic form (Fig. 188) consists of insulated wire wound spirally around a hollow cylinder of iron with taps taken from the winding at regular intervals and connected to commutator segments. This winding is simple, and has the advantage that a single winding is adapted to any number of poles, if the voltage limitations do not prevent. The portions of the conductors which lie inside the cylinder or ring cut practically no flux and act merely as connectors for the active portions of the conductors which lie on the outer surface of the cylinder or armature. Because of the small proportion of active conductor, a relatively large amount of copper is required in such a winding. In small machines, there is not sufficient room to carry the inactive conduc-

tors back through the armature core. In a gramme-ring winding, formed coils cannot be used, and this makes the winding expensive.

It will be noted that the electromotive force between brushes in a gramme-ring winding is the sum of the electromotive forces of all the coils that lie between brushes. When one coil passes a brush another moves forward to take its place. Figure 189

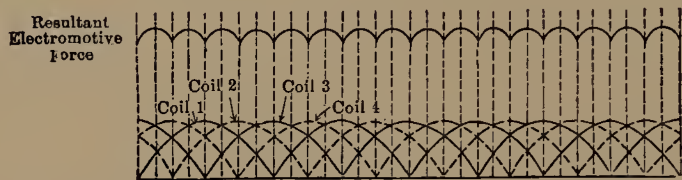


FIG. 189.—Resultant electromotive force due to four series-connected coils between brushes.

shows the electromotive force between brushes due to four coils, it being assumed that the voltage curve for each is a sine wave. The electromotive force of each coil is plotted separately. These electromotive forces do not all have their zero value at the same time, nor do they reach their maximum value at the same time, owing to the different positions of the individual coils. The resultant electromotive force at any point is the sum of these individual electromotive forces at that point. This voltage should be compared with the electromotive force obtained with the single-coil winding shown in Fig. 187 (*b*). It will be noted that a fairly smooth resultant electromotive force is obtained even with as few as four coils, the "ripples" being noticeable but comparatively small in magnitude.

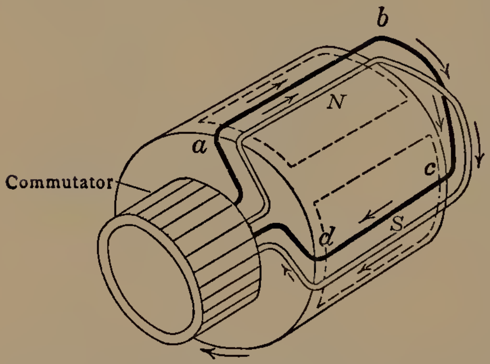


FIG. 190.—Two coils in place on a 4-pole, drum-wound armature.

168. Drum-winding.—The objections to the ring-winding are overcome by the use of the drum-winding. The conductors of this winding all lie on the surface of the armature and are connected to one another by front and back connections or coil

ends (*ad* and *bc*, Fig. 190, are coil ends). (Also see page 206, Fig. 182.) With the exception of these end connections, all the armature copper is "active," that is, it cuts flux and so is active in generating electromotive force.

The sides of each coil should be approximately one pole-pitch (the peripheral distance between centers of adjacent poles) apart. If one conductor is under a north pole, the other is then under a south pole, and as both move in the same direction, but under opposite poles, the electromotive forces of these two conductors will be in opposite directions (Fig. 190). (Also see page 206, Fig. 182.) Due to the manner in which these conductors are connected at their ends, the electromotive forces of the conductors add.

In most gramme-ring windings, and in the earlier drum-wound machines, the surface of the armature core was smooth. The conductors were held in position partly by projecting pins, and were prevented by binding wires from flying out under the action of centrifugal force. The smooth-core armature has been superseded by the "iron-clad" armature in which the conductors are embedded in slots as indicated in Fig. 193, page 215 The slots are lined with insulation and the conductors are held firmly in the slots by wooden or non-conducting wedges in the larger machines and by binding wires in the smaller machines (see Fig. 202, page 226). The iron-clad construction is much better mechanically than the smooth-core construction and it also permits a much shorter air-gap.

169. Simplex Lap-winding.—As a rule, direct-current armatures are wound with former-made coils (Fig. 191). These coils are usually wound with the necessary number of turns on machines, and are then taped with cotton or mica tape. They are then bent into proper shape by another machine. The two ends of the coil are left bare so that later they may be soldered to the commutator bars. The span of the coil, called the coil-pitch, should be equal or nearly equal to the pole-pitch, so that when one side of the coil is under the center of a north pole the other side is under a south pole. The span of the coil may be less than the pole-pitch, in which case the winding is called a *fractional-pitch winding.* A fractional pitch as low as eight-tenths is sometimes used. With the ordinary values of fractional pitch, the

induced electromotive force is reduced by a small amount and some saving of copper results from the shorter end-connections.

Usually, direct-current windings are *two-layer* windings. That is, each slot contains two coil-sides (Fig. 193). The coils are so placed on the armature that one side of each coil occupies the top

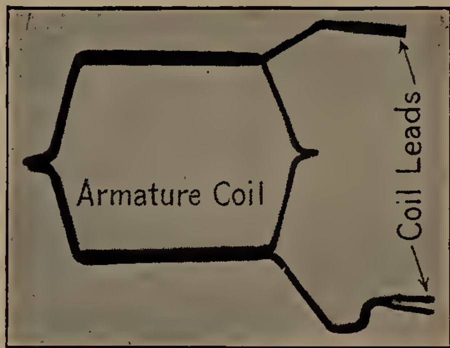


FIG. 191.—Formed armature coil.

of one slot and its other side occupies the bottom of another slot spaced approximately one pole-pitch away (Fig. 193). With this arrangement, the coils overlap one another, not unlike shingles on a roof, and the winding is made to fit readily on the armature. Also, with this type of winding, the end connections are easily

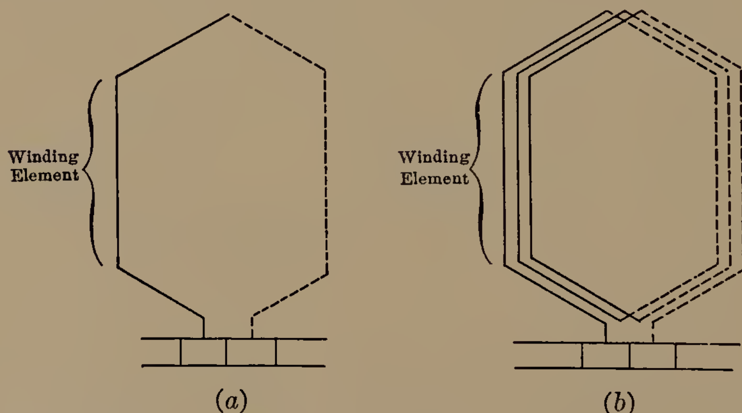


FIG. 192.—Single coil representing a three-turn coil of an armature winding.

made, as the coil-ends can be bent around one another in a systematic manner, passing from the bottom to the top layer by means of a peculiar twist in the ends of the coils.

If an armature is simplex *lap-wound* and each coil has a single turn, the two ends of the coil are connected to adjacent com-

mutator segments (Fig. 192 (*a*)). If each coil consists of several turns, the coil is taped up as a unit (Fig. 191) and the two end of the coil are connected to adjacent commutator segments (Fig. 192 (*b*)).

In the simplest form of two-layer winding, two coil-sides occupy a single slot (Fig. 193). It is customary in designing such a winding to number the coil sides, beginning with one at the top of a slot as 1, that directly under it in the same slot as 2, the one at the top of the adjacent slot to the right as 3, etc., as shown in

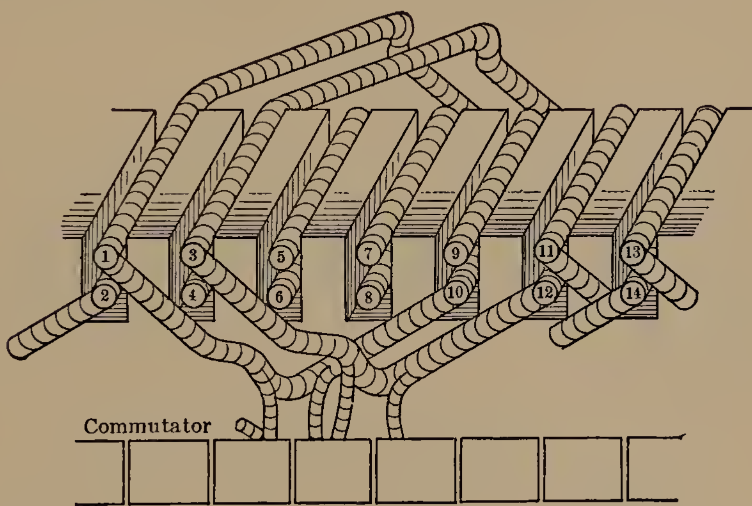


Fig. 193.—Simplex lap-winding having back pitch of 9 and front pitch of 7.

Figs. 193 and 194. The number of elements that a coil connection spans on the commutator end of the armature is called the *front pitch,* and the number that the coil connection spans at the back of the armature is called the *back pitch.* The average of the front and back pitch is called the *average* pitch. For example, in Fig. 193, the element 3 connects with element 10 at the front of the armature through a commutator segment, hence the front pitch is 7. The element 1 connects with the element 10 at the back of the armature, hence the back pitch is 9. This is also shown in Fig. 194. Since all connections are made from elements at the top of the slots to other elements at the bottom of other slots, both the front pitch and the back pitch must be *odd.* Also, the pitch must be such that if one side of a coil is under a north pole the other side is under a south pole.

A study of Figs. 193 and 194 shows that the connection on the back of the armature, whose span equals the back pitch, joins the two sides of any one coil together. The coil connection on the front of the armature whose span equals the front pitch, and which is ordinarily made through a commutator segment, is the means by which the armature coils are connected in series.

The winding in Fig. 194, a part of which is shown in Fig. 193, is designed for a 4-pole, 18-slot machine. There are, therefore, 36 elements. In order that the two sides of any coil may span a distance equal approximately to the pole-pitch, the average pitch

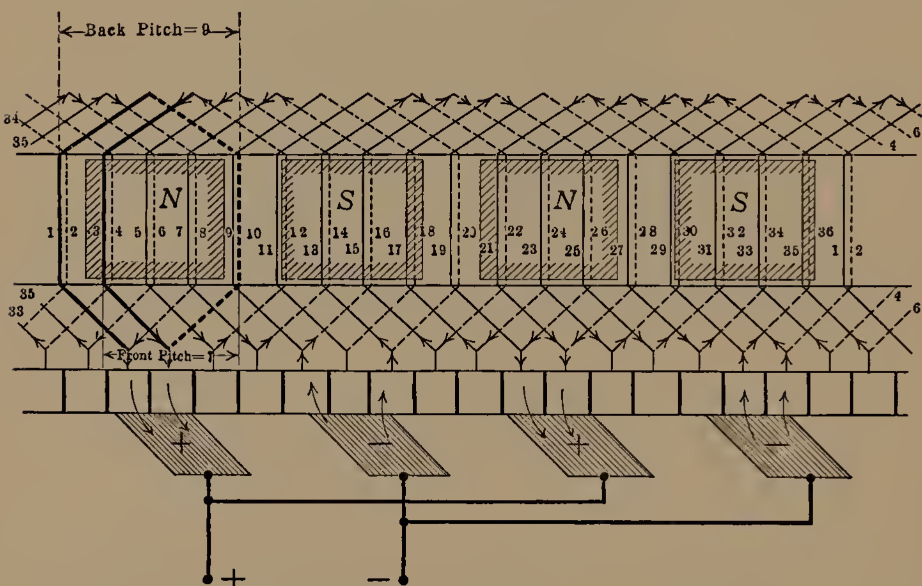


FIG. 194.—Development of a four-pole lap-winding.

must be in the neighborhood of $36/4$, or 9. In this winding the back pitch is chosen as 9 and the front pitch as 7, giving an average pitch of 8. Since the back pitch is 9, the coil must span a peripheral distance on the armature equal to one pole-pitch. Hence, this back pitch of 9 gives a *full-pitch winding*, as the electromotive forces in the two sides of the coil are additive at every instant of time. The fact that the average pitch is 8 and the front pitch is 7 does *not* mean that this is a fractional-pitch winding, since the front connection is merely the means by which adjacent coils are connected in series. Figure 194 shows the development of the entire winding.

If the back pitch is greater than the front pitch, as in Figs. 193 and 194, the winding advances clockwise when viewed from the commutator end and is called a *progressive* winding. If the back pitch is less than the front pitch, the winding advances counter-clockwise when so viewed, and is called a *retrogressive* winding.

It is obvious that the number of commutator segments is one-half the number of winding elements.

170. Lap-winding—Several Coil-sides per Slot.—In the larger sizes of machines, it is often necessary to place several coil-sides or elements in one slot, usually 4, 6, or 8. For a given armature,

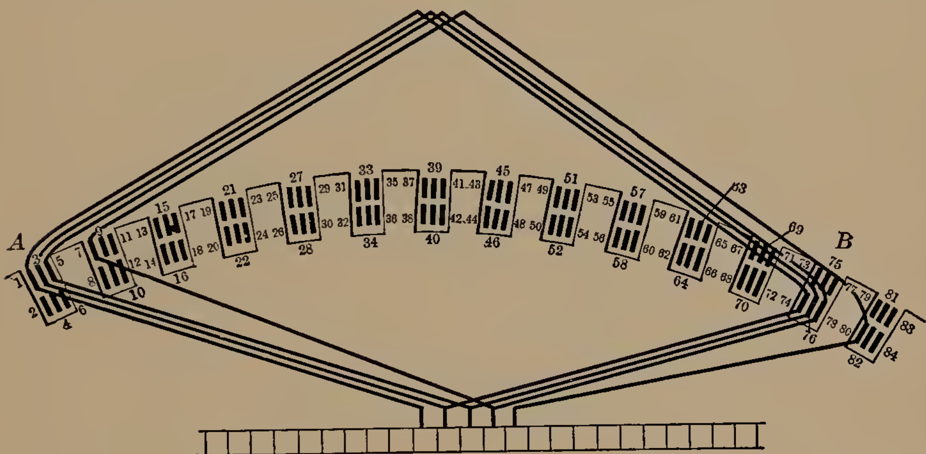


FIG. 195.—Method of connecting the conductors of a triple coil.

if two elements per slot were used, the number of slots would be excessive and the slots would be too small. Figure 195 illustrates the placing of six coil-sides in each slot. These coil-sides are taped together and placed in the slot as a unit. The coil so formed is called a *multiple coil.* The elements are numbered (Fig. 195) as with two coil-sides per slot. The pitch, however, must be so chosen that the elements 1, 3, and 5 lie in the top of a single slot and 74, 76, and 78 of the same coil lie at the bottom o another single slot.

171. Paths through an Armature.—If four batteries, each having an electromotive force of 2 volts and a current capacity of 10 amp., be connected in parallel (Fig. 196 (*a*)) there will be *four* paths for the current to follow in going through the batteries. The voltage of the combination will be 2 volts and the ampere

capacity will be 40 amp., making a total power capacity of 80 watts. If now these same batteries be arranged in two groups of two in series (Fig. 196 (*b*)) there will be but two paths for the current to follow, but the voltage is now 4 volts. The current capacity is now 20 amp., and the power capacity is $4 \times 20 = 80$ watts, the same as before.

Similarly, the conductors in an armature may be so connected that certain groups of conductors are in series. These groups may then be connected so that there are two or more paths in parallel. As with the batteries, the capacity of the machine is

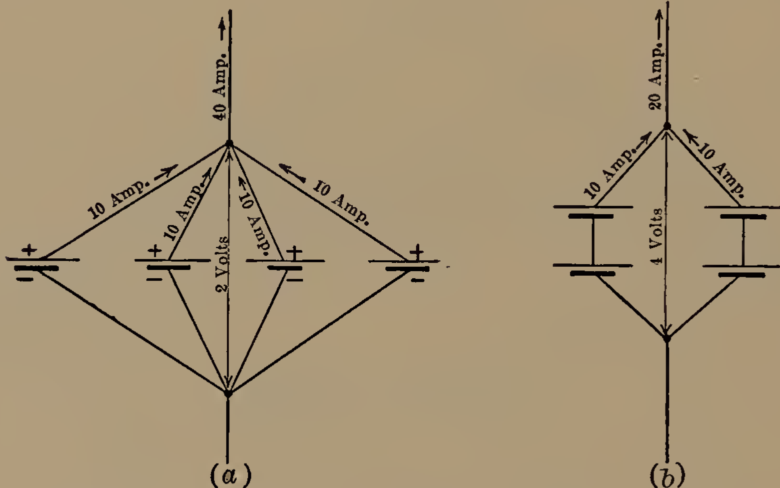


Fig. 196.—Parallel and series-parallel arrangement of batteries.

not changed by connecting the conductors in series and parallel groups, but the voltage and ampere ratings are changed. To determine the number of such parallel paths, start at one of the machine terminals, as, for example, the negative, and see how many different paths through the armature it is possible to follow in reaching the positive terminal.

To illustrate for a drum-wound armature having a lap-winding, the 18-slot winding of Figs. 193 and 194, developed in circular form, is shown in Fig. 197. Remembering that the two negative brushes are connected together to form the negative terminal of the machine and the two positive brushes are connected together to form the positive terminal of the machine, there are four distinct paths between the negative and positive terminals of the

machine. For the sake of simplicity, two paths are shown with heavy lines, one from brush *a* to brush *b*, and the other from brush *c* to brush *d*. These constitute two paths. By tracing through the lighter lines, two more paths may be found, one between brushes *c* and *b* and the other between brushes *a* and *d*, making four paths in all.

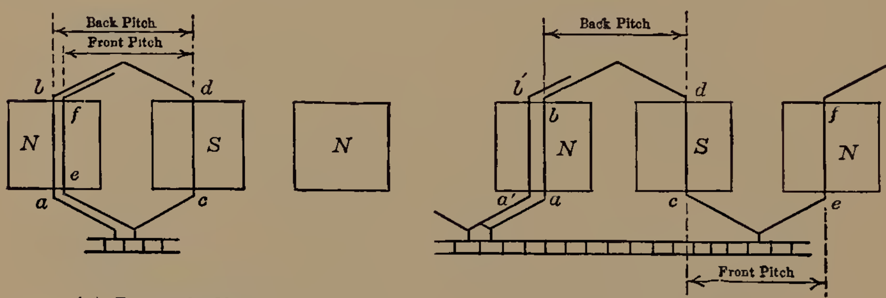
In all simplex lap-windings there are as many paths through the armature as there are poles.

Fig. 197.—Heavy lines show two of the four parallel paths of a lap-winding.

From the foregoing, it is clear that, with a given machine having a fixed number of armature conductors, the current-carrying capacity of the armature is directly proportional to the number of paths through the armature, and the voltage between the terminals of the machine is inversely proportional to the number of paths through the armature. The kilowatt rating remains unchanged.

172. Simplex Wave-winding.—It has been shown that with the *lap*-winding, a conductor under one pole is connected directly to

a second conductor which occupies a nearly corresponding position under the next pole. This second conductor is then connected *back* again to a conductor under the *original* pole, but removed two or more conductors from the initial conductor. This is shown in Fig. 198 (*a*), where conductor *ab* under a north pole is connected to conductor *cd* having a corresponding position under the next south pole. Conductor *cd* is then connected to *ef*, which is adjacent to *ab* under the original north pole. Obviously, it would make no difference, so far as the direction and magnitude of the induced electromotive force in the winding are concerned, if the connection, instead of returning back to the same



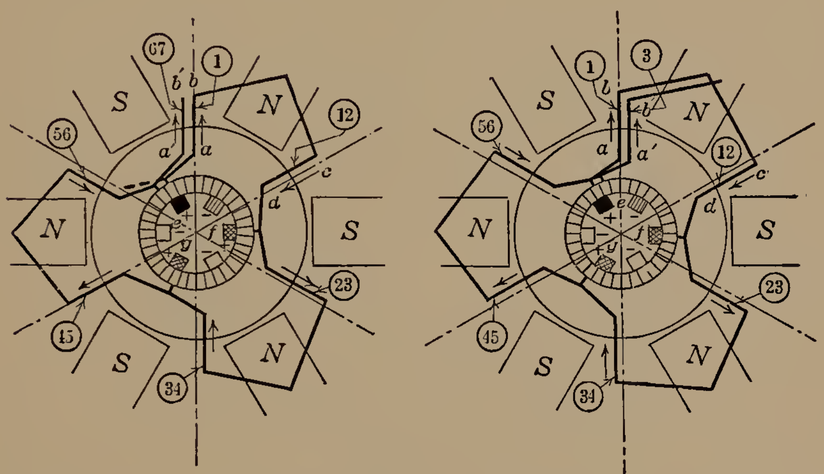
(*a*) Lap-winding (*b*) Wave-winding

FIG. 198.—Lap- and wave-windings.

north pole, advanced *forward* to the next north pole, as shown in Fig. 198 (*b*). When the connection is so made, the winding passes successively every north and south pole before it returns again to the original pole, as shown at *a′b′* in Fig. 198 (*b*). The winding, after passing once around the armature, reaches conductor *a′b′* lying under the same pole as the initial conductor *ab*. When a winding advances from pole to pole in this manner, it is called a *wave-winding*. The number of units spanned by the end connections on the back of the armature is called the *back pitch* (Fig. 198 (*b*)). This corresponds to the back pitch in the lap-winding (Fig. 198 (*a*)). The number of elements which the end connections span on the commutator end of the armature is the *front pitch* (Fig. 198 (*b*)). This should be compared with the front pitch in Fig. 198 (*a*). In the wave-winding, as in the lap-winding, the front pitch and the back pitch must both be odd in order that one side of a coil may lie in the top of a slot and the other side in the bottom of a slot. Unlike the lap-winding, the

front pitch may equal the back pitch in the wave winding. The average of the front pitch and the back pitch gives the *average pitch.*

The wave-winding is much more restricted than the lap-winding in the front and back pitch that can be used and in the number of slots that may be used with a given number of poles. This is illustrated in Fig. 199, which shows a six-pole machine. In (*a*) there are 34 commutator segments and slots, hence there are 68 elements. The average pitch should be $^{68}/_{6}$, or 11,



(*a*) Retrogressive wave-winding with 34 commutator segments.
(*b*) Progressive wave-winding with 32 commutator segments.

FIG. 199.

approximately. If both the front pitch and the back pitch be 11, and if element *ab* be numbered 1, the connections are made 1–12–23–34–45–56–67 or *a′b′*, which is two elements to the left of *ab* and in the top of the adjacent slot. This winding may be continued in this manner, until every slot is occupied by two elements and the winding closes on itself. Since this winding, after each passage around the armature, falls in a counter-clockwise direction from its starting point, it is a *retrogressive* winding.

In Fig. 199 (*b*), the machine is shown as having 32 commutator segments and slots, hence there are 64 winding elements. Let

conductor *ab* be numbered 1 and let both the front and back pitch be 11. The winding connections are then 1–12–23–34–45–56–3 or *a′b′*, which is two elements to the right of *ab* and in the top of the adjacent slot. Again, this winding will close on itself after every slot on the armature is filled with two elements. Since this winding, after each passage around the armature, falls in a clockwise direction from its starting point, it is a *progressive* winding.

If this armature had 33 slots or 66 winding elements, it would close on itself after one passage around the armature and hence would not be a possible winding. That is, the winding would close on itself when only one slot under each pole contained a single coil-side, whereas the winding should close on itself only after *every* slot contains *two* coil-sides. Again, if the winding closed on itself after a single passage around the armature, three coils, all of which are generating electromotive force (Fig. 199) would form a closed circuit. That is, the coils would be short-circuited, and a large current would circulate within the armature coils.

If a wave-winding were necessary under these conditions of 33 slots, a *dummy* or *idle* coil would be used. This consists merely of one of the regular coils which is *not* connected to the commutator, but has its ends taped. It merely acts as a filler. The using of such a coil in this case would reduce the active winding elements to 64, which, as has just been shown, makes a possible winding.

A complete wave-winding for a 4-pole, 17-slot machine is shown in Fig. 200.

173. Number of Brushes with Wave-winding.—In Fig. 199 (*a*), three positive brushes, *e*, *f*, and *g*, are shown. Each rests at the instant considered on commutator segments connected to the conductors *ab*, *cd*, etc. At this instant, these conductors lie practically in the neutral plane of the machine and there is no induced electromotive force in them. Therefore, brushes *e*, *f*, and *g* are connected together through the armature by *idle* conductors. Consequently, but one positive brush is necessary to carry the current from the armature, since any one positive brush is connected by idle conductors to all the points in the winding from which current should be collected (also see Fig. 200).

All three brushes would ordinarily be used, since more current could be collected with a commutator of given length.

The foregoing applies equally well to the number of negative brushes, as may be seen from a study of Fig. 200.

In railway motors, only two brushes are used, being situated near the top of the commutator, where they are accessible from the hand-holes in the motor frame directly under the floor of the car.

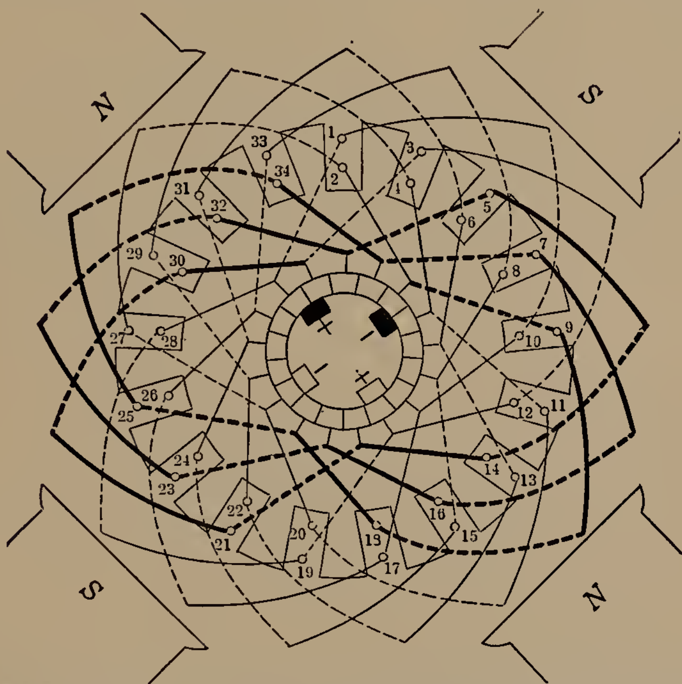


FIG. 200.—17-slot, 4-pole, simplex wave-winding; back pitch = 9, front pitch = 7; one of two parallel paths shown with heavy lines; the other with light lines.

174. Paths through a Wave-winding.—In a simplex wave-winding, there are but *two* parallel paths, regardless of the number of poles. Figure 200 shows a 4-pole, 17-slot, simplex wave-winding, having two coil-sides per slot. One of the two parallel paths is shown by the heavy lines. Approximately half the winding is shown heavy, the other half constituting the second path. (The coils short-circuited by the brushes are not included.)

175. Uses of the Two Types of Winding.—With a given number of poles and armature conductors, the wave-winding gives a higher voltage than the lap-winding. It is used, therefore, in

small machines, especially in those designed for 600-volt circuits, where a lap-winding would require a very large number of small conductors. This, in turn, would involve a higher winding cost and less efficient utilization of the space in the slots.

The wave-winding has the additional advantage that the electromotive force in each path is produced by series-connected conductors, which lie under successive north and south poles. Any magnetic unbalancing, therefore, due to such causes as air-gap variation and difference in pole strength, does not produce cross-currents, because the corresponding conductors of each and every path are moving by the same poles at every instant and the effect of such unbalancing will be the same in each path.

When large currents are required, the lap-winding is more satisfactory, since it gives a large number of paths. This is particularly true of large engine-driven, multipolar generators.

DYNAMO CONSTRUCTION

176. The Armature.—The cross-section of a typical dynamo is given in Fig. 201. The construction of the armature is plainly shown. A cast-iron spider is pressed or keyed to the shaft. The armature iron consists of circular sheet-iron stampings or laminations, shown at (*b*), which are assembled and clamped over the spider as shown in (*a*). Three or four ventilating ducts, held open by spacers, allow the passage of air in through the spider and radially outward through the ducts.

Figure 202 shows an armature in the process of winding. The slots are first lined with fish paper, which is a thin, tough fiber, and the coils, already formed and taped, are then forced into the slot. The coils are held in the slot either by wooden or fiber wedges or with banding wire (Fig. 202).

177. Frame and Cores.—The frame or yoke of a dynamo has two functions. It forms a part of the magnetic circuit, and it acts as a mechanical support for the machine as a whole. In small machines, where weight is of little importance, the yoke is often made of cast iron. The feet almost always form a part of the casting. In another type of construction, a steel plate is rolled around a cylindrical mandrel and then welded. The yoke of the dynamo (Fig. 203) is made in this manner. The feet in this

case are made of steel stampings and are riveted on (Fig. 203). In larger machines, the yoke is made of cast steel and is usually more or less oval in cross-section (Fig 204). The feet are a part of the yoke casting. The yoke for the larger machines is usually cast in two pieces which are bolted together. This facilitates the shipment of large machines and allows the armature to be removed easily.

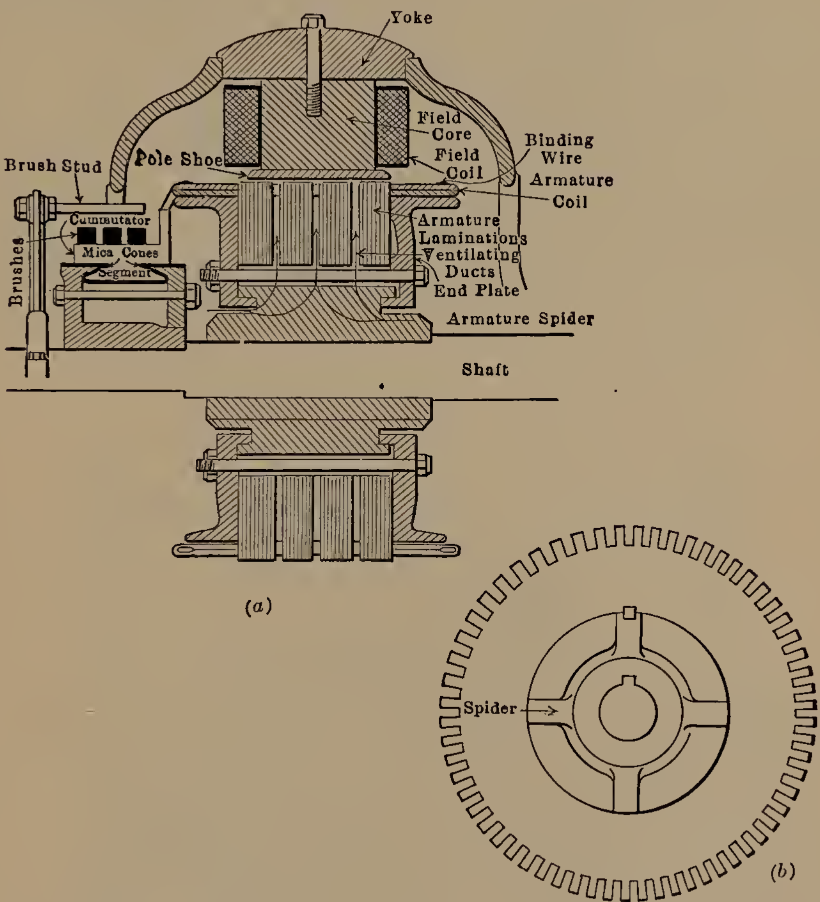


FIG. 201.—(a) Cross-section of a generator of moderate size; (b) armature stamping.

178. Field-cores and Shoes.—The field-cores are made of forged steel, cast steel, and steel laminations. When made of cast or forged steel, they are usually circular in cross-section, as such a section allows the minimum length of turn for a given core section. These cores are held to the yoke by bolts (Fig.

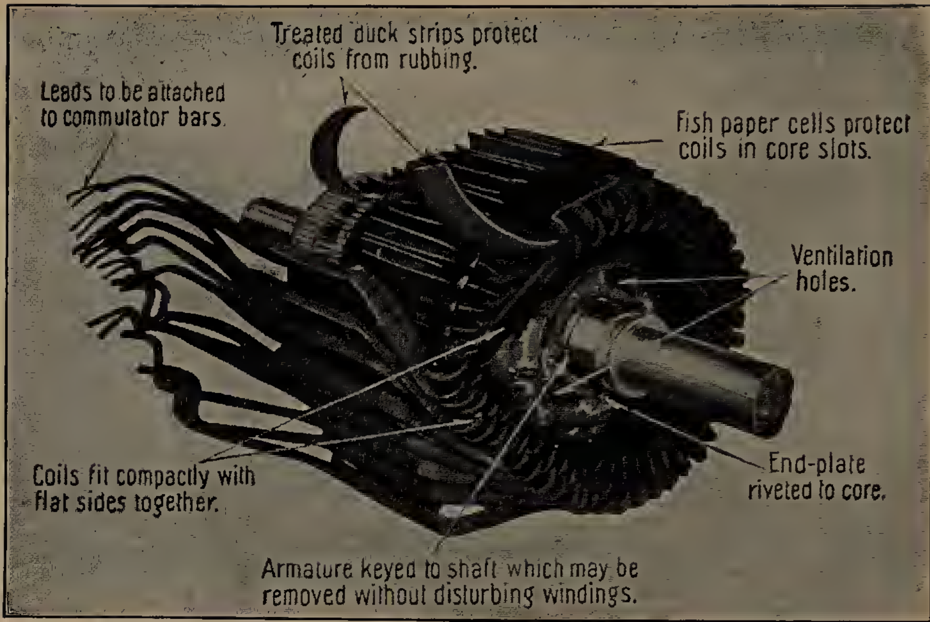


FIG. 202.—Partly wound armature, showing method of assembling coils (Westinghouse).

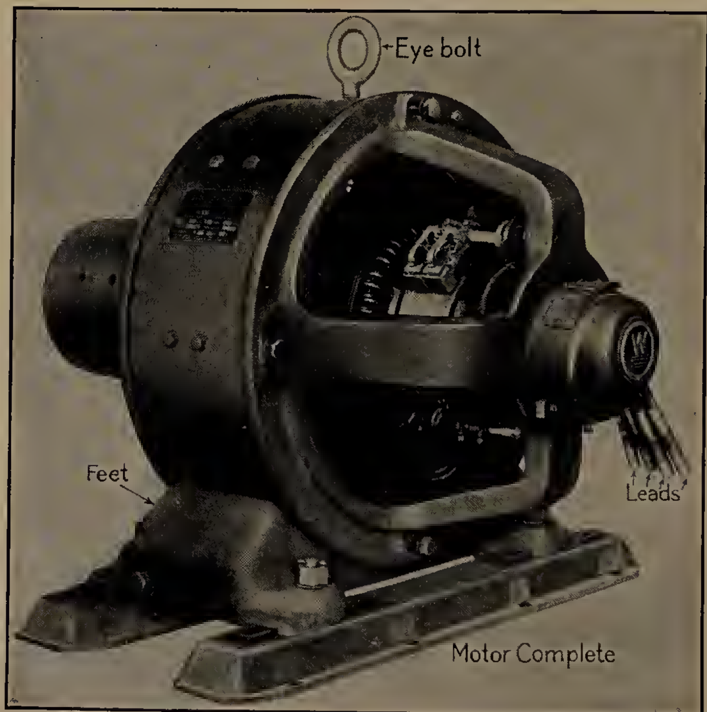


FIG. 203.—Westinghouse 230-volt, 35-hp., 850-r.p.m., shunt motor.

204). The laminated cores are built of sheet-steel stampings, (Fig. 205). They are stacked so that the pole tip comes alternately on one side and the other. This results in there being

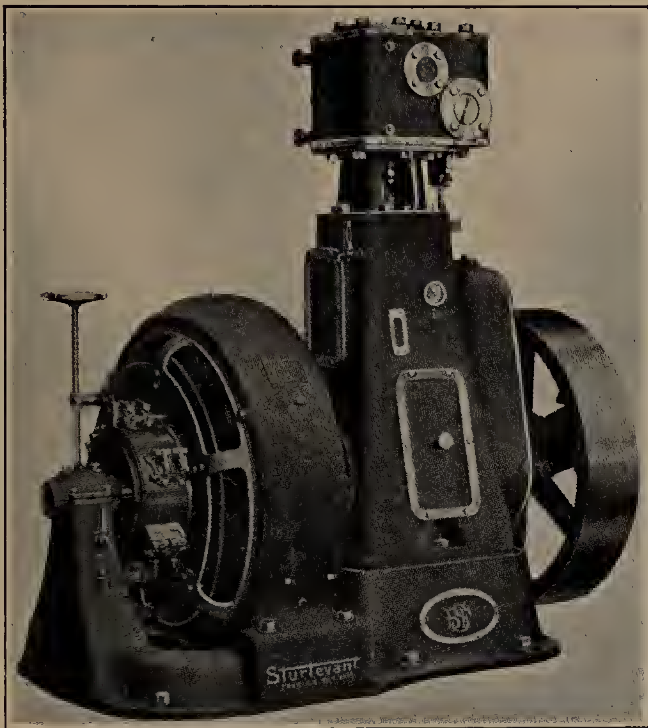


FIG. 204.—Sturtevant engine-generator set.

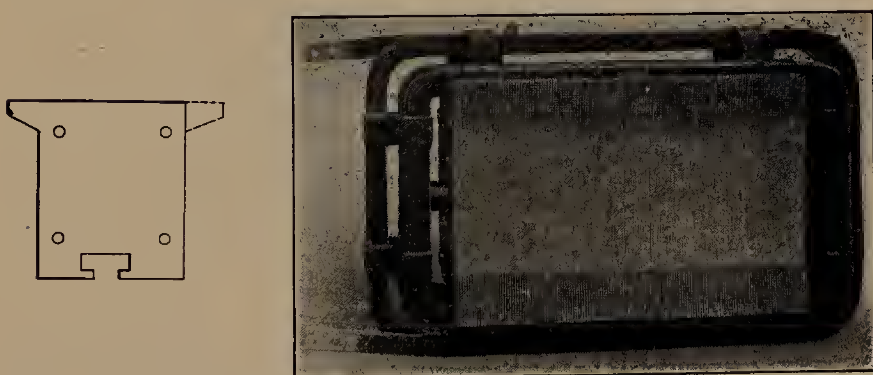


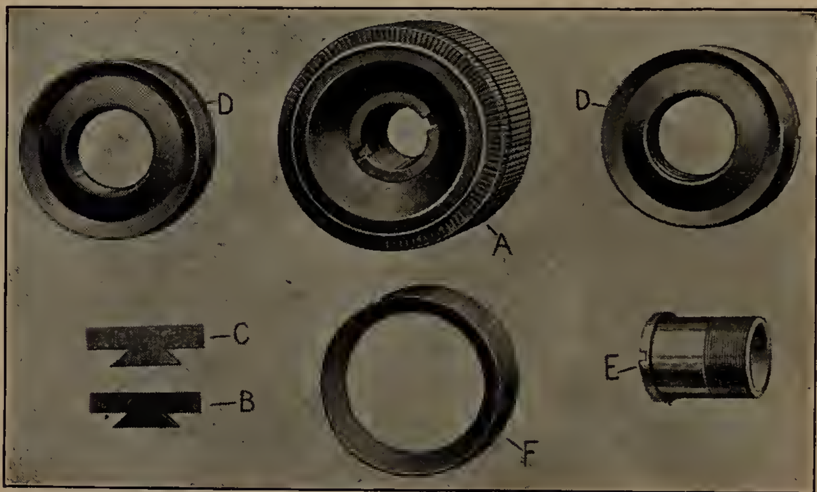
FIG. 205.—Field-core lamination and pole piece assembled—Westinghouse d.-c. motor.

but half the iron in a pole tip cross-section, thus producing a saturated pole tip, which assists commutation. When stacked

to the proper thickness, they are riveted together and dovetailed to the yoke.

Laminated pole-faces are used on the larger machines to reduce pole-face losses.

179. The Commutator.—The commutator is made of wedge-shaped segments of hard-drawn or drop-forged copper, insulated from one another by thin layers of mica. The segments are held



(*A*) Assembled commutator.
(*B*) Commutator bar.
(*C*) Mica commutator insulating strip.
(*DD*) Clamping flanges.
(*E*) Drawn steel tube.
(*F*) Insulation used between clamping flanges and commutator bars.

Fig. 206.—Crocker-Wheeler commutator and details.

together by clamping flanges (*DD*, Fig. 206), which pull the segments inward when the flanges are drawn together by through-bolts. These flanges are prevented from short-circuiting the segments by two cones of built-up mica (*F*, Fig. 206). This construction is illustrated by the commutator of the machine shown in Fig. 201, page 225.

The leads from the armature coils may be soldered into small longitudinal slits in the ends of the segments or the segments may have risers (Fig. 201), to which these leads are soldered.

180. Field-coils.—The field-coils are usually wound with double-cotton-covered (d.-c.-c.) wire. The coils are dried in a vacuum and then impregnated with an insulating compound. The outer cotton insulation is often protected by tape or cord on

the outside. In the larger machines, an air space is often left between layers for ventilating purposes. The coils are also wound on metal spools (Fig. 207). An edgewise series-winding, set some distance from the shunt-winding, also is shown here.

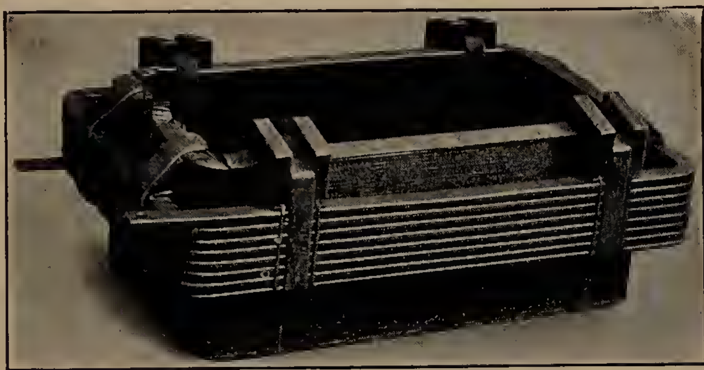


Fig. 207.—Shunt-field coil and edgewise series-winding.

181. The Brushes.—The function of the brushes is to carry the current from the commutator to the external circuit. They are usually made of carbon, although in very low-voltage machines they may be made of copper gauze, or patented metal compounds. The brush should be free to slide in its holder, in order that it may

Fig. 208.—Rocker ring and brush holder.

follow any irregularities in the commutator. The brush is made to bear down on the commutator by a spring (Fig. 208). The pressure should be from 1 to 2 lb. per square inch. To decrease the electrical resistance, the upper portion of the brush is copper-plated and this plating is connected to the brush holder by a pigtail made of copper ribbon.

CHAPTER XI

GENERATOR CHARACTERISTICS

The principles involved in the generation of electromotive force in an armature were discussed in Chap. X and the method by which direct current is obtained from an alternating electromotive force was described in some detail. It remains to consider the induced electromotive force from the *quantitative* point of view and to analyze the behavior of the various types of generator under load.

182. Electromotive Force in an Armature.—In Chap. X it was shown that, if a conductor *cuts* a magnetic field, an electromotive force is generated or induced in the conductor. It was also shown that, if conductors are placed on the surface of an armature and are properly connected, their induced electromotive forces add. Further, if unidirectional or direct current is desired from such an armature, the conductors on the armature surface must be connected to a commutator. From equation (83), page 207 ($e = Blv10^{-8}$ volts), the magnitude of the electromotive force between brushes can be computed. Equation (83), however, involves the flux density, the length of conductor, and its linear velocity. For a generator, these three factors can be determined only after some computation, whereas the total flux per pole and the rotational speed of the armature are usually known or can be determined. Therefore, by proper substitutions in equation (83), an equation may be derived which is readily applicable to a generator armature.

In a generator, the total flux per north pole ϕ is proportional to the average flux density B under the pole, the length l of the active conductors is a constant, and the linear velocity v of the conductor is proportional to the speed of the armature in r.p.m.

By substituting these values in equation (83), the following equation for the total induced electromotive force between brushes is obtained, where ϕ is the total flux in maxwells leaving

one north pole and entering the armature, S is the speed of the armature in r.p.m., P is the number of poles, Z is the total number of conductors on the surface of the armature, and p is the number of parallel paths through the armature.

$$E = \frac{\phi SPZ}{60p10^8} \text{ volts.} \tag{84}$$

With a simplex lap-winding, $p = P$; with a simplex wave-winding, $p = 2$.

Example.—A 900-r.p.m., 6-pole generator has a simplex lap-winding. There are 300 conductors on the armature.

The poles are 10 in. square and the average flux density under the poles is 50,000 lines per square inch. What is the voltage induced between brushes?

$$\phi = 10 \times 10 \times 50.000 = 5{,}000{,}000 \text{ lines}$$
$$S = 900 \text{ r.p.m.}$$
$$P = 6$$
$$p = 6 \text{ (see Par. 171)}$$
$$E = \frac{5{,}000{,}000 \times 900 \times 6 \times 300}{60 \times 6 \times 10^8} = 225 \text{ volts. } \textit{Ans.}$$

183. The Saturation Curve.—Equation (84) may be written as follows:

$$E = \left(\frac{PZ}{60p10^8}\right)\phi S. \tag{85}$$

For a given machine, the quantity within the parenthesis is constant and may be denoted by K.

Hence:

$$E = K\phi S. \tag{86}$$

The induced electromotive force in a machine, therefore, is *directly proportional* to the *flux* and to the *speed.*

If the speed be kept constant, the induced electromotive force is directly proportional to the flux, ϕ.

The flux is produced by the field ampere-turns and, as the turns on the field remain constant, the flux depends on the field current. It is not directly proportional to the field current because of the varying permeability of the magnetic circuit.

Figure 209 shows the relation existing between the field ampere-turns and the flux per pole. Ordinarily, the flux is

not zero when the field current is zero but is equal to some value, such as oa, because of the residual magnetism in the magnetic circuit. At first the line ab is practically straight, since most of the reluctance of the magnetic circuit is in the air-gap. At the point b the iron approaches saturation and the curve falls away from the straight line.

From b to c and beyond, the iron is saturated. The flux increases very slowly with further increase in field current. In

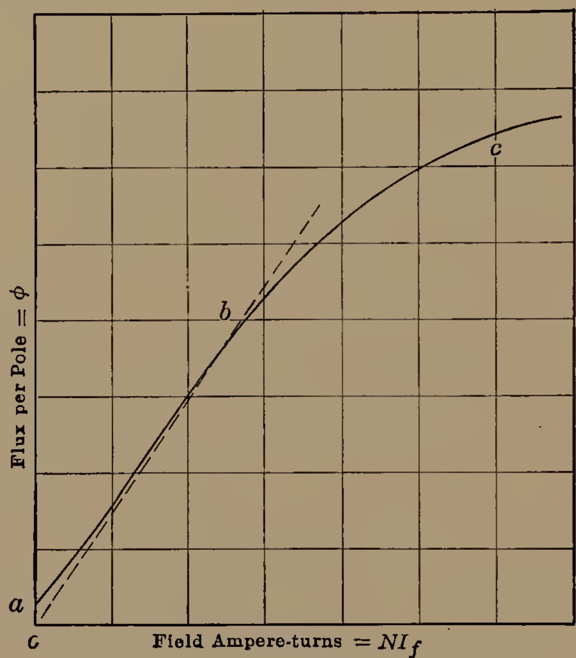


FIG. 209.—Saturation curve.

fact, beyond point c, it is practically impossible to obtain an appreciable increase in flux without an excessive increase of field current.

From equation (86), the induced voltage is proportional to the flux, if the speed is maintained constant. Therefore, if the induced voltage be plotted as ordinates with field current as abscissas, a curve similar to that of Fig. 209 is obtained. This is shown in Fig. 210, where a saturation curve for a 25-kw., 120-volt, compound generator is shown. This curve shows that, with a field current of 3.3 amp., an electromotive force of 100 volts is induced in the armature when the speed is 900 r.p.m. With a

field current of 4.4 amp., 120 volts is induced in the armature at this speed.

The saturation curve has a considerable effect on the operation of all generators and motors, as will be shown later.

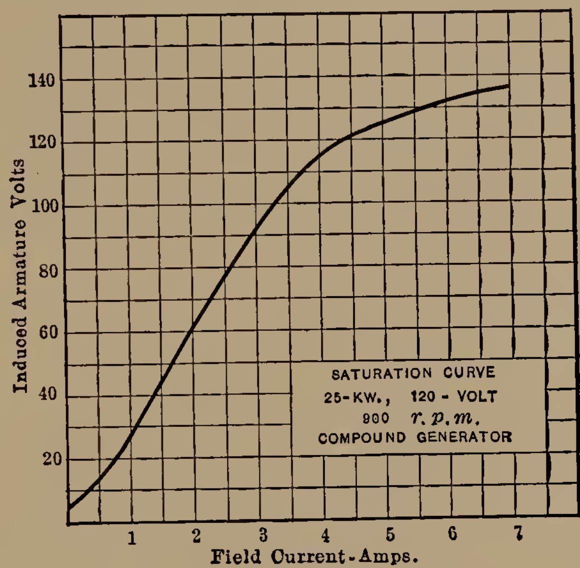


FIG. 210.—Saturation curve of a 25-kw., 120-volt, 900-r.p.m. generator.

184. Hysteresis in Dynamos.—The saturation curve Oab, (Fig. 211) is determined for *increasing* values of the field current. If when point b is reached, the field current be *decreased*, the curve baO will not be followed. For any given field current, the corresponding induced voltage will now be greater than it was for *increasing* field currents. This is shown by the curve bcd. This is due to hysteresis in the iron (see page 163, Par. 134).

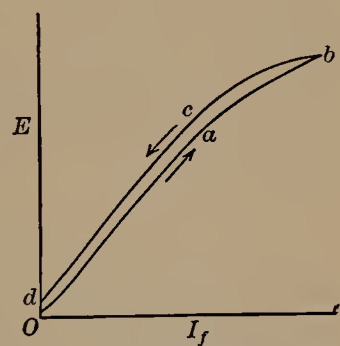


FIG. 211.—Hysteresis loop of magnetic circuit of dynamo.

For any given value of field current, there is no single value of flux. The value of flux for any given field current depends on whether the field current was *increased* until it reached the value in question or whether it was *decreased*. This characteristic of the magnetic circuit should be carefully kept in mind, for the operating characteristics of both generators and

motors are affected to a considerable degree by hysteresis in the magnetic circuit.

185. Determination of the Saturation Curve.—To determine the saturation curve experimentally, connect the field, in series with an ammeter, across a direct-current source of power. A voltmeter should be connected across the armature terminals. The ammeter measures the field current, values of which are plotted as abscissas; the voltmeter reads the values of induced armature voltage, which are plotted as ordinates. The connections are shown in Fig. 212. As the voltage-drop within the armature due to the voltmeter current is negligible, the terminal

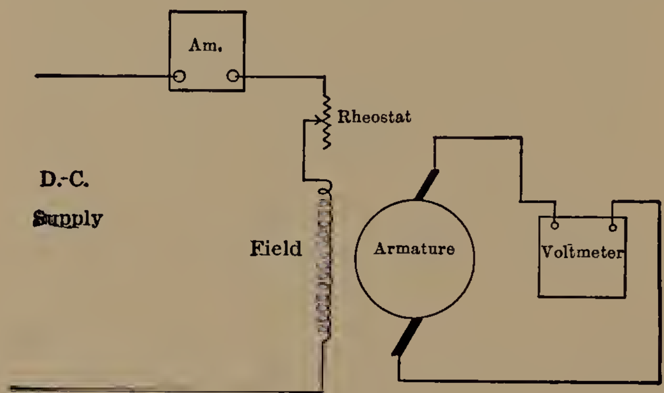


FIG. 212.—Connections for obtaining saturation curve.

volts and the induced volts under these conditions are identical. During the experiment, the speed should be determined each time the other readings are taken. If the speed cannot be maintained constant, corrections can be made for any variation, since the induced electromotive force is proportional to the speed (equation (86), page 231).

In determining the saturation curve experimentally, the field current should be varied continuously in one direction, either with increasing values or with decreasing values, as shown in Fig. 211. Otherwise, hysteresis loops will be introduced.

The field current in this experiment should be obtained from a supply other than the generator itself, for two reasons: If the generator excited its own field, the voltage and field current would be interdependent and it would be difficult to adjust the field current without the voltage in turn changing this

adjustment. Also a voltage-drop would exist in the armature due to the field current and the voltmeter would not give the true induced voltage, although the error from this cause would be slight.

186. Types of Generator.—There are three general types of generator in common use, the shunt, the compound, and the series. In the shunt type the field circuit is connected across the armature terminals, in shunt with the load (Fig. 213). A shunt-field rheostat is usually connected in series with the shunt field. The shunt field, therefore, must have a comparatively high resistance in order that it may not take too great a proportion of

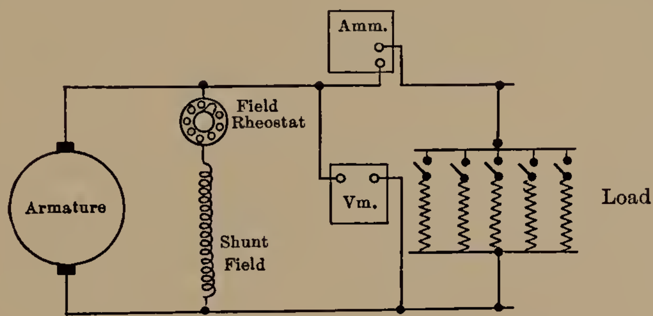


FIG. 213.—Shunt-generator connections.

the generator current. The compound generator is similar to the shunt type, but has an additional field-winding connected in series with the armature or load. This series-winding increases the flux with increase of load, Fig. 228, page 254. The series generator is excited by a winding of comparatively few turns connected in series with the armature and load (see page 252).

THE SHUNT GENERATOR

187. Building Up of Shunt Generator.—The shunt generator excites its own field. When the generator is started, there is obviously no voltage across its terminals, hence there is no field current. Ordinarily, however, there is some residual magnetism or flux in the magnetic circuit of the machine due to the retentivity of the iron. This is shown by the ordinate *oa* (Fig. 209) page 232. As the generator comes up to speed, a small induced voltage appears at its terminals due to the cutting of this residual

flux by the armature conductors. Therefore, a small current flows through the field. If the field resistance be sufficiently low and the connections of the field to the armature terminals be such that the small field current due to this small terminal voltage tends to *increase* the flux due to the residual magnetism, the voltage of the generator will build up of its own accord. Under the foregoing conditions, this initial field current, due primarily to residual magnetism, increases the flux in the magnetic circuit. This increase of flux causes an increase in the induced voltage. This, in turn, causes further increase in the field current.

These reactions are obviously cumulative. That is, an increase in field current produces an increase in terminal voltage and the increase in terminal voltage produces a further increase in field current. The terminal voltage therefore increases more or less rapidly.

It might seem from the foregoing analysis that the terminal voltage would build up indefinitely. That it does not do so is due to the saturation of the magnetic circuit. When the flux reaches the value indicated by *b* (Fig. 209) page 232, a given increase in field current produces a much smaller proportionate increase in flux. As the field current increases still further, the rate of increase of flux diminishes, until a point is reached where the increase of flux is extremely small, even with very large increase in field current. For this reason, the voltage to which a generator can build up is *limited by the saturation of the iron.* If the magnetization curve were a straight line, a shunt generator would build up indefinitely.

188. Failure of Generator to Build Up.—A shunt generator may fail to build up for one of the following reasons:

1. *Lack of Residual Magnetism.*—In time, the residual magnetism may diminish to a very small value, particularly if the machine has been idle and subjected to vibration. A reversed field-connection may have opposed or "bucked" the residual magnetism and caused it to diminish to a value so small that the field current resulting from the initial induced terminal voltage is insufficient to produce an appreciable increase in the flux due to residual magnetism. These conditions are indicated when a voltmeter connected across the armature terminals reads very nearly zero.

The remedy is to "flash" the generator field. The shunt field is connected across another source of voltage, to build up the flux due to residual magnetism. If the machine is a compound generator, a convenient method is to connect across the series field a dry cell in the smaller machines or a low-voltage storage battery in the larger machines, the shunt field remaining connected across the armature terminals. If the polarity of the battery is correct, the current which it sends through the series field may cause the machine to begin to build up.

2. *Too High Shunt-field Resistance.*—If the resistance of the shunt field be too great, the generator will not build up. When this condition exists, the voltmeter reading after the field circuit is closed increases but slightly above its initial reading and then ceases to increase. The remedy is to reduce the resistance of the shunt-field circuit by cutting out resistance in the field rheostat.

3. *Reversed Shunt-field Connections.*—If the connections of the shunt field are such that the initial field current, primarily due to residual magnetism, opposes or "bucks" the residual magnetism, the machine cannot build up. When this condition exists, the voltmeter reading *decreases* when the shunt-field connection is made. The remedy is to reverse the shunt-field connections. This condition of reversed field-connection rarely occurs with a shunt generator which is installed in service and in which the field-connection has already been properly made. With compound generators, a flashover, an open field in the generator, or other conditions, may cause the current to reverse and enter the armature at its positive terminal. The current in the series field is, therefore, reversed and as the current under these conditions is usually large, the series-field ampere-turns may exceed the shunt-field ampere-turns and cause a reversal of flux.

4. *Shunt-field Open-circuited.*—Occasionally the shunt-field circuit becomes open-circuited, due to a burnt-out rheostat, poor contact, etc. This condition is indicated by there being no change in the voltmeter reading when a field connection to the armature terminals is made and broken. The remedy is to test the field circuit and locate the open-circuit.

189. Armature Reaction.—Figure 214 (*a*) shows the flux passing from the field poles through an armature, when there is no current in the armature conductors. This flux is due entirely

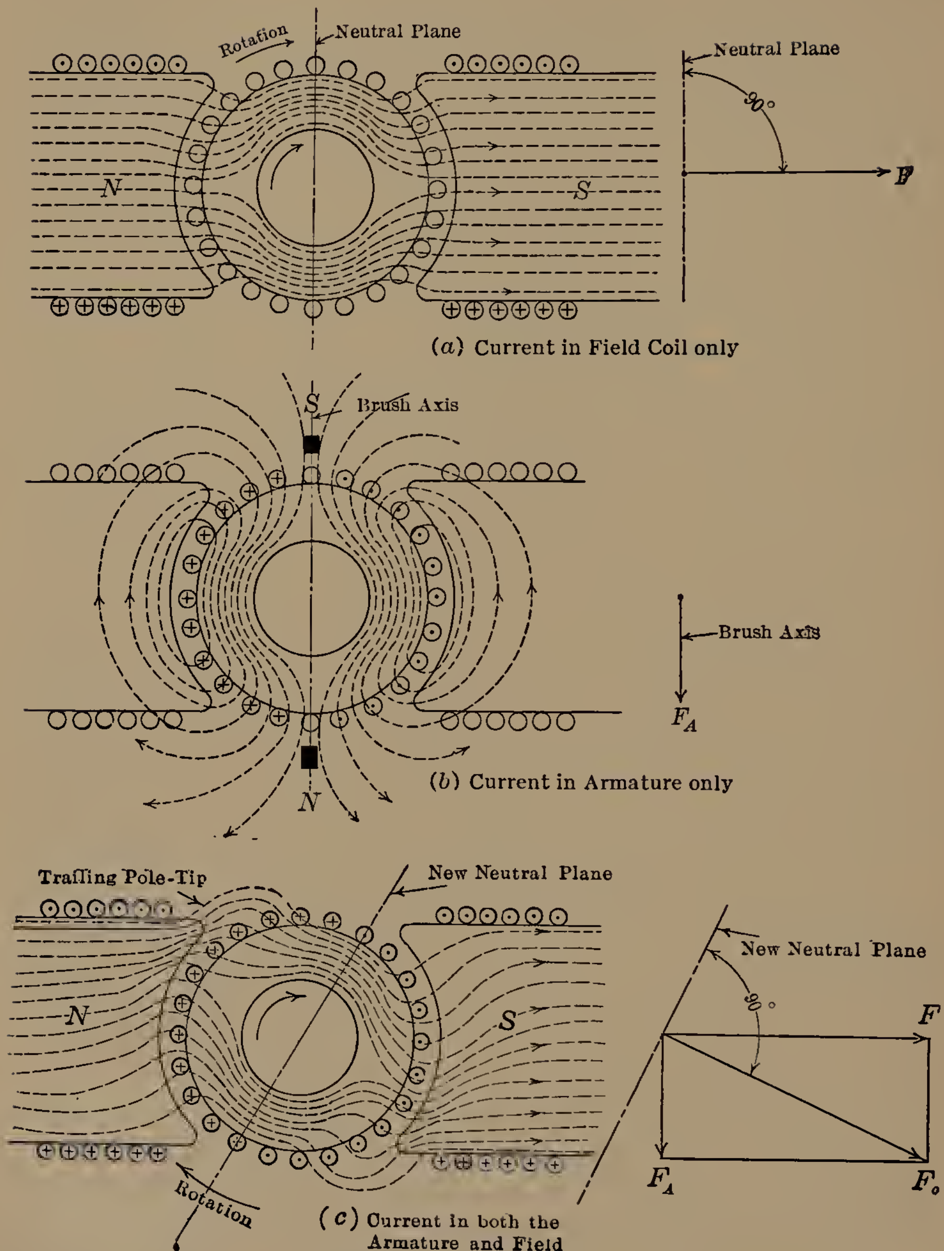


FIG. 214.—Effect of armature reaction on the field of a generator.

to the ampere-turns of the field. The neutral plane, which is the plane perpendicular to the flux, coincides with the geometrical neutral of the system. At the right is shown a vector F which represents in magnitude and direction the magnetomotive force producing this flux. At right angles to this vector F is the neutral plane.

In Fig. 214 (*b*), there is no current in the field coils, but the armature conductors are shown as carrying current. This current is in the same directions in the armature conductors as it would be were the generator under load, with the polarity and direction of rotation as shown in (*c*). The current flows in the same direction in all the conductors that lie under one pole. The current is shown as flowing into the paper on the left-hand side of the armature. (This current-direction may be checked by Fleming's right-hand rule, page 208.) The magnetomotive forces due to these conductors on the left-hand side of the armature combine to send a flux *downward* through the armature, as shown in the diagram, this direction being determined by the corkscrew rule. The conductors on the right-hand side of the armature are shown as carrying current coming out of the paper and the magnetomotive forces due to these conductors combine to send a flux *downward* through the armature. That is, the magnetomotive forces due to the conductors on both sides of the armature combine in such a manner as to send flux downward through the armature. The direction of this flux is perpendicular to the main polar axis. To the right of the figure the armature magnetomotive force is represented in direction and magnitude by the vector F_A.

Figure 214 (*c*) shows the result obtained when the field current and the armature current are acting simultaneously, which occurs when the generator is under load. The armature magnetomotive force crowds the symmetrical field flux shown in (*a*) into the upper pole-tip in the north pole and into the lower pole-tip in the south pole. As the generator armature is shown rotating in a clockwise direction, it will be noted that the flux is crowded into the *trailing* pole-tip of each pole. On the other hand, the flux is weakened in the *leading* pole-tip of each pole.

The effect of the armature current is to displace the field in the direction of rotation of the generator. It should be kept firmly

in mind that the flux is not pulled around by the mechanical rotation of the armature.

To the right of Fig. 214 (*c*) the effect of armature reaction is shown by vectors. The field vector F and the armature vector F_A combine at right angles to form the resultant field vector F_o. The direction of F_o is downward and to the right, which corresponds to the general direction of the resultant flux. The neutral plane must be at right angles to F_o, provided the direction of the resultant flux is the same as that of the resultant magnetomotive force.

As the neutral plane is perpendicular to the resultant field, it will be observed that it also has been advanced. It was shown in Chap. X that the brushes should be set so that they short-circuit the coil undergoing commutation as it is passing through the neutral plane. When the generator delivers current, the brushes should be set a little ahead of this neutral plane, as will be shown later. If the brushes are advanced to correspond to the advance of the neutral plane, all the conductors to the left of the two brushes must still carry current into the paper, and those to the right must carry current out of the paper.

Figure 215 (*a*) shows that portion of the ampere-conductors which are included within an angle β on each side of the geometrical neutral, where β is the angle of brush advance. The conductors at the top of the armature carry current into the paper, since they are to the left of the brushes (see Fig. 214). By the corkscrew rule their magnetomotive force through the armature is from right to left. The conductors at the bottom of the armature carry current out of the paper, since they are to the right of the brushes. Their magnetomotive force through the armature is also from right to left. Referring to Fig. 214 (*c*), the magnetomotive force of the main poles is from left to right. Hence the magnetomotive force of the ampere-conductors which are included within twice the brush angle at both the top and bottom of the armature *opposes* the magnetomotive force of the main field and therefore tends to reduce the flux through the armature. These conductors are called *demagnetizing ampere-conductors*.

Figure 215 (*b*) shows the armature ampere-conductors of Fig. 214 which are not included in Fig. 215 (*a*). By the corkscrew

rule, the magnetomotive force of the conductors on both the left-hand and the right-hand side of the armature is downwards, and hence acts at right angles to the magnetomotive force of the main field. Therefore, these conductors are called *cross-magnetizing ampere-conductors.*

When the generator is carrying load and the brushes are advanced, the armature ampere-conductors tend both to

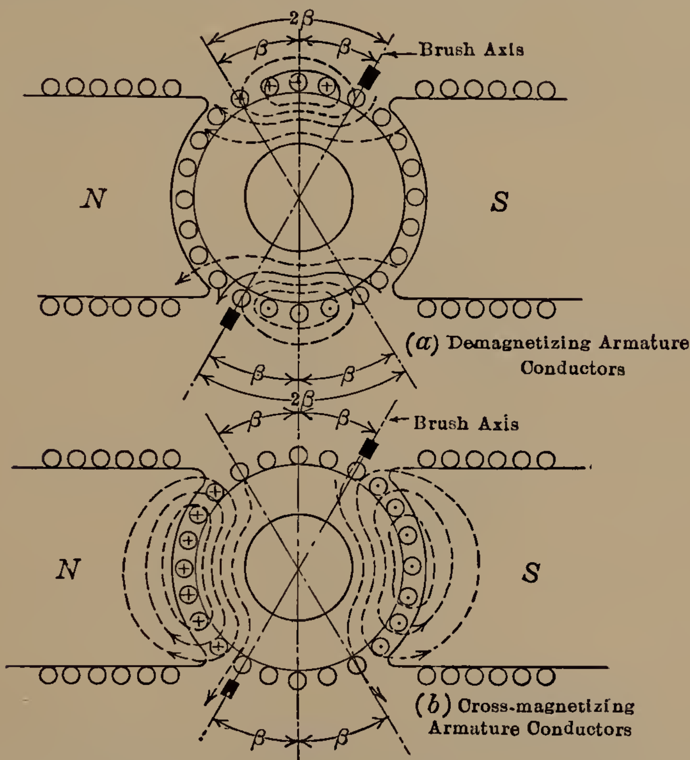


Fig. 215.—Demagnetizing and cross-magnetizing components of armature reaction.

demagnetize and to cross-magnetize the main magnetic field. The combination of the demagnetizing and cross-magnetizing magnetomotive forces is called *armature reaction.* Armature reaction has a very important influence on the operation of all generators and motors.

Armature reaction may be reduced by increasing the length of the air-gap. This increases the reluctance of the path of the armature flux, and of the field flux as well, and necessitates an increase in the field ampere-turns. Operating the tooth tips at

high saturation also reduces the effect of armature reaction. Extra windings on the field structure, which are connected in series with the armature and oppose the magnetomotive force of the armature, are sometimes used to reduce the effect of armature reaction.

190. Commutation.—It has been shown that the electromotive force induced in any single coil of a direct-current generator is alternating and, in order that the current may flow always in the same direction to the external circuit, a commutator is necessary.

Figure 216 shows diagrammatically a bipolar generator with a gramme-ring winding. The armature rotates in a clockwise

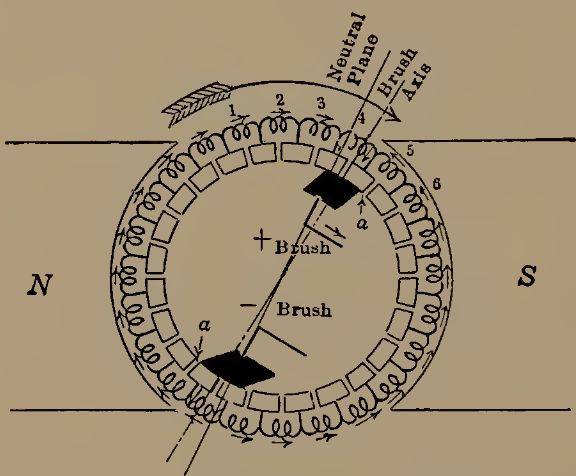


Fig. 216.—Commutation in a generator.

direction. The currents on both sides of the armature flow upwards in the diagram and combine to flow out of the positive brush to the external circuit. The figure shows the direction of the currents in all the coils at any one instant, and hence the direction of the current in any one coil at successive instants. For example, consider any coil *1* as it takes successive positions *2*, *3*, *4*, *5*, and *6*. For all positions between brushes on the left-hand side of the armature, the current is upwards. Likewise, for all positions of this coil between brushes, on the right-hand side of the armature, the current is upwards. Therefore, in the interval between positions *3* and *5*, the current in this coil must have been reversed. This reversal must occur in the very brief interval of time during which the commutator segments to which

this coil is connected are passing under the brush. Since the armature coils are embedded in slots and are nearly surrounded by the iron teeth of the armature, they have very appreciable self-inductance. As was shown in Chap. VIII, self-inductance *opposes* any change of current. When the current in the armature coil changes, an electromotive force of self-induction is produced which tends to *prevent* the change and to prolong the current flow in a given direction. Therefore, if the brushes are in the neutral plane (see page 211, Par. 166) and the generator is carrying load, sparks will ordinarily appear under the brush and particularly under the toe (a, a, Fig. 216) where the segments are breaking contact with the brush.

This electromotive force of self-induction may be neutralized in part if the brushes are advanced *ahead* of the neutral plane, as shown in Fig. 216, so that the coil will have induced in it, due to cutting flux, an electromotive force *opposed* to the electromotive force of self-induction. Even then it is usually impossible to neutralize completely this electromotive force of self-induction.

The foregoing, which refers to a single coil only, obviously applies to all the coils of the armature as they undergo reversal of current. Since both armature reaction and this electromotive force of self-induction are proportional to the current, theoretically the brushes should be shifted with every change of load. Practically, and particularly when carbon brushes are used, this is not necessary.

It is thus evident that commutation consists of two parts, the reversal of the current in the individual armature coils during the time required for a segment to pass under the brush; and the conduction of the useful current from the commutator to the external circuit.

191. Sparking at the Commutator.—The voltages induced in a coil due to the shifting of the neutral plane and also due to its own self-inductance are comparatively low in value, being of the order of magnitude from a few tenths of a volt to perhaps 4 or 5 volts. But they are acting in a circuit having a very low resistance. The resistance of the coil itself is extremely low, so that the greater part of the circuit resistance is at the brush contact. If the brush-contact resistance is too low, these short-circuit currents may reach such values as to produce severe sparking at

the brushes. Consequently, in most generators and motors, except in very low-voltage, high-current generators, carbon brushes are used on account of their greater contact resistance and because they are self-lubricating, due to their graphitic character.

The passage of the current from the commutator to the brush is in the nature of an arc phenomenon rather than one of pure conduction. A careful examination will show myriads of minute arcs existing between the brush surface and the commutator. These arcs burn away or volatilize the copper leaving "high mica" (Fig. 217). Hard brushes are used to prevent high mica because of their abrasive action.

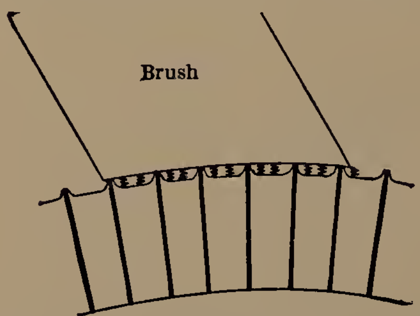


Fig. 217.—Commutator with high mica.

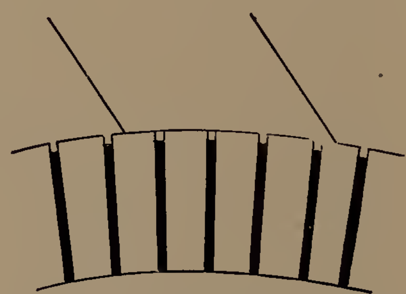


Fig. 218.—Undercut mica commutator.

If the carbon brushes are too hard, they cut the commutator. Different grades of carbon are required for different operating conditions. In modern practice, the mica is frequently undercut, that is, the top of the mica is lower than the commutator surface (Fig. 218). There is some disadvantage in this, in that small bits of copper, carbon, and dirt collect in the grooves and may ultimately short-circuit the segments. These grooves can be easily cleaned out, however. Undercutting gives better results with high-speed commutators, since the centrifugal force tends to throw out the particles that lodge between segments.

The result of any arcing under the brush is to pit the commutator. As irregularities and depressions in the commutator surface tend to prevent the brush making intimate contact with the commutator, arcs of increasing magnitude will be formed. Hence, any condition which produces sparking, and so roughens the commutator, increases the sparking and roughening, as these

actions are cumulative. If a commutator is sparking badly and the cause of the sparking is not corrected, the commutator will deteriorate very rapidly and soon become inoperative.

The brushes should be fitted very carefully to the commutator surface by grinding with sandpaper in the manner shown in Fig. 219. Carbon on the surface of the commutator should

Fig. 219.—Proper method of fitting brushes.

be removed with an oily cloth. Do not use waste. A slightly roughened commutator may be partially smoothed with fine sandpaper. Do not use emery, as the particles of emery are conducting and may short-circuit the commutator bars. Commutator compounds of a lubricating character are on the market. They often reduce sparking, particularly if the sparking is due to a slightly roughened commutator. If the commutator is badly grooved by the brushes, or is otherwise in poor condition, it may be necessary to turn it down in a lathe.

Other conditions, such as loose mica and loose segments, are more serious in character. It is often possible to remedy these by tightening up the commutator clamp-bolts when the commutator is hot.

192. Commutating Poles (Interpoles).—When load is applied to a generator, the flux is crowded into the trailing pole-tips by the magnetomotive force of the armature ampere-conductors or armature reaction (see Fig. 214 (*c*), page 238). If the brushes are allowed to remain in the *geometrical* neutral, sparking will occur, due to the fact that the coils undergoing commutation generate electromotive force and are simultaneously short-

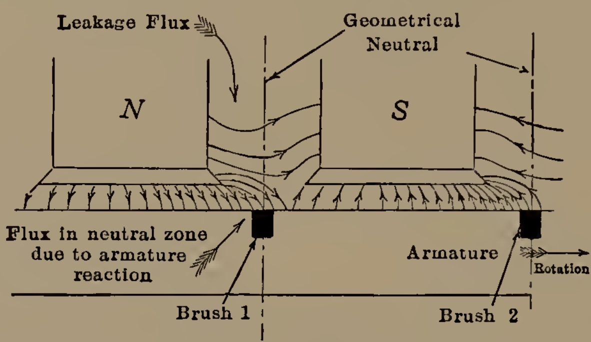


FIG. 220.—Flux distortion due to armature reaction.

circuited by the brushes. This effect of armature reaction is indicated diagrammatically in Fig. 220, where, for simplicity, the armature is shown as a flat surface and the armature conductors are omitted. The flux is shifted in the direction of rotation. The brushes, if allowed to remain in the geometrical neutral, will short-circuit coils whose active conductors are cutting flux, and severe sparking will result. One method of improving commutation, as has already been stated, is to move the brushes ahead, beyond the neutral zone into the flux of the next pole.

This result may be obtained without moving the brushes, and in a much more satisfactory manner, by introducing small poles, called *commutating poles* or *interpoles*, between the main poles.

In Fig. 220, a portion of the flux from the north pole enters the armature midway between the poles or at the geometrical neutral. The armature coils which are cutting this flux are simultaneously short-circuited by the brush. As a result, a large circulatory

current flows in the short-circuited coils and causes sparking at the brush. This short-circuit current is further increased by the electromotive force of self-induction (see page 243).

If a small south pole of the proper strength be placed in this interpolar region (Fig. 221), this north-pole flux which enters the armature midway between poles may not only be neutralized, but the south pole may be sufficiently strong to cause flux actually to *leave* the armature in this region. The armature conductors cutting this south-pole flux may also generate sufficient electromotive force to neutralize the electromotive force of self-induction. Therefore, if the commutating pole

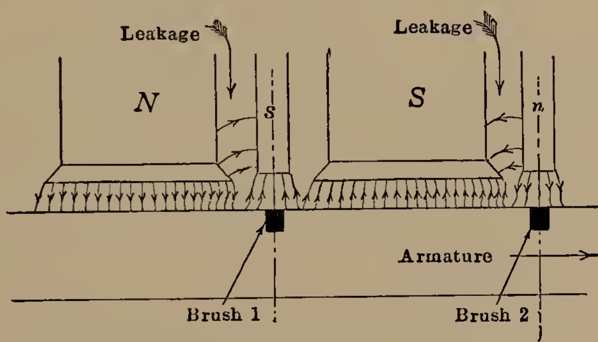


FIG. 221.—Effect of commutating poles on flux distortion.

be properly adjusted, good commutation at all loads is ordinarily obtainable.

Obviously, a commutating pole of north polarity should be placed in the commutating region of brush 2.

Armature reaction is practically proportional to the armature magnetomotive force, hence to the armature current. The electromotive force of self-induction is proportional to the self-inductance of the armature coils and to the rate of change of current (see equation 65, page 172). The self-inductance of the armature coils is practically constant, and for a given speed the time of complete reversal of current is inversely proportional to the brush width. Therefore, for a given speed, the electromotive force of self-induction is directly proportional to the armature current. Hence the flux which neutralizes these effects must be proportional to the armature current. The commutating poles, therefore, must be excited by a winding connected in *series* with the armature, as shown in Fig. 222. Com-

mutating poles are usually designed with a larger number of turns than are necessary for good commutation, and the poles are then adjusted to the proper strength by means of a shunt (Fig. 222). It should be noted that in a generator the sequence of main and commutating poles in the direction of rotation is *Ns Sn* where the capitals refer to the main poles and the small letters to the commutating poles.

If the commutating poles are properly adjusted, it is not necessary to shift the brushes with change of load.

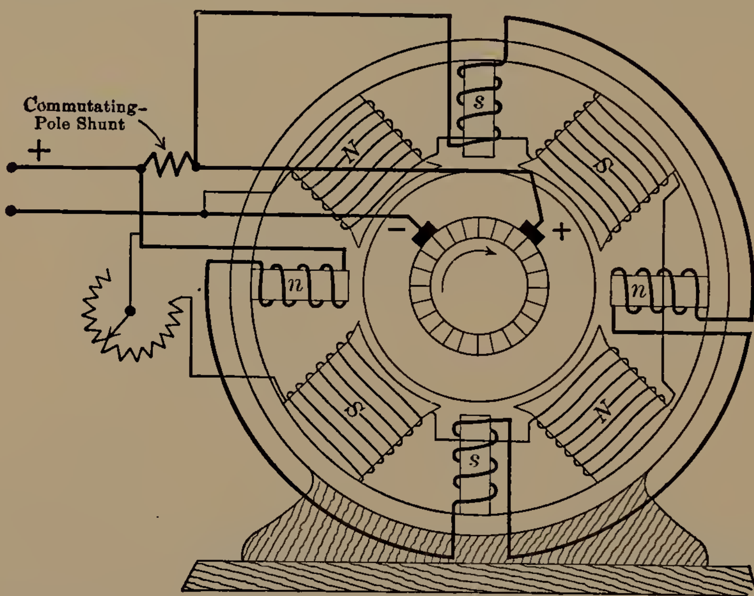


FIG. 222.—Connections of shunt field and commutating poles.

193. The Characteristics of the Separately-excited Generator. If either a separately-excited generator or a shunt generator, after building up to voltage, be loaded, the terminal voltage will drop. This drop in voltage will increase with increase of load. It is important to know the voltage at the terminals of a generator for each value of the load current, because the ability to maintain voltage under load conditions determines in large measure the suitability of a generator for a specific use.

Figure 223 shows the connections for determining the characteristics of a separately-excited generator. The field, with its rheostat and an ammeter in series, is connected across a constant-potential, direct-current supply. An adjustable load is con-

nected across the armature terminals. An ammeter in series with the line measures the armature or load current and a voltmeter across the armature terminals measures the armature terminal voltage. Throughout the test, the speed is maintained

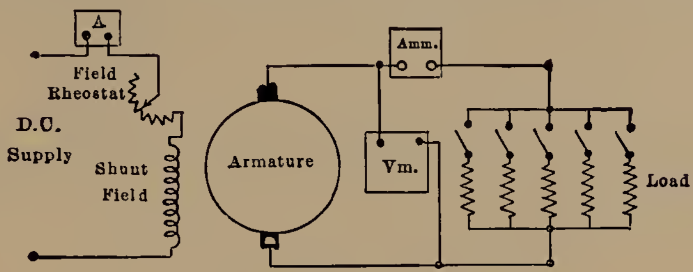


FIG. 223.—Separately-excited generator.

constant at its rated value. The field rheostat is adjusted until rated voltage at rated current is obtained, the load is then removed and the no-load voltage *oa* (Fig. 224), is thus obtained. Load is then gradually applied, the speed being maintained

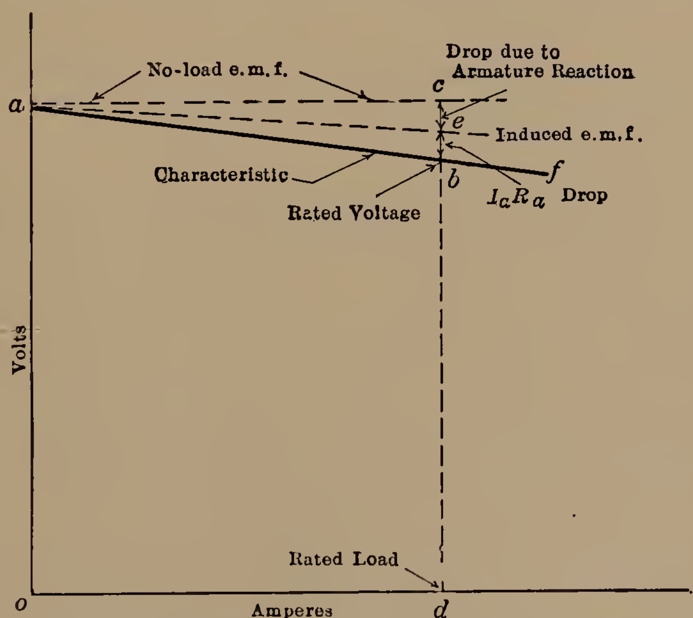


FIG. 224.—Characteristic of a separately-excited generator.

constant. At each load the ammeter and the voltmeter are read. The test is carried to approximately 125 per cent. of rated current.

If the results of this test are plotted, a curve similar to *abf* (Fig. 224) is obtained. The terminal voltage decreases with

increase of load for two reasons. *Armature reaction reduces the flux produced by the main field windings,* and therefore reduces the induced electromotive force, as shown by the line *ce* (Fig. 224) (see page 241). *There is a voltage-drop in the armature itself due to the current flowing through the resistance of the armature.* This also increases with increase of load. The net voltage at rated load is *bd.*

It is evident that in a generator the *induced* electromotive force exceeds the *terminal voltage* by the resistance drop in the armature.

That is, the induced electromotive force

$$E = V + I_aR_a \tag{87}$$

where V is the terminal voltage, I_a the armature current, and R_a the armature resistance.

Transposing equation (87), the terminal voltage

$$V = E - I_aR_a. \tag{88}$$

That is, the terminal voltage equals the induced electromotive force minus the armature-resistance drop.

Equations (87) and (88) should be compared with equations (35) and (36), page 71.

194. Characteristic of the Shunt Generator.—The operating or external characteristic of a shunt generator is obtained in a manner similar to that used in obtaining the characteristic of the separately-excited generator. The shunt field, in series with its rheostat, is now connected directly across the armature terminals and the machine excites itself. The connections for obtaining the characteristic are shown in Fig. 213, page 235. An additional ammeter connected in the field circuit is usually desirable. The procedure is the same as that followed in obtaining the characteristic of the separately-excited generator. The speed is held constant and the volts and amperes are recorded at various loads.

In Fig. 225, the curve *abedfa* shows a typical shunt characteristic. Three causes now contribute to the drop in terminal voltage as load is applied, the voltage-drop due to *armature resistance,* the voltage-drop due to *armature reaction,* and the voltage-drop due to *decrease in field current.* The decrease in

field current is due to the field being connected across the armature terminals. The voltage-drops due to armature reaction and to armature resistance result in lessened terminal voltage, hence decreased field current. Armature reaction and the lessened field current cause a further decrease in the induced electromotive force. Line *ag* gives the induced electromotive force with change of load.

When the break-down point *e* on the characteristic is reached, the voltage drops very rapidly almost to zero with but slight increase of current. Little or no additional current can be

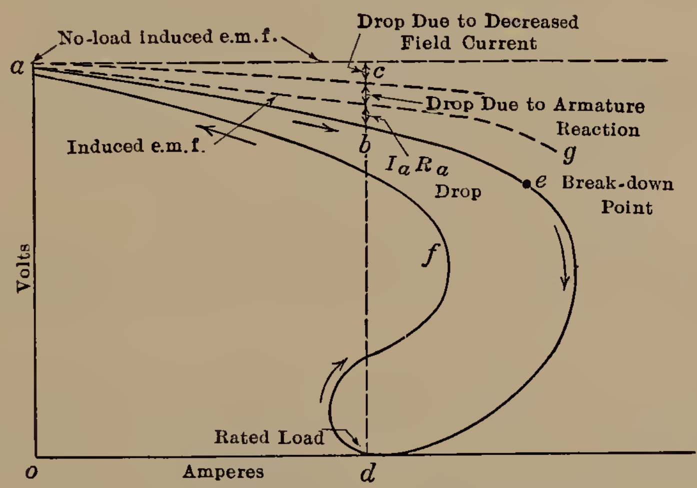


FIG. 225.—Characteristic of a shunt generator.

obtained, even if the machine be short-circuited. In large generators, this break-down point may occur at three or four times rated-load current. The break-down is due to the rapid drop in field current as the load is increased. Considerable current *od* flows at zero terminal voltage (short-circuit), due to residual magnetism. If the short-circuit is gradually removed by increasing the resistance of the load, the voltage returns to the no-load value in the manner given by the part *df* of the curve. This no-load value of voltage is slightly less than the initial value, *oa*. On account of the effect of hysteresis this return curve *df* lies below the curve *ab* (see Fig. 211, page 233). The machine is usually operated along the portion *ab* of the characteristic.

Example.—The no-load voltage of a 50-kw., 220-volt, shunt generator is 232 volts and the rated-load voltage is 220 volts. The armature resistance, including brushes, is 0.026 ohm and the resistance of the shunt-field circuit is 52 ohms. (*a*) What is the induced electromotive force at rated load? (*b*) What power is lost in the field at rated load? (*c*) What power is lost in the armature at rated load?

(*a*) The rated current

$$I = \frac{50{,}000}{220} = 227 \text{ amp.}$$

The field current

$$I_f = \frac{220}{52} = 4.23 \text{ amp.}$$

The armature current

$$I_a = 227 + 4.23 = 231 \text{ amp.}$$

The induced electromotive force, equation (87), page 250.

$$E = 220 + (231 \times 0.026) = 220 + 6.0 = 226.0 \text{ volts.} \quad \textit{Ans.}$$

(*b*) $$P_f = \frac{V^2}{R_F} = \frac{(220)^2}{52} = 931 \text{ watts.} \quad \textit{Ans.}$$

$$\text{or } 220 \times 4.23 = 931 \text{ watts (check).}$$

(*c*) $$P_a = I_a^2R_a = (231)^2 \times 0.026 = 1{,}387 \text{ watts.} \quad \textit{Ans.}$$

The induced electromotive force at rated load in (*a*) in less than the no-load voltage because of the effects of armature reaction and decreased field current.

195. The Compound Generator.—The drop in voltage with load, which is characteristic of the shunt generator, makes this type of generator undesirable where constancy of voltage is essential. This applies particularly to lighting circuits, where a very slight change of voltage makes a material change in the candle-power of incandescent lamps. A generator may be made to produce a substantially constant voltage, or even a rise in voltage as the load increases, by placing on the field-core a few turns which are connected *in series* with the load. These turns are connected so as to *aid* the shunt turns when the generator delivers current (Fig. 226). As the load increases, the current through the series turns also increases and, therefore, the flux through the armature increases. The effect of this increased flux is to increase the induced electromotive force. By proper adjustment of the series ampere-turns, this increase in induced electromotive force or armature voltage may be made to balance the combined drop in voltage due to armature reaction and to the resistance of the armature. If the terminal voltage is maintained

substantially constant, the field current will not drop as the load increases. Therefore, the three causes of voltage-drop, namely, armature reaction, I_aR_a drop, and drop in field current (Fig.

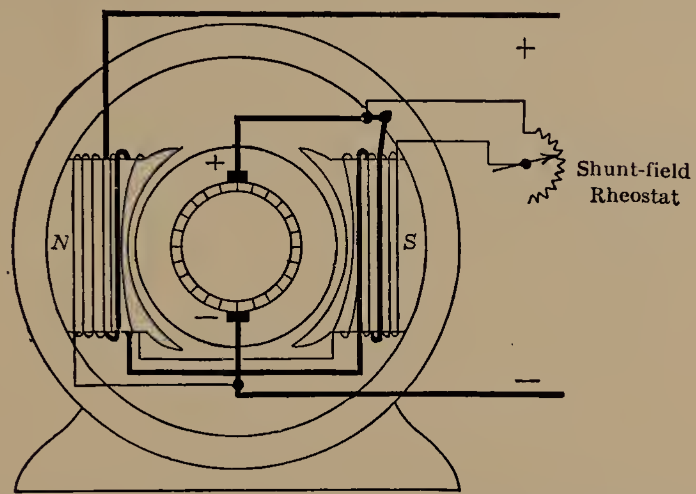


FIG. 226.—Connections of a compound generator (short shunt).

225), are neutralized more or less completely by the effect of the series ampere-turns.

The shunt field may be connected directly across the armature terminals (Fig. 227 (*a*)) in which case the machine is *short*

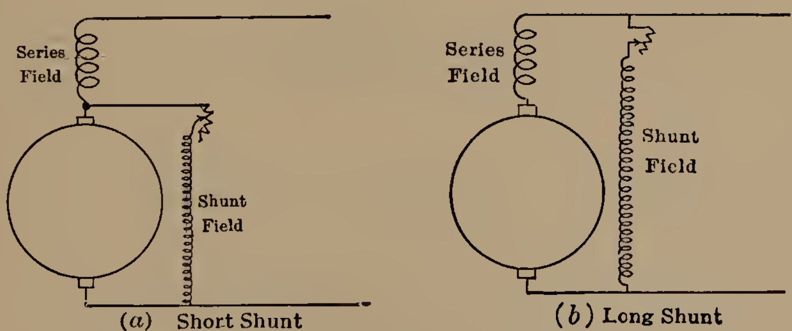


FIG. 227.—Compound generator connections.

shunt. If the shunt field be connected across the machine terminals outside the series field (Fig. 227 (*b*)) the machine is *long* shunt. The operating characteristic is practically the same in either case.

If the effect of the series turns is to produce the same voltage at rated load as at no load, the machine is said to be *flat* compounded (see Fig. 228).

When the rated-load voltage is greater than the no-load voltage, the machine is said to be *overcompounded*. When the rated-load

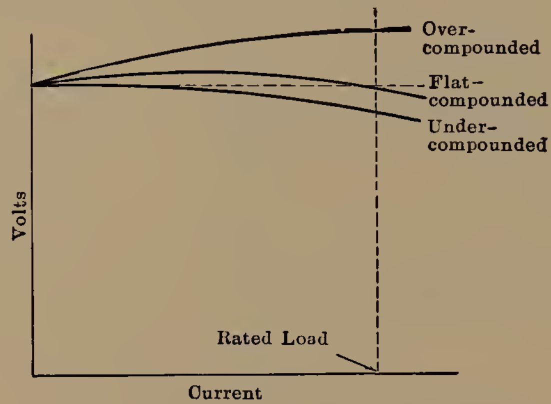


FIG. 228.—Compound-generator characteristics.

voltage is less than the no-load voltage, the machine is said to be *undercompounded*. Generators are seldom undercompounded.

All three characteristics tend to droop, due to saturation of the iron. The series ampere-turns do not increase the flux at full load so much proportionately as they do at light load (see Fig. 210, page 233).

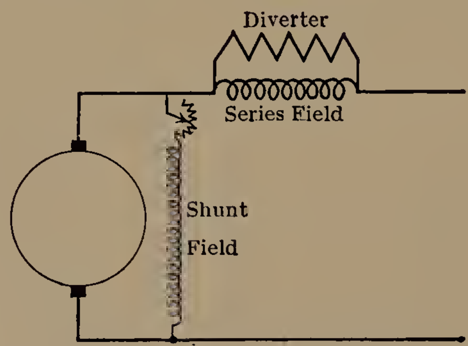


FIG. 229.—Series-field diverter.

Compound generators are usually wound so as to be somewhat overcompounded. The degree of compounding can then be regulated by shunting more or less current away from the series field. To do this, a low-resistance shunt, called a *diverter*, is used (Fig. 229).

196. Industrial Applications of Compound Generators.—Flat-compounded generators are used principally in isolated plants, such as hotels and office buildings, where the circuit runs are short and where it is desirable that the voltage be maintained constant at all loads without adjustment of the shunt-field rheostat.

Overcompounded generators are used where the load is located at some distance from the generator. As the load increases, the voltage at the load tends to decrease, due to the voltage-drop in the feeder. If, however, the generator voltage rises just enough to offset this feeder drop, the voltage at the load remains constant. Railway generators are typical of this class of service. They are usually so compounded that the no-load terminal voltage is 550 volts and the rated-load terminal voltage is 600 volts.

197. The Series Generator.—In the series generator the field winding is connected in series with the armature and the external circuit. It must consist necessarily of a comparatively few turns of wire having a sufficiently large cross-section to carry the rated current of the generator.

In most instances the *series* generator is used for *constant-current* work, in distinction to the *shunt* generator, which maintains *constant voltage*. For a number of years the series generator[1] was used successfully to supply series arc lights for street lighting. At the present time it has been almost entirely replaced by constant-current transformers used in conjunction with mercury-arc rectifiers, if constant-current direct-current service is necessary (see Part II).

198. The Tirrill Regulator.—It has been pointed out that the voltage of a generator varies with the load, speed, etc. By means of a Tirrill regulator the voltage of a generator can be maintained constant even under rapid fluctuations of load. In addition, compensation may be made for line drop. The voltage is controlled by small relay contacts, which short-circuit the shunt-field rheostat, the duration of the short-circuit depending on the amount of regulation required. If the generator voltage drops, the time during which the shunt-field rheostat

[1] For a more complete description see Dawes, "A Course in Electrical Engineering," Vol. I.

is short-circuited increases, that is, the armature of the relay vibrates more slowly. If the generator voltage rises, the armature of the relay vibrates more rapidly. The field rheostat is

Fig. 230.—The Tirrill regulator.

usually set so that the generator voltage is 35 per cent. below normal when the regulator is disconnected. Figure 230 shows a Tirrill regulator and its mounting.

CHAPTER XII

THE MOTOR

A generator is a machine for converting mechanical energy into electrical energy.

A motor is a machine for converting *electrical* energy into *mechanical* energy. The same machine, however, may be used either as motor or generator.

199. Principle.—Figure 231 (*a*) shows a magnetic field of uniform intensity, in which is placed a conductor carrying no current. In (*b*) the conductor is shown as carrying a current into the paper, but the field due to the *N* and *S* poles has been removed. A cylindrical magnetic field exists about the conductor, due to the current in it. The direction of this field, by the corkscrew rule, is clockwise.

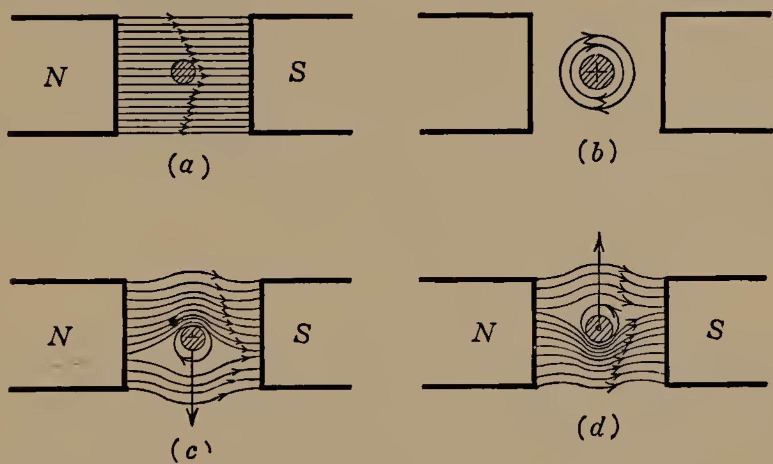


Fig. 231.—Force acting on a conductor carrying current in a magnetic field.

Figure 231 (*c*) shows the resultant field obtained by combining the main field and that due to the current in the conductor. The field due to the current in the conductor acts in conjunction with the main field above the conductor, whereas it opposes the main field below the conductor. The result is to crowd the flux *above*

the conductor and to reduce the flux density in the region *below* the conductor.

It will be found that a force acts on the conductor, tending to push the conductor *down*, as shown by the arrow attached to the conductor.

It is convenient to think of this effect as due to the crowding of the lines on one side of the conductor. Magnetic lines of force may be considered as acting like elastic bands under tension. These lines always try to contract so as to be of minimum length. The tension in these lines on the upper side of the conductor tends to pull the conductor down, as shown in the figure.

If the current in the conductor be reversed, the crowding of the lines will occur *below* the conductor, which will tend to move *upward*, as shown in Fig. 231 (*d*).[1]

The operation of the electric motor depends on the principle that *a conductor carrying current in a magnetic field tends to move at right angles to the field.*

200. Force Developed with Conductor Carrying Current.—The force acting on a conductor carrying a current in a magnetic field is directly proportional to three quantities: the strength of the field, the magnitude of the current, and the length of the conductor lying in the field. The force in *dynes* is given by

$$F = \frac{BlI}{10} \text{ dynes} \tag{89}$$

where B is the flux density in gausses or lines per square centimeter, l the active length of the conductor in centimeters, and I the current in amperes. The direction of the field, the conductor, and the direction of the force are perpendicular to one another.

Example.—A flat coil of 20 turns lies with its plane parallel to a magnetic field (see Fig. 235 (*a*)), the flux density in which is 3,000 lines per square centimeter. The length of the coil along its axis is 8 in. (20.32 cm.). The

[1] When the conductors are embedded in slots, as is customary in the usual machine, the actual force is transferred to the armature teeth. However, such force exists only in virtue of the current in the armature conductors, and the armature behaves as if the force acted on the conductors themselves. Therefore, for most purposes, motor phenomena may be treated as if the electromagnetic forces acted on the conductors themselves.

current per conductor is 30 amp. Determine the force in pounds which acts on each side of the coil (see arrows in Fig. 235 (*a*)).

$$B = 3{,}000$$
$$l = 8 \times 2.54 = 20.32 \text{ cm.}$$
$$I = 30$$
$$F_1 = 3{,}000 \times 20.32 \times \frac{30}{10} = 182{,}900 \text{ dynes.}$$

As there are 20 turns,

$$F = 20 \times 182{,}900 = 3{,}658{,}000 \text{ dynes}$$
$$\frac{3{,}658{,}000}{981} = 3{,}730 \text{ grams}$$
$$= 3.73 \text{ kg.}$$
$$3.73 \times 2.204 = 8.23 \text{ lb.} \quad \textit{Ans.}$$

201. Fleming's Left-hand Rule.—The relation among the direction of a magnetic field, the direction of motion of a conductor in that field, and the direction of the *induced* electromotive force is given by Fleming's right-hand rule (see page 208).

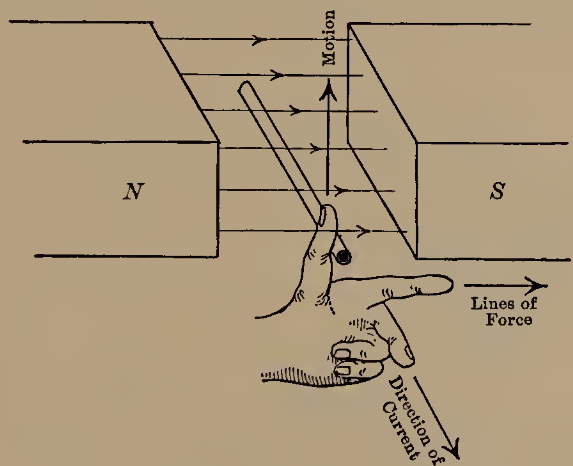


FIG. 232.—Fleming's left-hand rule.

In a similar manner, the relation among the direction of a magnetic field, the direction of a current in that field, and the direction of the resulting force can be determined by using Fleming's left-hand rule.

Fleming's Left-hand Rule:

Point the forefinger in the direction of the field or flux, the middle finger in the direction of the current in the conductor, and the thumb will point in the direction in which the conductor tends to move. This is illustrated by Fig. 232.

If Fleming's right-hand rule (for generator action) be applied to Fig. 232, the direction of motion is downwards or opposite to that for the motor.

Hence in a generator the conductor must move *against* a force tending to oppose its motion and so the conductor requires a driving force to keep it in motion. This driving force is supplied by the prime mover to which the generator is connected.

It will be recognized that the force, due to motor action, which opposes the motion of a conductor in a generator is in accordance with the law of the conservation of energy. The electrical energy delivered by the generator results from the action of the prime mover in overcoming this force.

Thus motor action exists in a generator. It will be shown later that generator action exists in a motor.

202. Torque.—When an armature, a flywheel, or any other device is revolving about an axis, a tangential force is necessary to produce and maintain rotation. This force may be

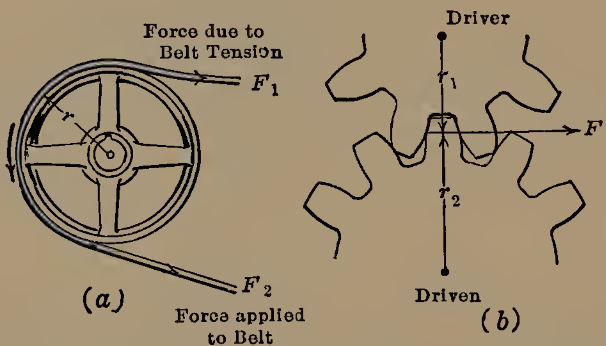


FIG. 233.—Torque developed by a belt and by gears.

developed within the machine itself, as in a motor or steam engine, or it may be applied to a driven device, such as a pulley, a shaft, a generator, the driving gears on the wheels of a street car, etc. (Fig. 233). The total effect of the force is determined not only by its *magnitude* but also by its *arm*, or radial distance from the center of the pulley or gear to the line of action of the force.

The product of the force and its perpendicular distance from the axis measures the *torque*.

Torque may be considered as a mechanical couple tending to produce rotation. It is expressed in units of force and length.

In the English system, torque is usually expressed in pounds-feet. (This distinguishes it from foot-pounds which represents *work*.)

In the c.g.s. system, the unit of torque is the dyne-centimeter (a very small unit), and in the metric system the unit is the kilogram-meter. A kilogram-meter equals 7.23 lb.-ft.

In Fig. 233 (*a*), the tight side of the belt is pulling on the rim at the bottom of the pulley with a tangential force F_2. The loose side of the belt is pulling on the rim at the top of the pulley with a tangential force F_1. The net tangential force acting on the rim of the pulley is F_2-F_1. The radius of the pulley is r. Hence the torque being applied to the pulley is $(F_2\text{-}F_1)r$.

Figure 233 (*b*) shows a driving gear having a radius r_1 to its line of action or pitch circle, and a driven gear having a radius r_2 to its pitch circle. Neglecting friction, the torque being applied by the driver is the product of the tangential force F and the radius r_1, or Fr_1. The torque available in the driven gear and shaft is Fr_2. Since F is the same in both cases, and r_2 is greater than r_1, the torque of the driven shaft is greater than that of the driver. The angular speeds of the two shafts are inversely as their radii, and hence as their torques. Therefore, if friction be neglected, the power is the same in the two cases. This must follow from the law of the conservation of energy.

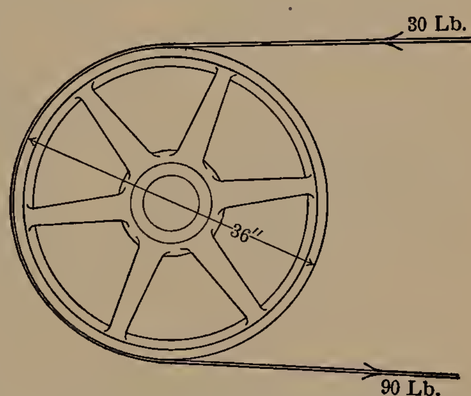


FIG. 234.—Example of torque produced upon a pulley by a belt.

Example.—A belt is driving a 36-in. (91.4 cm.) pulley, as shown in Fig. 234. The tension in the tight side of the belt is 90 lb. (40.8 kg.) and that in the loose side is 30 lb. (13.6 kg.) Determine the torque applied to the pulley.

The two sides of the belt are acting in opposition so that the net tangential pull on the rim of the pulley is

$$90 - 30 = 60 \text{ lb.}$$

This force is acting 18 in. (45.7 cm.) or 1.5 ft. from the center of the pulley. Therefore the torque

$$T = 60 \times 1.5 = 90 \text{ lb.-ft.} \quad Ans.$$

or

$$T = (40.8 - 13.6)\ 0.457 = 12.43 \text{ kg-m.} \quad Ans.$$

203. Torque Developed by a Motor.—Figure 235 (*a*) shows a coil of a single turn, whose plane lies parallel to a magnetic field. Current flows into the paper in the left-hand side of the coil and out of the paper in the right-hand side of the coil. Therefore, the left-hand conductor tends to move downward with a force F_1 and the right-hand conductor tends to move upward with an equal force F_2. These two forces form a couple tending to rotate the coil about its axis in a counter-clockwise direction. Torque is thus developed. As the current in each of the conductors is the same and they lie in the same magnetic field, the force F_1 equals the force F_2. In (*a*), the coil is in the position of maximum torque because the perpendicular distance from the axis to the lines of action of the forces F_1 and F_2 is a maximum.

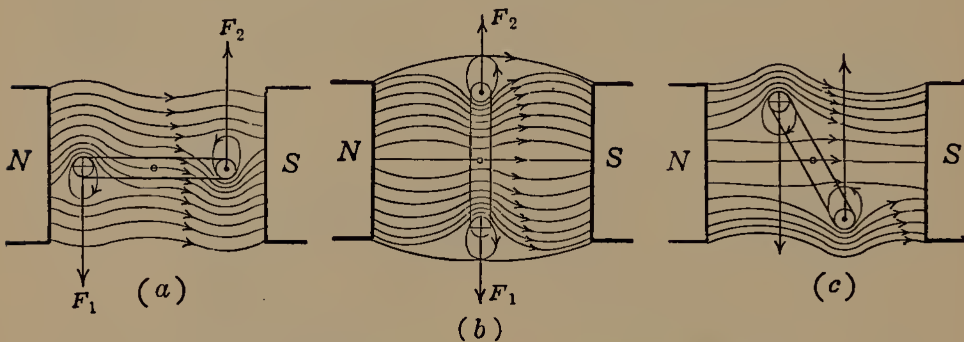


FIG. 235.—Torque developed at different positions of a coil.

When the coil reaches the position (*b*), neither conductor can move any farther unless the coil spreads. This is a position of zero torque because the perpendicular distance from the axis to the lines of action of the forces is zero.

If, however, the current in the coil be reversed when the coil reaches position (*b*), and the coil be carried slightly beyond the dead center, as shown in (*c*), a torque is developed which tends to turn the coil still in the counter-clockwise direction.

To develop a continuous torque in a motor, the current in each coil on the armature must be reversed just as the coil is passing through the neutral plane or plane of zero torque. A commutator is therefore necessary. This is analogous to using a commutator in connection with a generator in order that the current delivered to the external circuit may be unidirectional.

A single-coil motor, like that shown in Fig. 235, is impracticable, as it has dead centers and the torque which it develops is pulsating. A two-coil armature would eliminate the dead centers, but the torque developed would still be pulsating in character.

The best results are obtained when a large number of coils is used, just as in the armature of a generator. In fact, there is no difference in the construction of a motor armature and a generator armature. In Fig. 236, an armature and a field are shown for a two-pole machine and the force developed by each individual conductor is indicated by the arrow attached to that conductor.

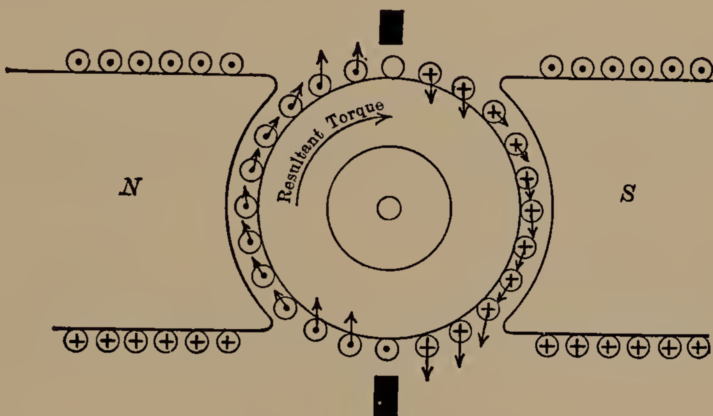


Fig. 236.—Torque developed by belt conductors in motor armature.

In armatures of this type, a very small proportion of the total number of coils is undergoing commutation at any one instant. Therefore, the variation in the number of active conductors is so slight that the torque developed is substantially constant, for constant values of armature current and main flux.

From equation (89), page 258, the torque developed by any armature can be shown to be

$$T = K_t I \Phi \tag{90}$$

where K_t is a constant of proportionality, I the armature current in amperes, and Φ the flux entering the armature from one north pole.

That is, in a given motor, *the torque is proportional to the armature current and to the strength of the magnetic field.*

This is a very important relation to keep in mind, for by its use the variation of torque with load, in the various types of motors, can be readily determined.

Example.—When a certain motor is taking 50 amp. from the line, it develops a torque of 60 lb.-ft. (8.30 kg-m.). If the field strength is reduced to 75 per cent. of its original value and the current increases to 80 amp., what is the new value of the torque developed?

If the current remained constant, the new value of torque, due to the weakening of the field, would be

$$0.75 \times 60 = 45 \text{ lb.-ft. } (6.21 \text{ kg-m.}).$$

Due to the increase in the current, however, the final value of torque will be

$$\frac{80}{50} 45 = 72 \text{ lb.-ft. } (9.96 \text{ kg-m.}). \quad \textit{Ans.}$$

It must be remembered that the torque expressed by the above equations is the entire torque *developed* by the armature. The torque available at the pulley will be slightly less than this, due to the torque lost in overcoming friction and in supplying the iron losses of the armature.

204. Counter Electromotive Force.—The resistance of the armature of the ordinary 10-h.p., 110-volt motor is about 0.05 ohm. If this armature were connected directly across 110-volt mains, the current, by Ohm's law, would be

$$I = \frac{110}{0.05} = 2{,}200 \text{ amp.}$$

This value of current is not only excessive but unreasonable, especially when one considers that the rated current of such a motor is in the neighborhood of 90 amp. When a motor is in operation, *the current through the armature is evidently not determined by its ohmic resistance alone.*

The armature of a motor is in every way similar to that of a generator. The conductors on its surface, in addition to carrying current and so developing torque, are *cutting flux.* Therefore, they *must* be generating an electromotive force.

Figure 237 shows a single conductor of a motor armature at the instant when it is directly in front of a north pole. This conductor is free to move. The direction of the applied voltage, hence the direction of the current, is shown by the left-hand arrow. If Fleming's *left-hand rule* be applied, it is found that the

relation of direction of current and flux is such that the conductor tends to move downwards under the action of the force developed.

As the conductor moves downwards, it cuts flux, and an electromotive force is generated in the conductor. If the *right-hand rule* be applied to determine the direction of this *induced* electromotive force, it is found to be acting from right to left (Fig. 237). Hence it is in *opposition* to the applied voltage and to the current. It follows that the induced electromotive force in a motor armature must always be in opposition to the impressed voltage and to the current. Hence, this *induced* electromotive force is called a *counter electromotive force* or a *back electromotive force*. Obviously, the counter electromotive force *opposes* the current entering the armature. Since the counter electromotive force opposes the line voltage also, the net voltage acting in the armature circuit is the difference of the line voltage and the back electromotive force. Let V equal the line voltage and E the back electromotive force. The *net* voltage acting in the armature circuit is

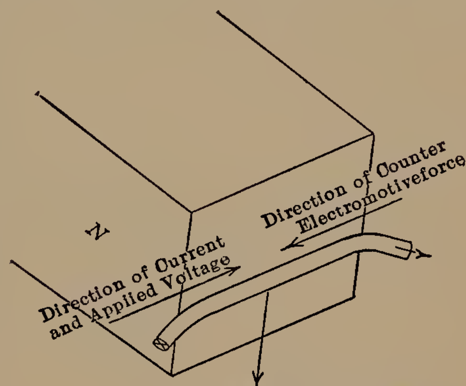


FIG. 237.—Relation of the direction of currents and voltages in a motor conductor.

$$V - E.$$

The armature current follows Ohm's law and is

$$I_a = \frac{V - E}{R_a} \tag{91}$$

where R_a is the armature resistance.

This equation may be transposed and written

$$E = V - I_aR_a. \tag{92}$$

This should be compared with equation (87), page 250, which is the corresponding equation for a generator.

In a generator the induced electromotive force is equal to the terminal voltage *plus* the armature resistance drop. In a motor

the induced electromotive force is equal to the terminal voltage *minus* the armature resistance drop. The counter electromotive force must always be less than the terminal or impressed voltage if current is to flow *into* the armature at the positive terminal.

Example.—Determine the back electromotive force of a 10-h.p. motor when the terminal voltage is 110 volts and its armature is taking 90 amp. The armature resistance is 0.05 ohm.

$$E = 110 - (90 \times 0.05) = 110 - 4.5 = 105.5 \text{ volts.} \quad \textit{Ans.}$$

An interesting experiment for demonstrating the existence of counter electromotive force is shown in Fig. 238. A lamp bank

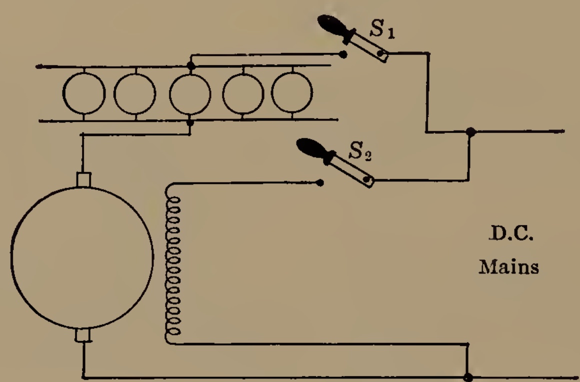


Fig. 238.—Demonstration of counter electromotive force.

is connected in series with the armature of a shunt motor. First close switch S_2 which closes the field circuit. Then close S_1. At the instant of closing S_1 the lamps will burn brightly, being practically up to candle power. As the armature speeds up, these lamps will become dimmer and dimmer, showing that the armature is generating a *counter* electromotive force, which opposes the line voltage and so results in less voltage and less current for the lamps. When the armature is up to speed, the lamps will be very dim. If, however, the field switch S_2 now be opened, the flux and, therefore, the counter electromotive force will be immediately reduced to zero practically, which will be shown by the lamps again coming up to full candle power. (In practice, when a motor is in operation, the field circuit should *not be opened under any conditions whatsoever.*)

Equation (86), page 231, for the induced electromotive force in a generator will obviously apply to a motor. That is, the counter electromotive force

$$E = K\phi S$$

where K is a constant; ϕ is the total flux entering the armature from *one* north pole; and S is the speed of the armature in revolutions per minute.

205. Speed of a Motor.—If equation (86) be solved for speed, the speed

$$S = K_1 \frac{E}{\phi} \tag{93}$$

where K_1 is a constant equal to $\frac{1}{K}$.

The speed of a motor is directly proportional to the counter electromotive force and inversely proportional to the strength of field.

Substituting for E in (93) its value given in (92), the speed becomes

$$S = K_1 \frac{V - I_a R_a}{\phi}. \tag{94}$$

This is a very important equation, for it shows the law of speed variation of a motor with changes of load.

Example.—A 15-h.p., 110-volt motor, having an armature resistance of 0.08 ohm, runs at 1,000 r.p.m. when the *armature* current is 50 amp. What is its speed when the *armature* current is 120 amp., provided the field strength remains unchanged?

Let S_2 be the speed when the armature current is 120 amp. Applying equation (94),

$$\frac{S_2}{1{,}000} = \frac{K_1 \dfrac{110 - (120 \times 0.08)}{\phi}}{K_1 \dfrac{110 - (50 \times 0.08)}{\phi}}.$$

As ϕ is the same in both cases,

$$S_2 = 1{,}000 \frac{110 - 9.6}{110 - 4.0} = 1{,}000 \frac{100.4}{106.0} = 947 \text{ r.p.m.} \quad \textit{Ans.}$$

To illustrate the effect on speed of changing the field current, the following example is given:

Example.—The field current of this motor is reduced so that the field strength is decreased 10 per cent. What is the speed when the armature current is 50 amp.?

Since the armature current does not change, the counter electromotive force E does not change. Hence (94) may be applied directly

$$\frac{S_2}{947} = \frac{K_1\dfrac{E}{\phi_2}}{K_1\dfrac{E}{\phi_1}} = \frac{\phi_1}{\phi_2}. \qquad \phi_2 = 0.9\,\phi_1.$$

Hence

$$S_2 = 947\frac{\phi_1}{0.9\phi_1} = \frac{947}{0.9} = 1{,}052 \text{ r.p.m.} \quad \textit{Ans.}$$

206. Armature Reaction and Brush Position in a Motor.—Figure 239 (*a*) shows the currents in the conductors of the armature of a motor, the brushes being in the geometrical neutral. The direction of the current in each half of the armature corresponds to the polarity and direction of rotation shown in (*b*). Due to the armature ampere-turns, a magnetomotive force F_A exists in the armature. The direction of the flux produced by this magnetomotive force is upwards and is at right angles to the polar axis.

Figure 239 (*b*) shows current both in the armature conductors and in the field turns.

The combined effect of the ampere-turns of field and armature causes the resultant flux to have a direction diagonally upwards to the right and to be crowded into the *leading* pole-tips. That is, it is distorted in the direction *opposite* to that of rotation. Since the neutral plane is perpendicular to the direction of the resultant flux, it also moves backwards. Hence, the brushes must be moved *backwards* by an angle β.

Therefore, in a motor it is necessary to move the brushes *backward* with increase of load (as shown in (*b*)), whereas in a generator the brushes are moved *forward* with increase of load. Were it not for the electromotive force of self-induction (see page 243), the brush axis would coincide with the neutral plane. Due, however, to the necessity of counteracting this electromotive force, the brushes are set behind this load neutral plane, as is shown in Fig. 239 (*b*). That is, in both the motor and the generator it is necessary to set the brushes beyond the load neutral plane in order to counteract the electromotive force of self-induction. (Fig. 239 should be compared with Fig. 214, page 238.)

This backward movement of the brushes is accompanied by a demagnetizing action of the armature upon the field, as may be

seen from a study of Fig. 239 (*b*). If β is the angle between the brush axis and the geometrical neutral, the armature ampere-conductors included in the angle 2β *oppose* the field ampere-turns. Therefore, as the load is increased, armature reaction

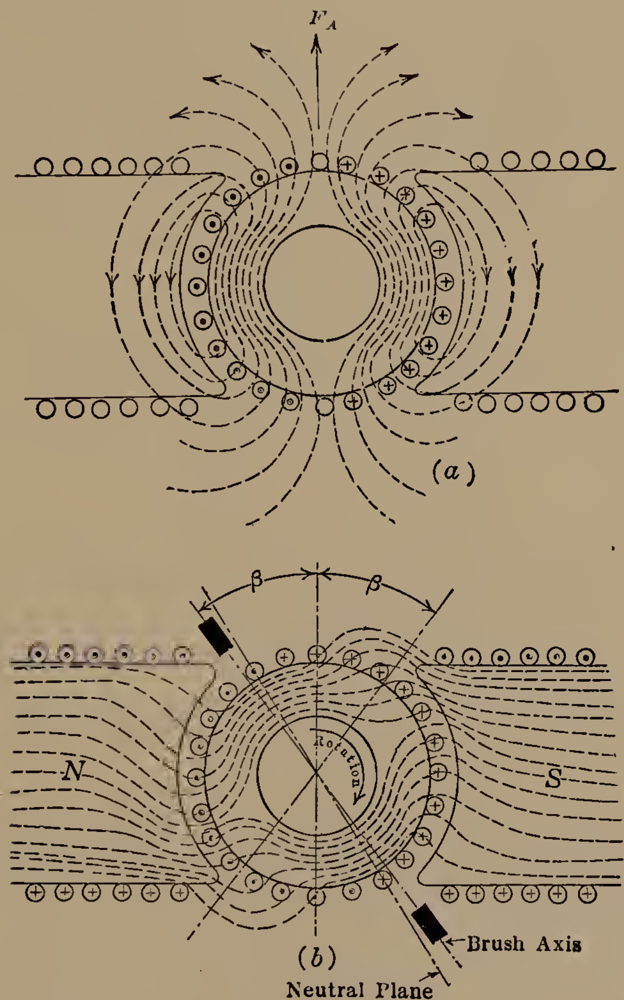


Fig. 239.—Armature reaction in a motor.

tends to increase the speed of a motor. In fact, instances are known where motors with short air-gaps (thus having high armature reaction) have run away when the load was applied.

207. Commutating Poles.—Commutating poles are used with motors, as well as with generators, for annulling armature reaction *in the neutral plane* and also for inducing in the armature coils

undergoing commutation an electromotive force to overcome the electromotive force of self-induction.

Figure 240 shows diagrammatically a bipolar motor. The left-hand pole is north and the right-hand pole is south. The current flows outwards in the conductors on the left-hand side of the armature and inwards in the conductors on the right-hand side of the armature. The armature therefore rotates in a clockwise direction (Fleming's left-hand rule). The field magnetomotive force F acts from left to right and the armature mag-

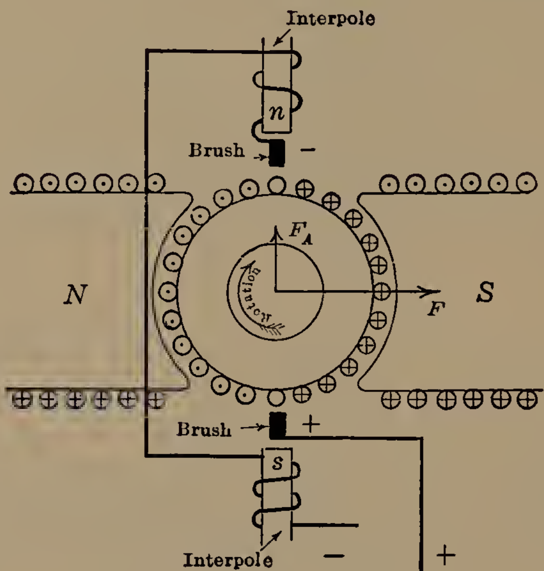


FIG. 240.—Interpoles in a motor.

netomotive force acts upwards (see Fig. 239). These directions are determined by the corkscrew rule.

Since the interpoles must oppose the armature reaction in the neutral plane, the interpole at the top of the armature must be north and that at the bottom of the armature must be south, as shown in the figure. Therefore, in motors the relative polarity of the main poles and the interpoles taken in the direction of rotation is *NnSs*, which is opposite to that for generators (see Fig. 222, page 248). The interpoles are connected in series with the armature, as shown in Fig. 240.

If a motor happens to be sparking badly from some unknown cause, the polarity of the interpoles should be carefully investi-

gated with a compass, as the sparking may be due to their being incorrectly connected.

208. The Shunt Motor.—The shunt motor is connected in the same manner as a shunt generator, that is, its field is connected directly across the line in parallel with the armature.

A field rheostat is usually connected in series with the field.

If load is applied to any motor, the motor immediately tends to slow down. Unless the reactions developed within the motor as a result of this slowing down are such that the torque increases with the increase of load, the motor stops. With the shunt motor this decrease of speed lowers the back electromotive force, as the flux remains substantially constant (see equation (93), page 267). If the back electromotive force is decreased, the *difference* between the terminal voltage and the back electromotive force is increased and more current flows into the armature (see equation (91), page 265). This continues until the increased armature current produces sufficient torque to meet the demands of the increased load.

The suitability of a motor for any particular duty is determined almost entirely by two factors, the variation of its *torque* with load and the variation of its *speed* with load.

In the shunt motor, the flux is substantially constant. Therefore, from equation (90), page 263, the torque will vary almost directly with the *armature* current. If the torque is plotted against current, the resulting graph is practically a straight line (Fig. 241). For example, in Fig. 241, when the line or motor current is 10 amp., the motor develops 42 lb.-ft. (5.80 kg-m.) torque, and when the line or motor current is 20 amp., the motor develops 85 lb.-ft. (11.8 kg-m.) torque. That is, when the current doubles the torque practically doubles. The torque at the pulley, not the internal torque, and the line current, rather than armature current, are plotted in Fig. 241. If the armature rotates, the torque at the pulley is slightly less than the internal torque by the amount necessary to overcome friction and to supply the iron losses.

The speed of a motor varies according to equation (94), page 267, where

$$S = K_1 \frac{V - I_a R_a}{\phi}.$$

In the shunt motor, K_1, V, R_a, and ϕ are substantially constant. Therefore, the only variable is I_a. As the load on the motor increases, I_a increases and the numerator of this equation decreases. As a rule, the denominator changes only by a small amount. The speed of the motor will then drop with increase of load, as shown in Fig. 241. Since I_aR_a is ordinarily from 2 to 6 per cent. of V, the percentage drop in speed of the motor is of this order of magnitude. For this reason, the shunt motor is con-

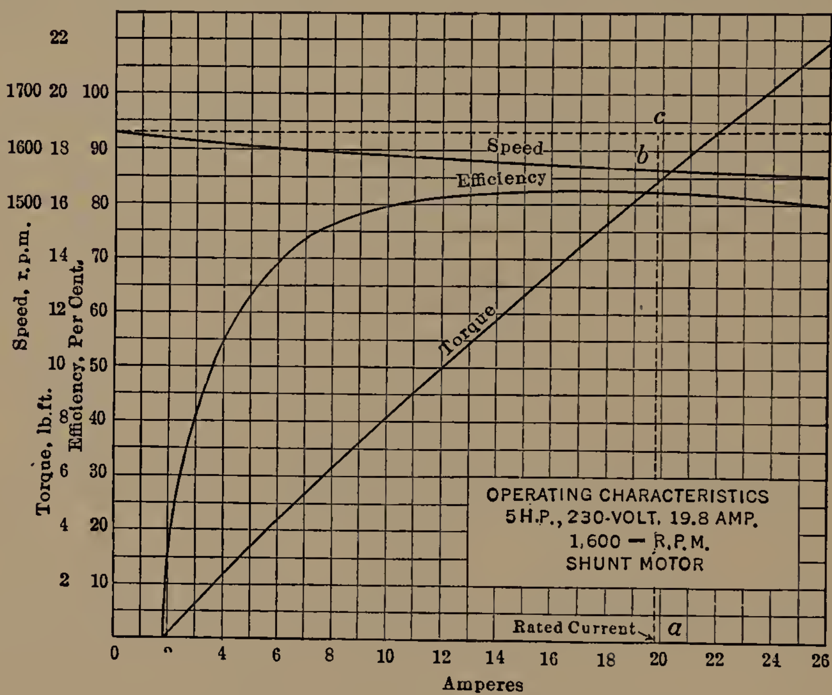


FIG. 241.—Typical shunt-motor characteristics.

sidered a constant-speed motor, even though its speed does decrease slightly with increase of load (see problem, page 267).

Owing to armature reaction, the resultant field flux ϕ ordinarily decreases slightly with increase of load, and this tends to maintain the speed constant. Occasionally, the armature reaction is sufficiently great to give a rising speed characteristic with increase of load.

Figure 241 shows the three essential characteristics of a shunt motor, the torque, the speed, and the efficiency, each plotted against current. The torque is practically proportional to the

current, and the speed drops slightly with load, as has already been demonstrated.

Shunt motors are used where a substantially constant speed is required, as in machine-shop drives, spinning frames, blowers, etc.

The shunt motor is able to develop full-load torque or even more on starting, provided the starter has sufficiently high carrying capacity.

203. The Series Motor.—In the series motor, the field is connected in series with the armature (Fig. 242). The field has comparatively few turns of wire and this wire must be of sufficient cross-section to carry the rated armature current of the motor.

In the series motor the flux, ϕ, depends entirely on the armature current. If the iron of the motor is operated at moderate saturation, the flux will be almost directly proportional to the armature current. Therefore, in the expression for torque,

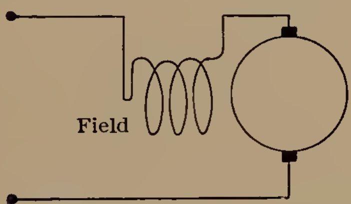


Fig. 242.—Connections of a series motor.

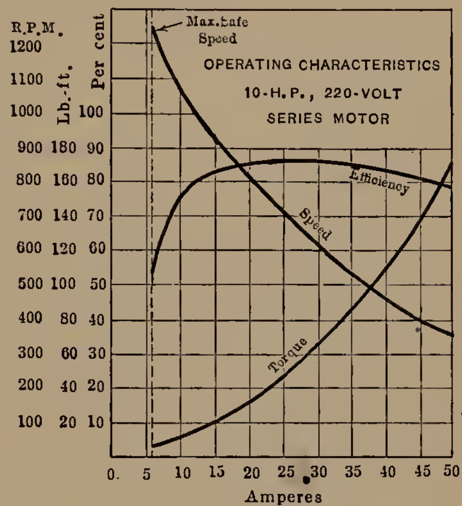


Fig. 243.—Typical series-motor characteristics.

$$T = K_t I \phi$$

if ϕ is assumed to be proportional to I, the torque is given by

$$T = K'_t I^2 \tag{95}$$

when K'_t is a constant.

The torque under these conditions is nearly proportional to the *square* of the armature current (Fig. 243). When the current is 20 amp., the torque is slightly over 30 lb.-ft. (4.15 kg-m.); at 40 amp., the torque is 110 (15.2) lb.-ft. That is, the doubling of the armature current results in nearly quadrupling the torque. This characteristic of the series motor makes its use

desirable where large increases of torque are desired with moderate increases in current. In practice, saturation and armature reaction both tend to prevent the torque increasing as rapidly as the square of the current.

When equation (94), page 267, is applied to the series motor, the speed

$$S = K_1 \frac{V - I_a(R_a + R_s)}{\phi} \tag{96}$$

where K_1 is a constant, V the terminal voltage, I_a the motor current, R_a the armature resistance including brushes, R_s the series-field resistance, and ϕ the flux entering the armature from a north pole. R_s, the resistance of the series field, is now added to the armature resistance in order to obtain the total motor resistance. Both I_a and ϕ vary with the load.

As the load increases, the voltage-drop due to the resistance of the field and armature increases, because this voltage-drop is proportional to the current. Therefore, the back electromotive force becomes less, which causes the motor to run more slowly, although this effect is only of the magnitude of a few per cent. The flux ϕ, however, increases almost directly with the load. Therefore the speed must drop, in order that the back electromotive force be of the proper value, which is usually a few per cent. less than the terminal voltage. Both effects tend to slow down the motor. The resistance-drop is ordinarily from 2 to 6 per cent. of the terminal voltage V, so its effect on the speed is only of this magnitude. As the speed is inversely proportional to the flux ϕ, a given percentage change in ϕ produces the same percentage change in the speed. Therefore, the speed of the motor is, almost entirely determined by the flux.

When the load is decreased, the flux ϕ decreases correspondingly and the armature must speed up in order to develop the required back electromotive force. If the load be removed altogether, ϕ becomes extremely small, resulting in a very high speed. It is dangerous to remove the load from a series motor, as the armature is almost certain to reach a speed at which centrifugal force will wreck it.

Figure 243 shows the characteristic curves of a 10-h.p., 220-volt series motor, plotted with current as abscissas. The torque curve concaves upward, for the reasons which have just been stated.

The torque at the pulley increases less rapidly than the square of the current, because of armature reaction, saturation, friction, and iron losses. The speed is practically inversely as the current, that is, at large values of current the speed is low and at small values of current the speed is high. The characteristics cannot be determined experimentally for small values of current, as the speed becomes dangerously high.

Series motors are used for work which demands large starting torque, such as street cars, locomotives, cranes, etc. They are particularly well adapted to electric railway work, where cars or trains must accelerate rapidly and are often required to start on grades.

210. The Compound Motor.—A shunt motor may have an additional series winding in the same manner as a shunt generator. This winding may be connected so that it aids the shunt winding, in which case the motor is said to be *cumulative-compound;* or the series winding may oppose the shunt winding, in which case the motor is said to be *differential-compound.*

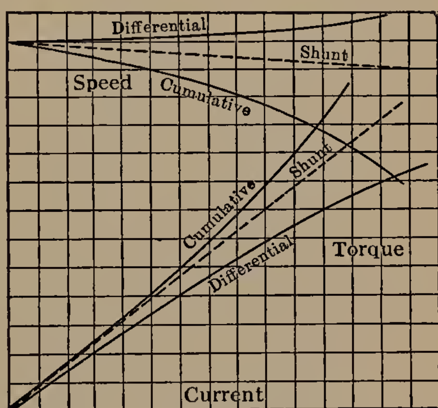


Fig. 244.—Torque and speed characteristics of shunt and compound motors.

The characteristics of the cumulative-compound motor are a combination of the shunt and series characteristics. As the load is applied, the series turns increase the flux, causing the torque for any given current to be greater than it would be for the simple shunt motor. On the other hand, this increase of flux causes the speed to decrease more rapidly than it does in the simple shunt motor. These characteristics are shown in Fig. 244. The cumulative-compound motor develops a high torque with sudden increase of load. It also has a definite no-load speed, so does not run away when the load is removed.

Its field of application is principally in driving machines which are subject to sudden heavy loads, such as occur in rolling mills, shears, punches, etc. This type of motor is used also

where a large starting torque and constant running speed is desired, as with elevators.

In the *differential-compound* motor, the series field opposes the shunt field, so that the flux is decreased as the load is applied. This results in the speed remaining substantially constant or even increasing with increase of load. This speed characteristic is obtained with a corresponding decrease in the rate at which the torque increases with load. Because of the substantially constant speed of the shunt motor, the differential-compound motor is little used. Typical torque and speed curves of the differential-compound motor are shown in Fig. 244.

To reverse the direction of rotation in any motor, either the armature alone or the field alone must be reversed. If both are reversed the direction of rotation remains unchanged. Therefore, in so far as the direction of rotation of the motor is concerned, it is immaterial which line is positive.

211. Motor Starters.—It was shown in Par. 204, page 264 that, if a 10-h.p., 110-volt motor were connected directly across 110-volt mains, the resulting current would be $\frac{110}{0.05}$, or 2,200 amp. Such a current would not be permissible under commercial conditions. Hence, resistance should be connected in series with the motor *armature* when starting. This resistance may be gradually cut out as the armature comes up to speed and develops back electromotive force.

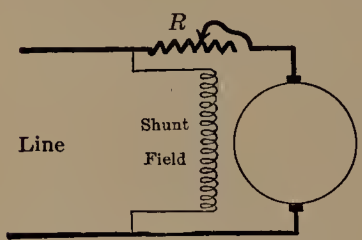
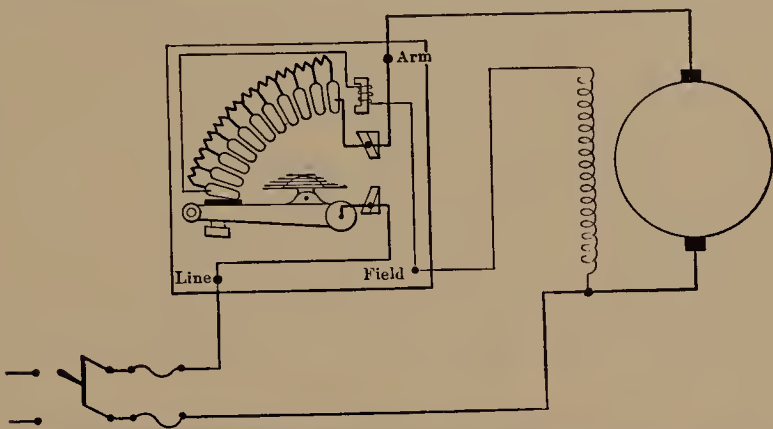


Fig. 245.—Resistance used for starting purposes.

Figure 245 shows the use of a simple resistance R for starting a motor. It will be noted that this resistance is in the *armature* circuit and that the field is connected directly across the line and outside the resistance. If the field were connected across the armature terminals, putting the resistance R in series with the entire motor, there would be little or no voltage across the field at starting. There would be little torque developed and difficulty in starting would be experienced.

Figure 246 shows a three-point starter. This does not differ fundamentally from the connections shown in Fig. 245. One line connects directly to an armature and a field terminal connected

together. It makes no connection whatever with the starting box. The other line connects to the line terminal of the starting box, which is connected directly to the starting arm. The armature terminal of the starting box, which is the right-hand end of the starting resistance, is connected to the other armature



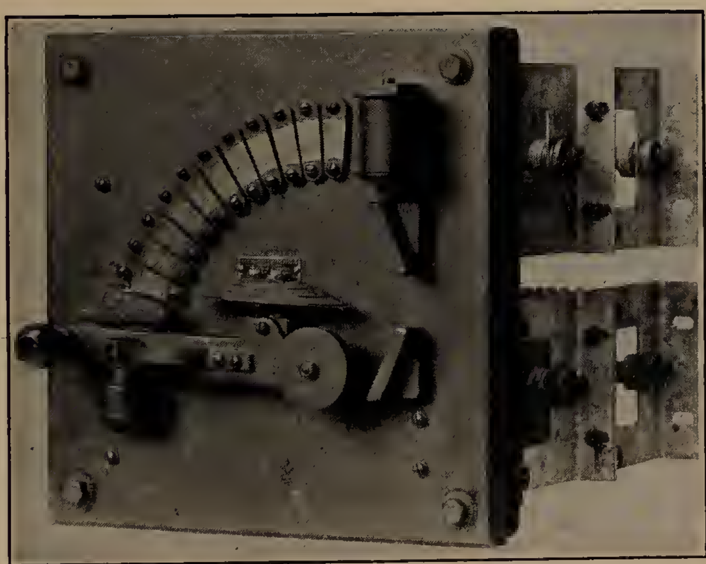


FIG. 246.—Three-point starting box.

terminal of the motor. The field connection in the starting box is from the first starting contact, through the hold-up magnet, to the field terminal of the box. This field terminal is connected directly to the other terminal of the shunt field.

When the starting arm makes connection with the first contact, the field is connected directly across the line and at the same time

the entire starting resistance is in series with he armature. As the arm is moved over the contacts, the starting resistance is gradually cut out. When the arm reaches the running position, the starting resistance is all cut out and, to insure good contact, the line and armature conductors frequently are connected directly, by a laminated copper brush, shown in Fig. 246. The field current now feeds back through the starting resistance. This resistance is so low compared with the resistance of the field itself that it has no material effect on the value of the field current. A spring tends to pull the starting arm back to the starting position. When the arm reaches the running position, it is held against the action of this spring by a soft-iron magnet (hold-up magnet), connected in series with the shunt field. If, for any reason, the line is without voltage, the starting arm springs back to the starting position. Otherwise, if the voltage returned to the line after a temporary shut-down, the stationary motor armature would be connected directly across the line and a short-circuit would result.

The advantage of connecting the hold-up coil in series with the field is that, should the field circuit become opened, the arm springs back to the starting position and so prevents the motor running away.

The three-point starting box cannot be used to advantage on variable-speed motors having field control. Such motors frequently have a speed variation of 5 to 1. This results in the field current having approximately this same variation. The hold-up magnet may be too strong, therefore, at the higher values of field current and too weak at the lower values. To obviate this difficulty a four-point box is used (Fig. 247). It is similar to the box shown in Fig. 246, except that the hold-up coil is of high resistance and is connected *directly across the line.* The only difference is that the negative line terminal (line —) must be connected to the side of the line which runs directly to the common armature and field terminals. When the voltage leaves the line, the hold-up coil becomes dead and allows the arm to spring back to the starting position.

Sometimes the field resistance is contained within the starting box. The box then has two arms of different lengths. The shorter arm is pushed up by the longer arm and cuts out the

armature resistance in the ordinary manner. When the starting resistance is entirely cut out, the shorter arm is held by the magnet, and the longer arm, which has no spring, inserts resistance in the field circuit when moved backward.

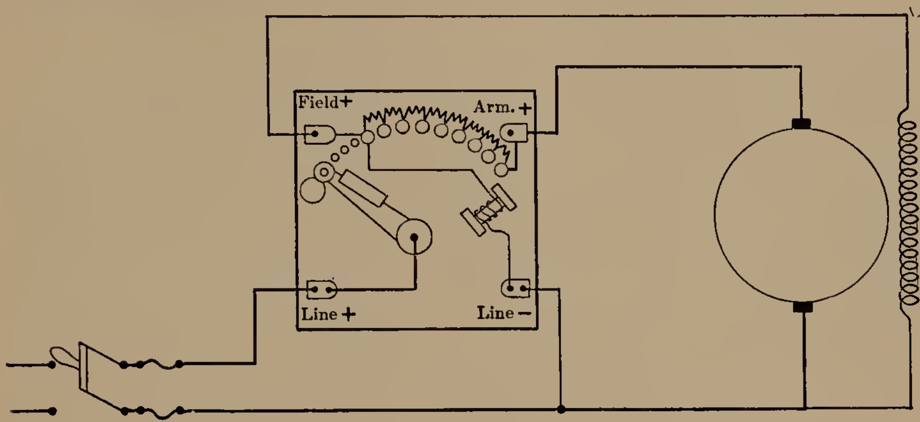


FIG. 247.—Connections for a four-point starting box.

The line switch should always be opened to stop the motor. Do not stop the motor by throwing back the starting arm, because the field discharge burns the contacts. With shunt motors, the line switch can be opened with no appreciable arc, since the motor has a high back electromotive force and the field can discharge gradually through the armature.

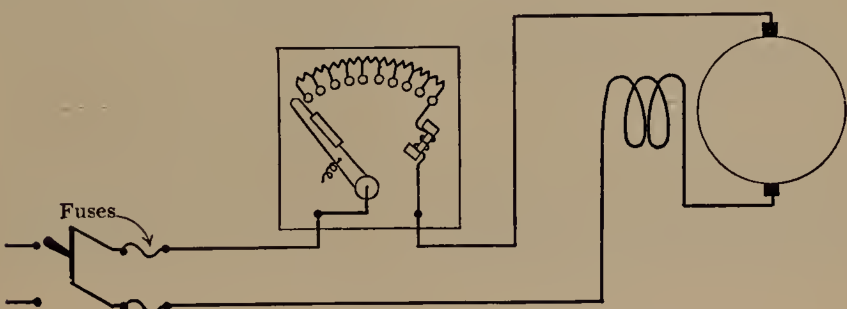


FIG. 248.—Series-motor starter, no-load release.

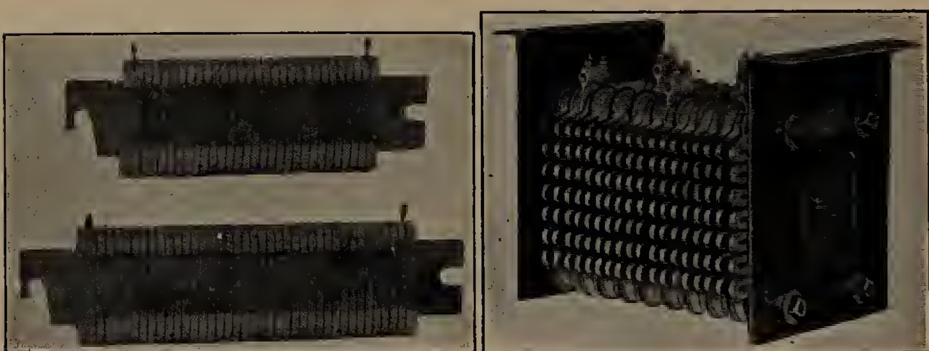
The series-motor starter needs no shunt-field connection. There are two principal types, one having a no-voltage release, and one having a no-load release, shown in Fig. 248. In the former type, the hold-up coil is connected directly across the line and releases the arm when the voltage goes off the line. In the latter type, the hold-up coil consists of a few turns in series

with the motor. When the motor current falls below the desired value, the starting arm is released. This latter type is particularly adapted to series motors where there is a possibility of the load dropping to such a low value that the motor speed may become dangerous. Some series-motor starters combine a shunt coil with the series coil.

Controllers are used where the operation of the motor is continually under the direct control of an operator, as in street car, crane, and elevator motors. The controller must be more rugged than the starting box, since the controller is in continual use for starting, stopping, and reversing the motor. Such controllers usually have an external resistance, which is cut in and out by fingers in the controller. A shunt-motor field-rheostat may also be incorporated in the controller. Controllers are usually fitted with a "reverse," so that the motor may be run in either direction.

Some types of starter are built in the form of controllers. The resistance, usually of the grid type, is designed to carry the rated current of the motor continuously so that it may be used to secure speed control.

212. Resistance Units.—Starting boxes are usually designed for starting duty only. They can carry the starting current



(*a*) (*b*)

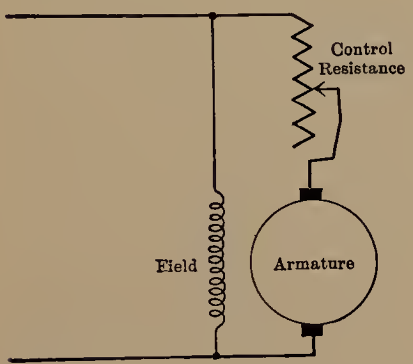
FIG. 249.—Starting-box resistance units.

of the motor safely for the short period of starting, but they cannot carry such a current continuously. The box resistance units are usually of the type shown in Fig. 249. In the smaller types, the wire if wound in the form of a helix. It may be self-supporting or it may be wound on asbestos or porcelain forms (Fig. 249 (*a*)). In the larger types, cast-iron grids are used (Fig.

249 (*b*)). These grids are bolted together. Current lugs are clamped on at suitable points so that the desired ranges of resistance are readily obtainable.

213. Speed Control.—In the equation for motor speed, $S = K_1\frac{E}{\phi}$ (see equation (93) page 267), there are but two factors which can be changed to secure speed control without making changes in the motor construction. These factors are the back electromotive force E and the flux ϕ.

Armature Resistance Control.—In this method, the speed control is obtained by connecting a resistance directly in series with the motor *armature*, keeping the field across the full-line potential (Fig. 250 (*a*)). A wide range of speed can be obtained by this method and at the same time the motor will develop any desired torque over its working range, for the torque depends only upon the *flux* and armature *current*.



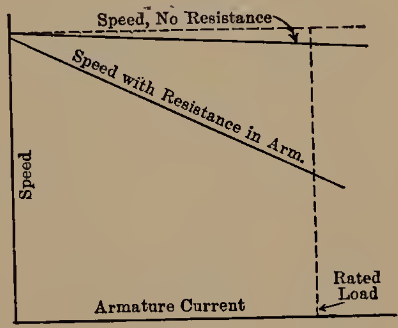


FIG. 250.—Speed control and regulation—armature resistance method.

The principal objections to this method of speed control are that an excessive amount of power is lost in the armature series-resistance, and the speed *regulation* is very poor. In Fig. 250 (*b*), there is shown for comparison the speed-load curves of a shunt motor with and without resistance in series with the armature. The speed-load curve with series resistance in the armature circuit shows that half speed is obtained at rated load. It will be observed that the speed at no load rises to a value which is practically equal to the speed of the motor when there is no resistance in series with the armature. With resistance in the

armature circuit, the change of speed with load is excessive. Furthermore with 50 per cent. reduction in speed about 50 per cent. of the power supplied to the armature circuit is lost in the series resistance. Without series resistance the speed change from no load to full load is the usual 3 or 4 per cent.

Multivoltage System.—In this system, several different voltages are available at the armature terminals of the motor, while the shunt field of the motor is connected permanently across one of these voltages. Owing to the fact that this system requires several wires for each motor, it is little used in this country.

Field Control.—In the foregoing methods of speed control, the voltage impressed on the armature has been varied. Change of speed may also be obtained by varying the flux ϕ by means of a field-rheostat. Since the speed varies *inversely* as the flux (equation (94), page 267), the field is *weakened* to *increase* the speed and is *strengthened* to *decrease* the speed. This method is very efficient, since the power lost in the field is slight, and for any particular field adjustment, the speed variation from no load to full load is slight. With the ordinary motor, the range of speed obtainable by this method is limited by commutation difficulties.

The main field is weakened to increase the speed. For a given power load, the armature ampere-turns remain constant. A study of Fig. 239, page 269, shows that with a constant armature flux and a decreasing field flux, the resultant flux tends more and more to take the direction of the armature flux. As a result, the neutral plane shifts further and further backwards. In order to commutate properly, the brushes must likewise be shifted backwards. This increases the demagnetizing effect of the armature, which results in still further backward shifting of the neutral plane. Shifting the brushes with the neutral plane to improve commutation merely results in increased demagnetization, etc., and ultimately in vicious sparking. There is also a tendency for the motor to become unstable and to blow the fuses. Hence with ordinary motors, field control cannot be used to obtain a wide range of speed, since satisfactory commutation at the higher speeds is practically impossible to secure.

With commutating poles, however, the brushes remain in the geometrical neutral, the demagnetizing action of the armature is small, and good commutation may be obtained over a wide range

of speed. A range of 5 to 1 in speed variation is obtainable with properly designed machines having commutating poles.

Because of its efficiency and simplicity, this method of speed control is ordinarily used.

214. Railway Motor Control.—In a two-motor trolley car, two different speeds can be efficiently obtained. The motors are first connected in series through a starting resistance R (Fig. 251 (*a*)). This resistance is gradually cut out by the controller as the car comes up to speed, and then each motor receives one-half the line voltage. This is the first running position. For any given value of armature current each motor will run at half its rated speed. As there is no external resistance in the circuit, the motors are operating at an efficiency very nearly equal to that obtainable with full-line voltage across the terminals of each.

When it is desired to increase the speed of the car, the two motors are connected in parallel with each other and in series with a portion of the resistance R. This resistance is gradually cut out and, when the running position is reached, each motor receives full-line voltage (Fig. 251 (*b*)).

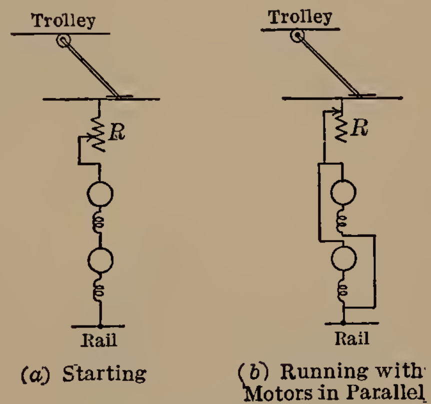


FIG. 251.—Series-parallel control of series motors.

Multiple-unit Control.—In the larger electric cars and in electric locomotives, the currents become so great that direct platform control is out of the question from the standpoint of the size of controller, safety, and expense. Moreover, when cars are operated in trains, it is necessary that the motors on all the cars shall be under a single control and that each step in the control shall operate simultaneously for all motors.

In the multiple-unit system, all the heavy-current switching is done by solenoid-operated contactors located beneath the car. These contactors, in turn, are operated by an auxiliary circuit called the *train line*, which receives its power through the master controller operated by the motorman. After the motorman has moved his controller handle to the running position, the closing

of the contactors, and hence the rate of acceleration, etc., are automatic.

215. Motor Testing—Prony Brake.—It is often necessary to determine the efficiency of a motor at certain definite loads, and frequently over its entire range of operation. This gives data for obtaining characteristic curves (see Fig. 241, page 272, and Fig. 243, page 273). A knowledge of the efficiency may be

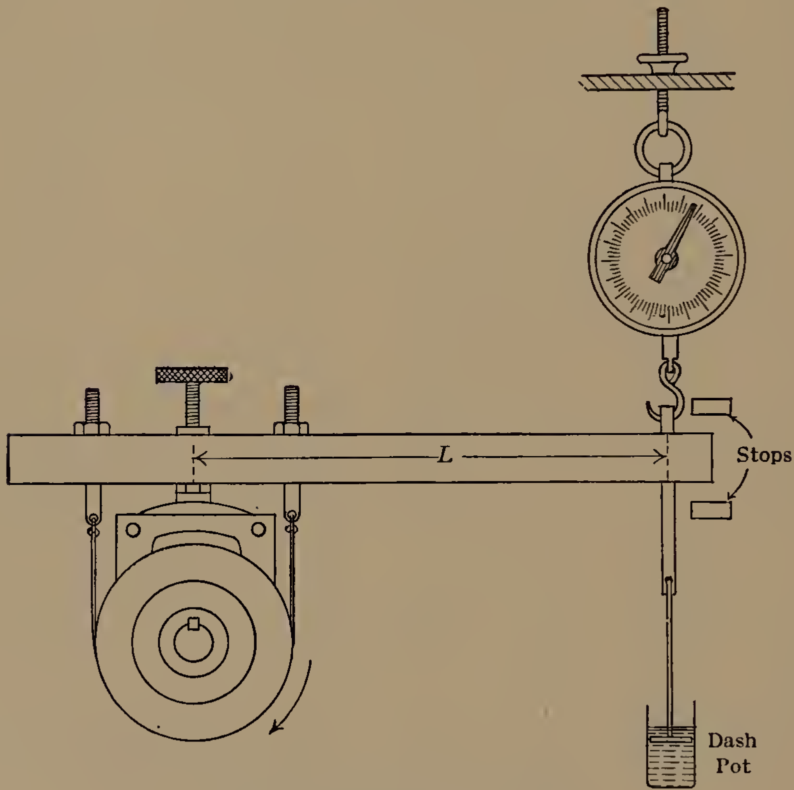


FIG. 252.—Typical prony brake.

necessary, as in an acceptance test; further, the motor may be used as a power-measuring device for determining the power taken by some machine, such as a generator, pump, blower, etc. Knowing the motor input, which can be measured with an ammeter and a voltmeter, and knowing the motor efficiency, the output for any given input can be computed. This output will be the power delivered to the generator, the pump, etc.

The most common method of making direct measurements of efficiency in motors up to about 50 h.p. is to use a prony brake. Such brakes are made in various forms. One typical form is shown in Fig. 252. It consists of a wooden arm of the proper length, a canvas brake-band, and a hand wheel for applying tension to the brake-band. By means of this hand wheel the motor load can be controlled. An oil dash-pot is advisable, to prevent vibrations of the brake arm.

The balance measures the pull on the arm due to the rotation of the drum, plus the dead weight of the arm. By multiplying the net balance reading by the distance L, the torque of the motor is determined.

There are two simple methods for determining the dead weight or tare of the brake arm. The brake band is loosened and some sort of knife edge, such as a pencil, is placed between the top of the drum and the brake carriage. This acts as a substantially frictionless fulcrum, so that the balance reads the dead weight of the arm alone. Another and easier way is to turn the drum toward the balance by hand, stop, and read the balance. In this case the friction of the brake causes the balance to read too high. If this operation be repeated by rotating the drum in the opposite direction, the balance reading will be too low, due to the same friction. The average of these two balance readings will give very nearly the correct value for the dead weight of the arm.

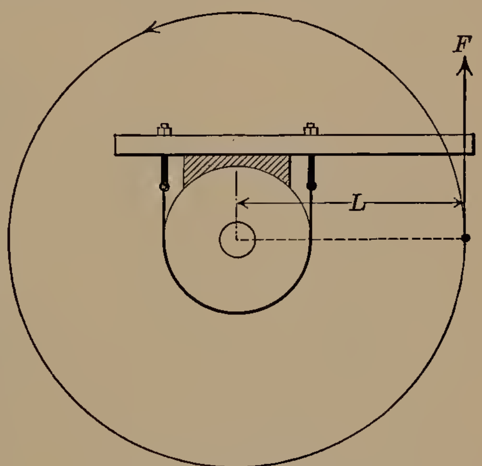


FIG. 253.—Work developed by a prony brake.

Brakes of this type are cooled ordinarily by pouring water into the hollow brake drum. This water prevents the drum from becoming excessively hot. As the maximum temperature which water can reach in the open air is 100° C., the drum temperature cannot much exceed this. The heat developed in the drum is utilized in converting the water into steam. As a considerable

number of heat units are required to convert a small amount of water into steam, a moderate amount of water will keep the drum comparatively cool.

To determine the equation for the horsepower absorbed by such a brake, consider Fig. 253. Let F be the net force in pounds acting at a perpendicular distance L feet from the center of the drum. First assume that the drum is stationary and that the arm is pulled around the drum by means of the force F. The distance per revolution through which the force F acts is $2\pi L$. The work done in one revolution of the arm around the drum is the force times the distance $= F\,(2\pi L)$.

The work done in S revolutions $= F(2\pi L)S$.

If S is the revolutions per minute, the horsepower

$$\text{H.p.} = \frac{2\pi(FL)S}{33{,}000}$$

But FL is the torque T, therefore

$$\text{H.p.} = \frac{2\pi TS}{33{,}000}$$

$$\frac{2\pi}{33{,}000} = 0.00019.$$

Therefore,
$$\text{H.p.} = 0.00019\,TS. \tag{97}$$

Obviously, the same amount of work is done on the brake surface whether the drum is stationary and the arm rotates, or the arm is stationary and the drum rotates. Therefore, equation (97) applies to brakes of the type shown in Figs. 252 and 253. It will be noted that in this particular type of brake the horsepower is independent of the diameter of the drum.

Example.—In a brake test of a shunt motor, the ammeter and voltmeter measuring the input read 34 amp., 220 volts. The speed of the motor is found to be 910 r.p.m. and the balance on a 2-ft. (0.61 m.) brake arm reads 26.2 lb. (11.9 kg.). The dead weight of the arm is found to be +2.4 lb. (1.09 kg.). (*a*) What is the output of the motor? (*b*) What is its efficiency at this particular load?

(*a*) Net reading of balance $= 26.2 - 2.4 = 23.8$ lb. (10.8 kg.).
The torque $T = 23.8 \times 2 = 47.6$ lb.-ft. (6.58 kg-m.).
H.p. output $= 0.00019 \times 47.6 \times 910 = 8.23$ h.p. *Ans.*

(*b*) Output $= 8.23 \times 746 = 6{,}140$ watts.
Input $= 220 \times 34 = 7{,}480$ watts.

$$\text{Efficiency, } \eta = \frac{6{,}140}{7{,}480}\,100 = 82.1 \text{ per cent.} \quad \textit{Ans.}$$

In brakes of this type, the brake arm should be kept approximately level.

Another simple type of brake is the rope brake shown in Fig. 254. A rope is given a turn and a half around a drum and the two free ends are held each by a spring balance. The larger balance is on the end of the rope which is being pulled downward by the rotation of the drum. Let F_1 be the reading of the larger balance and F_2 that of the smaller balance. As F_1 and F_2 pull in opposite directions with respect to the rotation of the drum, the net pull at the drum periphery is $F_1 - F_2$.

The torque in lb.-ft. is

$$T = (F_1 - F_2)R \tag{98}$$

where R is the radius of the pulley in *feet.*

Example.—In a rope brake of the type shown in Fig. 254, F_1 = 32.4 lb. (14.7 kg.) and F_2 = 8.2 lb. (3.72 kg.). The drum is 10 in. (0.254 m.) in diameter. If the motor speed is 1,400 r.p.m., what horsepower does the motor develop?

The torque

$$T = (32.4 - 8.2)\,{}^5\!/\!_{12} = 24.2 \times {}^5\!/\!_{12} = 10.08 \text{ lb.-ft. } (1.394 \text{ kg-m.}).$$

The horsepower

$$\text{H.p.} = 0.00019 \times 10.08 \times 1{,}400 = 2.68 \text{ h.p.}$$

Ans.

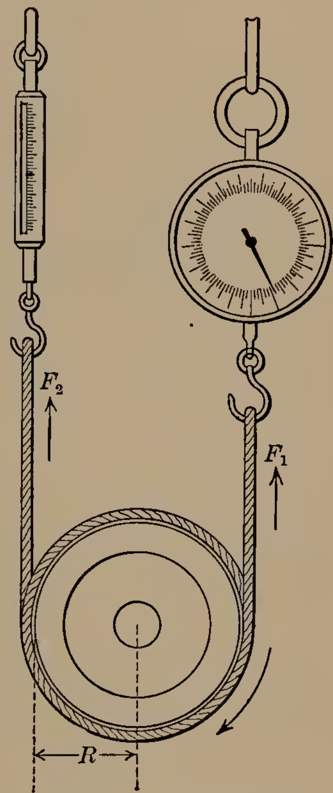
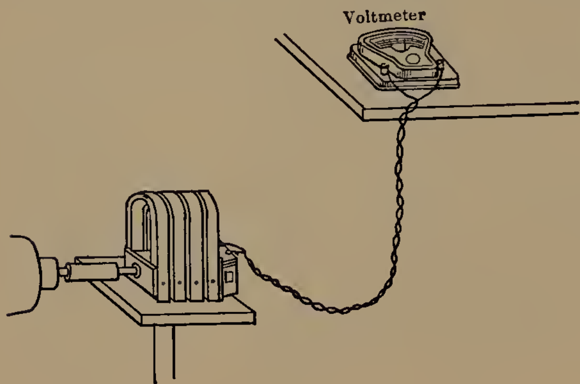


Fig. 254.—Rope brake.

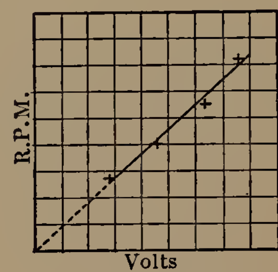
216. Measurement of Speed.—As a rule, the measurement of the speed of machines is much simpler than the measurement of torque. The most common method is to use a simple revolution counter having a conical rubber tip which fits into the countersink of the shaft.

Tachometers indicate the instantaneous value of speed. There are mechanical tachometers, where the indicator is actuated by centrifugal action. This type should be carefully checked at each occasion of use, as it is especially subject to error after having been in service for some time.

A simple and convenient type of tachometer is the combination of a direct-current magneto and a voltmeter (Fig. 255 (*a*)). In the magneto, the flux is produced by permanent magnets and so is constant. Therefore, the voltage induced in the magneto armature is directly proportional to the speed. If this voltage be measured with a voltmeter, the voltmeter reading multiplied



(*a*)



(*b*) Speed-voltage curve of magneto.

FIG. 255.—Speed measurement with magneto and voltmeter.

by a constant gives the speed directly. The relation of speed to volts may be plotted, as shown in Fig. 255 (*b*), and the speed read directly from the plot. This plot is ordinarily a straight line through the origin, so one point is accurately determined. A piece of rubber tubing is conveniently used to attach the magneto to the shaft of the machine whose speed is being measured. It is usually necessary to thread a small stud into the end of the shaft whose speed is to be measured, as shown in Fig. 255 (*a*).

CHAPTER XIII

LOSSES; EFFICIENCY; OPERATION

217. Dynamo Losses.—A part of the energy delivered to any motor or generator is lost within the machine itself, being converted into heat, and therefore wasted. This represents not only energy lost, but has the further objection that it heats the machine and so limits its output. If the energy loss in the machine becomes excessive, the resulting temperature rise may injure the insulation by carbonizing it.

As a motor and a generator are similar, they have the same types of losses throughout. Therefore, the following applies to either motors or generators.

COPPER LOSSES

Armature.—The armature windings have resistance and when current flows through them a certain amount of power must be lost. In addition to the loss in the armature copper, there is an electrical loss in the brushes and in the commutator. Let this total power loss be P_a. Then,

$$P_a = I_a^2 R_a \quad (99)$$

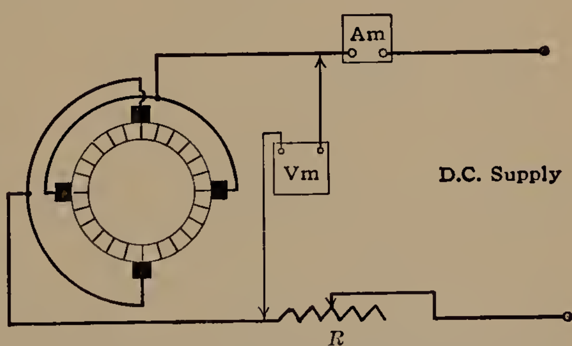


Fig. 256.—Measurement of armature resistance.

where I_a is the armature current and R_a is the armature resistance measured between the terminals of the machine, and including, therefore, the brushes and their contact resistance. The resistance measurement is often made by the connections shown in Fig. 256, readings being taken with the armature in three or four different positions (also see Figs. 120 and 121, pages 127 and

128). The resistance R is inscrted to limit the current flowing through the stationary armature (see Par. 211).

Shunt Field.—The field takes a current I_f at the terminal voltage V of the generator or motor. Therefore, the power lost in the field, including the rheostat, is

$$P_f = VI_f. \tag{100}$$

The foregoing losses are all copper losses and can be either measured directly or calculated with a high degree of precision from instrument readings.

IRON LOSSES

Eddy Currents.—As the armature iron rotates in the same magnetic field as the copper conductors, voltages are also induced in this iron. As the iron is a good conductor of electricity and the current paths are short and of large cross-section, large currents would be set up in the armature iron were it a solid mass as shown in Fig. 257 (*a*). These currents represent an excessive

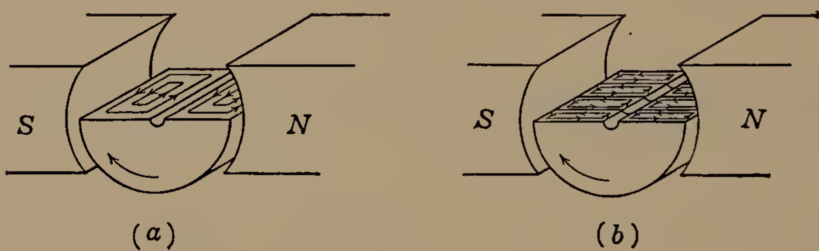


Fig. 257.—Eddy currents in armature iron without and with laminations.

power loss which cannot be tolerated in commercial machines. By laminating the armature iron in the manner indicated in Fig. 257 (*b*), the paths of these currents are broken up and their magnitude is reduced to a very low value. Laminating does not entirely eliminate these eddy-current losses, but it does reduce them to a small value. Although the laminations break up the eddy-current paths, they do not interpose reluctance in the magnetic circuit, since they are parallel to the direction of the magnetic flux.

Hysteresis.—It was shown in Chap. VIII that when iron is carried through a cycle of magnetization (page 163), there results an energy loss proportional to the area of the hysteresis loop.

As the armature rotates, all parts of the laminated structure pass alternate north and south poles and their magnetization obviously undergoes reversals. These reversals of magnetization must be accompanied by hysteresis loss. Laminating the iron does not affect this hysteresis loss.

FRICTION LOSSES

These losses consist of bearing friction, brush friction, and windage.

STRAY POWER

Unlike the copper losses, the iron and friction losses cannot be calculated readily. Also, they are supplied mechanically and not electrically. In a generator they are supplied directly by the driving torque of the prime mover; in a motor they cause a torque in opposition to the internal torque, and hence reduce the torque at the pulley. Since these losses all depend on either the speed or flux, or both, and because of their more or less indeterminate nature, they are all combined into one loss, called *stray-power* loss.

218. Efficiency.—The efficiency of a machine is the ratio of output to input. Thus:

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}$$

This may also be written in either of the following ways:

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{Losses}} \tag{101}$$

$$\text{Efficiency} = \frac{\text{Input} - \text{Losses}}{\text{Input}}. \tag{102}$$

Therefore, if the losses in a machine be known, the efficiency may be found for any given input or output.

As electrical units rather than mechanical quantities are ordinarily used in efficiency determinations, equation (101) is used for generators (output is electrical) and equation (102) for motors (input is electrical).

Example.—A 50-kw., 220-volt generator is delivering its rated load of 50 kw. (227 amp. at 220 volts). At this load the total generator losses are 4,100 watts. (*a*) What is the generator input in watts? (*b*) What is the

generator efficiency at rated load? (c) What horsepower is required to drive the generator?

(a) Input = 50,000 + 4,100 = 54,100 watts. *Ans.*

(b) Efficiency, $\eta = \dfrac{50{,}000}{50{,}000 + 4{,}100} = 0.924$, or 92.4 per cent. (equation (101)). *Ans.*

(c) H.p. $= \dfrac{54{,}100}{746} = 72.5$ h.p. *Ans.*

Example.—A shunt motor takes 52 amp. at 232 volts. The total motor losses at this load are 2,000 watts. What is the motor efficiency?

Using equation (102),

$$\text{Efficiency, } \eta = \frac{(232 \times 52) - 2{,}000}{232 \times 52} = \frac{10{,}060}{12{,}060} = 0.835 \text{ or } 83.5 \text{ per cent.}$$ *Ans.*

219. Efficiencies of Motors and Generators.—The efficiency of electrical apparatus is high as a rule. For example, a 1-h.p. motor has an efficiency of about 65 per cent.; a 5-h.p., 75 per cent.; a 10-h.p., 82 per cent.; and a 20-h.p., 88 or 89 per cent. A 500-kw. machine may have an efficiency of 94 per cent.

The efficiency of a motor may be determined from simultaneous measurements of its input and output, as was shown in Par. 215, where a prony brake is used. With larger motors it becomes difficult to construct a brake capable of absorbing the power.

It is difficult to make direct measurements of generator efficiency. The output is readily measured with an ammeter and a voltmeter, but a direct measurement of input can be made only by some such special device as a cradle dynamometer.

In any direct measurement of efficiency, any percentage error in the measurement of either output or input introduces the same percentage error in the efficiency.

In the direct measurement of efficiency, the power necessary for the test must be equal to the rating of the machine. In addition to *supplying* this power, there must be means for *absorbing* it. This is not a serious matter with small machines, but when large machines are tested, supplying and absorbing the necessary power may be difficult, if not quite impossible.

Because of the foregoing reasons, it is often desirable and even necessary to obtain the efficiency by determining the losses.

Example.—A 250-kw., 230-volt compound generator is delivering 800 amp. at 230 volts. The field current is 20 amp. The armature resistance is 0.005 ohm and the series-field resistance is 0.002 ohm. The stray power at

this load is 2,500 watts. The generator is connected long shunt. What is the generator efficiency at this load?

Output $= 230 \times 800 = 184{,}000$ watts.
Shunt-field loss $= 230 \times 20 = 4{,}600$ watts
Armature loss $= 820^2 \times 0.005 = 3{,}360$ watts
Series-field loss $= 820^2 \times 0.002 = 1{,}340$ watts
Stray power $= 2{,}500$ watts

Total loss $= 11{,}800$ watts

$$\text{Efficiency} = \frac{184{,}000}{184{,}000 + 11{,}800} = \frac{184{,}000}{195{,}800} \text{ or 94 per cent.} \quad \textit{Ans.}$$

220. Measurement of Stray Power.—It was pointed out in the preceding paragraph that stray power is a function of flux and speed only. If it is desired to determine the stray power of a motor or of a generator, the machine is connected as shown in Fig. 258, and run as a *motor* without load. The field current is adjusted to the value existing under operating conditions. Neglecting armature reaction, this gives the proper value of flux.

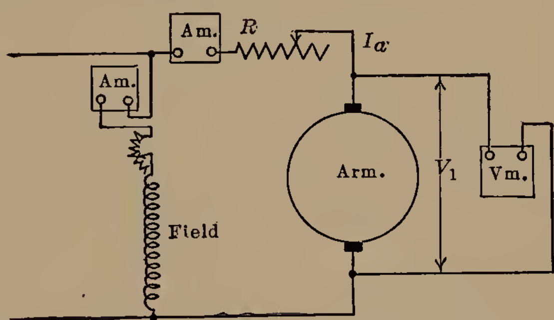


Fig. 258.—Connections for stray-power measurement.

The resistance R is adjusted until the speed is the same as under operating conditions. Then the stray power

$$S.P. = V_1 I_a - I_a^2 R_a. \tag{103}$$

That is, the stray power is equal to the *armature* input minus the armature resistance loss, which at no load is almost negligible.

Since the stray power is but a small percentage of the total power, when the machine is operating under load, a considerable error in the determination of stray power causes only a small error in the efficiency.

Example.—A 50-kw., 230-volt, 800-r.p.m. shunt generator delivers rated load at rated voltage and speed. The armature resistance is 0.032 ohm and the shunt-field current is 4.2 amp. The machine is then run as a motor without load, from 240-volt mains (see Fig. 258). The field current is maintained at 4.2 amp. When the resistance R is adjusted to give a speed of 800 r.p.m., the voltmeter reads 237 volts ($= V_1$). The armature cur-

rent is then 8.2 amp. (*a*) What is the stray power of the machine? (*b*) What is the armature loss at rated load? (*c*) What is the field loss at rated load? (*d*) What is the total loss at rated load? (*e*) What is the machine efficiency at rated load?

(*a*) From equation (103)

$S.P. = 237 \times 8.2 - (8.2)^2\, 0.032 = 1{,}940 - 2 = 1{,}940$ watts (nearly). *Ans.*

(*b*) The rated current

$$I = \frac{50{,}000}{230} = 217 \text{ amp.}$$

The armature current

$$I_a = 217 + 4.2 = 221 \text{ amp. (nearly).}$$

The armature resistance loss

$$P_a = (221)^2\, 0.032 = 1{,}560 \text{ watts.} \quad \textit{Ans.}$$

(*c*) $P_f = 230 \times 4.2 = 966$ watts. *Ans.*

(*d*) $P_e = 1{,}940 + 1{,}560 + 966 = 4{,}470$ watts (nearly). *Ans*

(*e*) Using equation (101), page 291, since the output is electrical,

$$\text{Efficiency, } \eta = \frac{50{,}000}{50{,}000 + 4{,}470} = \frac{50{,}000}{54{,}470} = 0.918, \text{ or } 91.8 \text{ per cent.} \quad \textit{Ans.}$$

The measurements necessary for determining the efficiency of the generator are made *without loading the machine*. This method gives a good degree of accuracy, since large errors in the determination of the losses produce but small errors in the resulting efficiency.

221. Ratings and Heating.—*Electrical apparatus* is usually rated at the load which it can safely carry without *overheating*. (Commutation may at times limit the output of direct-current machines.)

If the temperature of electrical apparatus becomes too high, the cotton insulation on the armature and field conductors and the insulating varnishes become carbonized and brittle. This may result ultimately in grounds and short-circuits within the machine. The A. I. E. E.[1] Standardization Rules specify the maximum safe temperatures of cotton and other fibrous insulations as 95° C. (203° F.).

It is very important, therefore, to be able to test a machine in order to determine whether it is operating within safe temperature limits. The difficulty in making such tests lies in the fact that the highest temperatures are within the coils, at points which are not easily accessible. The highest temperature within the machine is called the "hot-spot" temperature.

[1] American Institute of Electrical Engineers

The temperature at the surface of the winding may be measured by placing a thermometer bulb against the surface and covering it with a small pad of cotton. It has been found that 15° C. added to this reading will give an approximate value of the hot-spot temperature.

It has already been shown that the resistance of copper conductors changes with the temperature. By utilizing this principle, some idea of the average temperature within a winding may be obtained. For copper, the increase of resistance per degree rise of temperature may be obtained from the formula[1] $\frac{1}{(234.5 + t)}$, where t is the surrounding or ambient temperature. For example, at an ambient or room temperature of 30° C., the increase of resistance per degree rise is $\frac{1}{264.5} = 0.00378$.

Example.—With an ambient temperature of 30° C., the resistance of the field of a shunt generator increases from 104 to 112 ohms. What is its temperature rise?

The fractional change in resistance is $\frac{112 - 104}{104} = 0.077$.

Temperature *rise* $= \frac{0.077}{0.00378} = 20°.4$ C. *Ans.*

In measuring armature resistance to determine temperature rise, it is essential that the resistance of the *copper alone* be

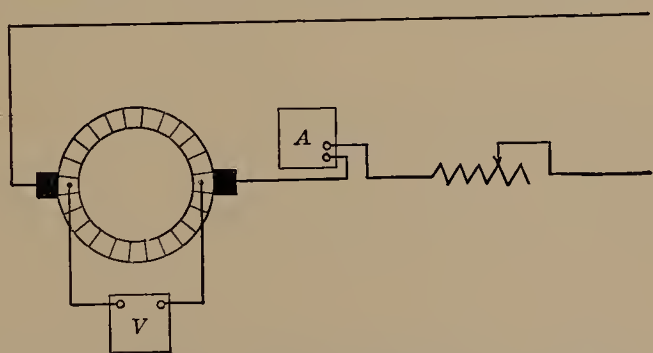


FIG. 259.—Measurement of armature resistanee for temperature test.

measured and that the current path through the copper be the same in every measurement. To exclude all resistance except that of the copper, the brush and contact resistances must not be included in the measurement. Therefore, the voltmeter leads

[1] See Par. 45, p. 43.

must be held on the commutator segments inside the brushes (Fig. 259). Moreover, these segments should be marked and in every subsequent measurement they should be directly under the same brushes.

In measuring the shunt-field resistance, the voltmeter should be connected directly across the winding so as to exclude the drop in the rheostat.

This resistance method gives an *average* value of the temperature of the windings. To find the hot-spot temperature, 10° C. should be added, according to the A.I.E.E. rules.

222. Parallel Running of Shunt Generators.—In most power plants it is necessary and desirable that the power be supplied by several smaller units rather than by a single large unit. Several smaller units are more reliable than a single large unit. With several smaller units, if one unit is disabled the entire power supply is not cut off. The smaller units may be connected in service and taken out of service to correspond with the load on the station. Additional units may be installed to correspond with the growth of station load.

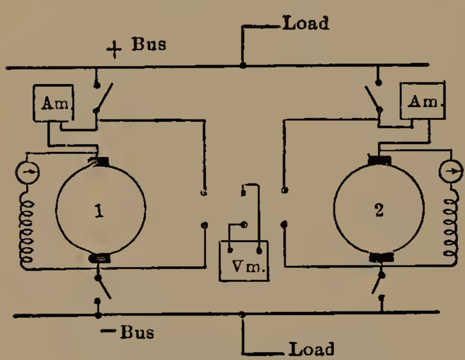


FIG. 260.—Two shunt generators in parallel.

Shunt generators, because of their drooping characteristic, are particularly well suited for parallel operation. In Fig. 260 is shown a diagram of connections of two shunt generators which may be operated in parallel and which are designated as 1 and 2. The characteristics of these two generators are shown in Fig. 261. It will be noted that generator 1 has the more drooping characteristic.

If the two generators are connected in parallel, their terminal voltages must be the same, neglecting any very small voltage-drop in the connecting leads. Therefore, for a common terminal voltage V_1 (Fig. 261) generator 1 delivers I_1 amp. and generator 2 delivers I_2 amp. That is, the machine with the more drooping characteristic carries the smaller load.

Hence two shunt generators of the same rating should have similar characteristics if they are to divide the load equally at all

times when operating in parallel. If two such machines have different ratings, the voltage-drop from no load to full load should be the same for each.

When operating in parallel, each generator should have its own ammeter. A common voltmeter is sufficient for all machines. The individual machines can be connected to the voltmeter or potential bus through suitable plug connectors or selective switches. A circuit breaker should be connected in the circuit of each generator. For simplicity these are omitted in Fig. 260.

Assume that 2 is out of service and that 1 is supplying all the load. It is desired to put 2 in service. The prime mover of 2 is started and 2 is brought up to speed. Its field is then adjusted so that its voltage is just equal to that of the bus-bars, which condition may be determined by the voltmeter. The breaker and switch are now closed and 2 is connected to the system. Under these conditions, however, it is not taking any load, as its *induced* voltage is just equal to the bus-bar voltage and no current will flow between points at the same potential.

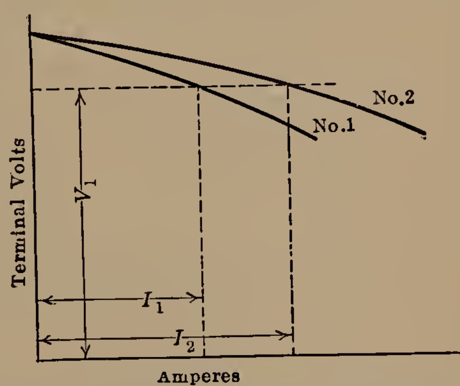


FIG. 261.—Characteristics of shunt generators in parallel.

That is, machine 2 is just "floating." Its induced voltage must be greater than that of the bus-bars in order that it may deliver current to the load. Hence, the field of 2 is strengthened until the generator takes its share of the load. It may be necessary to weaken the field of 1 simultaneously in order to maintain the bus-bar voltage constant.

To take a machine out of service, its field is weakened and that of the other machine is strengthened until the load of the first machine is zero. The breaker and then the switch are opened, clearing the machine. Connecting in and removing a machine from service in this manner will prevent any shocks or disturbance to the prime mover or to the system.

If the field of one generator be weakened too much, current will be delivered to this generator, which will run as a motor and tend to drive its prime mover.

223. Parallel Running of Compound Generators.—Figure 262 shows two overcompounded generators connected to the bus-bars, positive and negative terminals being properly connected as regards polarity. By proper adjustment of its field, each generator may be made to take its proper share of the load at the time the machines are connected in parallel.

Assume that for some reason, such as a slight change in speed, generator 1 takes a slightly increased load. The current in its series-field winding must increase, which strengthens its field and raises its electromotive force, thus causing it to take still more load. On the other hand, as the system load is assumed to be fixed, generator 2 will at the same time drop some of its load, resulting in a weakening of its series field and a consequent further dropping of its load. In a very short time 1 will be driving 2 as a motor, and ultimately the breaker of at least one of the machines will open.

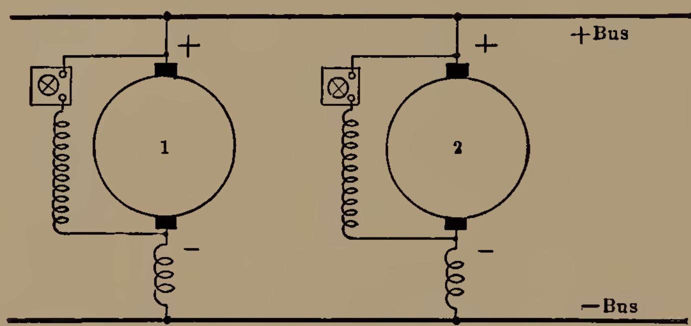


Fig. 262.—Compound generators in parallel.

These two compound generators may be considered to be in *unstable* equilibrium. That is, any action tending to throw the machines out of equilibrium is accentuated by the resulting reactions.

The machines may be made stable by connecting the two series fields in parallel (Fig. 263). This connection, which ties the two negative brushes together, is a conductor of low resistance, and is called the *equalizer*. Its operation is as follows: Assume that generator 1 starts to take more than its proper share of the load. The increase of current will cause an increase of current not only in the field of generator 1 but also, by means of the equalizer,

in the field of generator 2. Therefore, both machines are affected in a similar manner and 1 is unable to take the entire load.

It should be noted that the desired division of the load among either shunt or compound generators at any one load may be obtained by adjusting their field rheostats. However, it is usually desirable that this division remain constant at all loads, especially if an operator is not in continuous attendance. Therefore, it is desirable that generators operating in parallel have similar characteristics.

The load ammeter in a compound generator should always be connected between the *armature* terminal and the bus-bars. If it is connected in the series-field circuit, the ammeter may not indicate the generator current, due to the fact that some of the generator current may be passing through the equalizer.

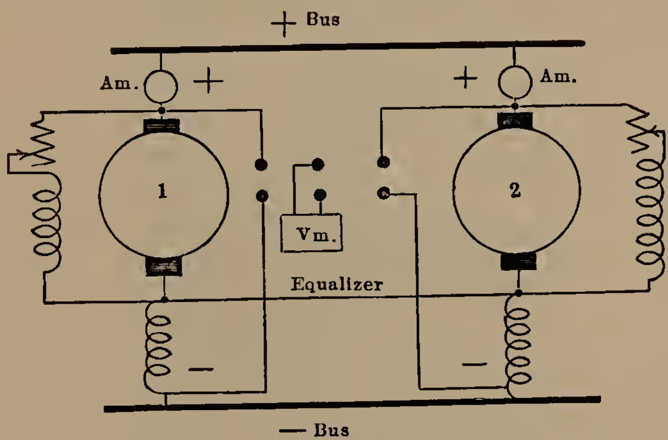


FIG. 263.—Two compound generators in parallel.

Compound generators are put in service and taken out of service in the same manner as shunt generators. The load is adjusted and shifted by means of the shunt-field rheostat.

224. Circuit Breakers.—Generators, motors, and electric circuits in general require protection from short-circuits and overloads. The sudden load imposed by a short-circuit may injure the generator or its prime mover. Wires may overheat under the short-circuit current, resulting in fire hazard. Two common devices are used for opening short-circuits and overloads, the *fuse* and the *circuit breaker*. The fuse has a much lower first cost and occupies less space. On the other hand, it is worthless

after being blown (unless it is of the refillable type) and considerable inconvenience often results from not having spare fuses at hand. The circuit breaker (Fig. 264) has a higher first cost and requires more space. On the other hand, it operates an indefinitely great number of times without injury and is readily reset. The action of a breaker is more rapid than that of a fuse.

Circuit breakers should always be mounted at the top of the switchboard. If they are placed at the bottom, the arc which rises may cause personal injury or may damage the switchboard equipment.

Fig. 264.—Two-pole, 2,000-amp. circuit breaker (Condit).

225. Automobile Starting and Lighting Systems.—Motor vehicles which are propelled by internal combustion engines are largely dependent on electrical energy for their successful operation. Electrical energy is required for starting, lighting, ignition (see page 178), and other purposes. A miniature power plant is necessary for supplying this energy. This power plant is ordinarily in the hands of persons not skilled in the operation of electrical machinery, and even if the operator is skilled it is impossible for him to attend to the power plant while driving. Therefore, this power plant must operate automatically. The source of energy is obviously the engine. The engine drives a

generator which, in turn, supplies electrical energy for lights, ignition, and other purposes. In addition, the generator must supply energy for charging the storage battery so that the battery may supply energy for starting, and for lights, etc., when the engine is not running.

The electrical power systems in automobiles operate usually at a constant potential of 6 or 12 volts. In a central station the generator operates at constant speed and little difficulty is experienced in maintaining constant voltage. In the automobile power plant, the generator speed is determined by the engine speed, which varies continually over wide limits. When the engine is not running or when its speed is so low that the generator electromotive force is less than the battery electromotive force, the generator must be disconnected from the system or allowed to run *idle* as a motor, as otherwise the battery will attempt to drive the engine through the generator operating as a motor. The generator may be disconnected electrically by a cut-out relay or it may run idle as a motor by driving it with an over-running clutch. When the engine reaches high speeds, the generator must be so regulated that it does not deliver too much current, thus overheating itself, overcharging the battery, and burning out the lamps.

With a self-starter the motor is usually of the series type, since it must develop large torque with the limited current which the battery can supply during the time required for starting.

There are several types of starting and lighting systems which employ different methods for accomplishing the desired results.

Since the Delco system[1] is in common use, and in addition has unique features, a description of it is given as an example of a successful system.

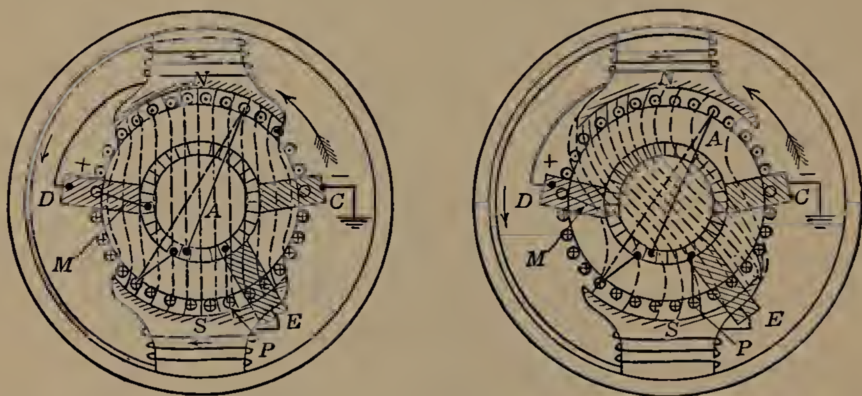
THE DELCO SYSTEM

226. The Delco Generator.—The Delco system operates at 6 volts. In the Delco two-unit system the motor and the generator are entirely separate. In Delco single-unit system the generator and motor are embodied in the same armature. The generator and motor have separate windings and different commutators. (A machine of this type is called a *dynamotor*.)

[1] The Dayton Engineering Laboratories Co., Dayton, Ohio.

The regulation of the generator will first be discussed. As the speed increases, the generator must develop a reaction which will counteract its tendency to deliver more current, as otherwise the generator will be overloaded. This result is accomplished in the Delco generator by means of "third-brush regulation," which depends on armature reaction for its operation.

A diagrammatic sectional view showing the armature, field, commutator, and shunt-field connection of the generator is given in Figs. 265 (*a*) and (*b*). In (*a*) the condition for low speed and light loading is shown. In (*b*) the condition for high speed and normal loading is shown. The main positive and negative



(*a*) Light load. (*b*) Normal load.

FIG. 265.—Third-brush regulation.

brushes are shown at *D* and *C*, the negative brush being grounded to the frame of the machine. The third or regulating brush *E* is set at a considerable angle from the negative brush *C* in a direction opposite to that of rotation. The shunt-field connection is made between the positive brush *D* and the third or regulating brush *E*.

In both figures a single armature coil *A* is shown. This coil is connected across two adjacent commutator segments lying between the positive brush *D* and the regulating brush *E*.

A study of Fig. 265 (*a*) or (*b*), with particular reference to the connection of coil *A* to the commutator and the manner in which the other armature coils must be similarly connected to commutator segments, will show that the electromotive force between brushes *D* and *E* is proportional to the number of armature

conductors between conductors P and M, to the flux cut by the conductors between P and M, and to the speed. With light loads, as in (*a*), the voltage between brushes D and E is approximately 5 volts when the voltage between main brushes D and C is 7 volts. Under these conditions the field current is approximately 1.25 amp. As the speed increases, the load on the generator tends to increase. The field becomes distorted, due to armature reaction, as shown in Fig. 265 (*b*) (compare with Fig. 214, page 238). The number of magnetic lines entering the armature between conductors P and M is considerably *decreased*, as a study of Fig. 265 (*b*) shows. This tends to decrease the voltage across the shunt field, and hence the shunt-field current, and counteracts in large measure the tendency for the voltage to increase with increase of speed. Overloading of the generator is thus prevented.

If the generator is found to be charging the battery at too high a rate, the regulating brush E should be moved slightly *against* the direction of rotation; if the charging rate is too low, the brush E should be moved slightly *in* the direction of rotation.

227. Delco Starting and Lighting System.—The complete connections of the Delco single-unit starting and lighting system, as used in the latest Buick models, are shown in Fig. 266. The system operates at 6 volts and with the negative side grounded to the frame of the machine. As was pointed out in Par. 226, the starting and lighting are accomplished by a single unit "motor-generator" or dynamotor (Fig. 266), having one armature and one field structure but having two distinct armature windings, two distinct commutators, and two distinct field windings, all of which are properly insulated. The motor and generator are connected through the ground connection only.

The heavy lead from the battery positive terminal to the motor-generator carries the large starting current, as well as any other current that may be flowing into or out of the positive terminal of the battery. From the junction *1-A* at the motor a smaller lead is carried through the zero-center ammeter to the connection *1* on the combination switch. From *1* it passes directly through the circuit breaker to the lighting switch. The circuit breaker is a series-connected solenoid operating a make-and-break contactor through an iron armature. Normal values of current do not affect the breaker. When the current

reaches a value of from 25 to 30 amp., the circuit breaker opens and closes the circuit more or less rapidly, thus reducing the effective current and warning the operator by the audible clicking, this circuit breaker operating on the principle of the buzzer or electric bell (see page 24).

The operation of the right-hand or lighting switch is as follows: Direct connections are taken from *7* and *9* for the tonneau and cowl lights, respectively. In the position shown, with the lowest contact arm at *4*, the lights are all off. When the switch is turned

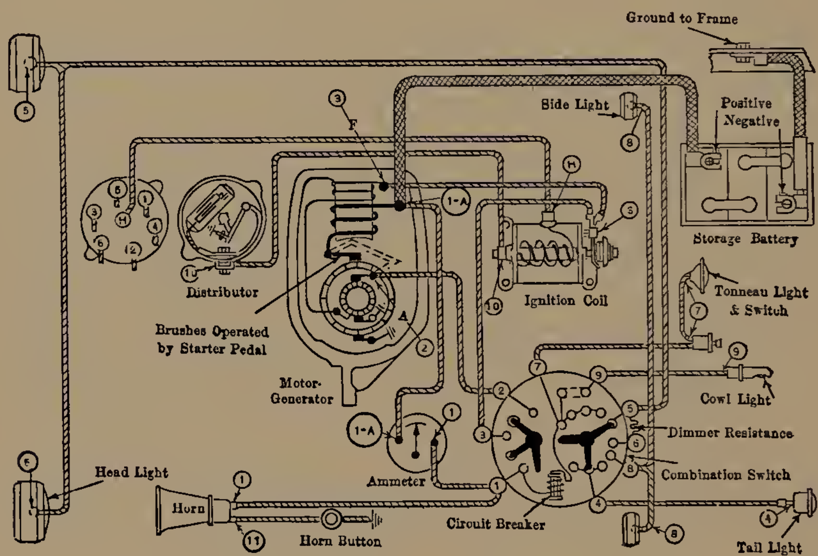


FIG. 266.—Delco starting and lighting system.

counter-clockwise to the next contact, the tail light only is connected. When the lower contactor arm is turned to *8*, the side lights and tail light are connected; when it is turned to *6*, the head lights are dimmed, and the tail light is connected; when it is turned to *5*, the head lights are at full brightness and the tail light is connected.

When the left-hand or starting switch is raised, contacts *2* and *3* are made alive by being connected to *1*. From *3* on the switch the current goes to *3* on the ignition coil. From this point a portion of the current passes through the ignition coil resistance (the small button at the right-hand end of the coil), thence through the primary, the interrupter at *10* and to ground, unless the interrupter contacts happen to be open (see page 178).

From *3* current also passes through the shunt field of the generating unit and to the third brush. From *2* on the switch the current passes directly to the generator commutator. The generator then runs as a motor but cannot drive the engine, since it is mechanically connected to the engine through an overrunning clutch. The generator when running as a motor facilitates the meshing of the starting gears.

Thus, when the starting switch is closed, the ignition system is connected, the gears may be readily meshed and the entire system is ready for starting.

When the starting pedal is depressed, the starting gears, between the flywheel and armature-shaft pinions, are slid into mesh. Further movement of the starting pedal raises the positive generator brush from the commutator by means of the mechanism shown dotted. This is necessary in order to prevent the generator delivering current at the high starting speed, hence putting load on the starting motor. The same movement of the starting pedal which raises the upper generater brush withdraws the brush-lifting pin, causing the upper motor brush to lower to the motor commutator. The motor armature and its series field are then connected from *1-A* to ground, and so are connected in series directly across the battery and with heavy leads. The cranking operation which results necessitates heavy current, so that all connections should be clean and tight. Because of the high gear-ratio, the engine, after it has started, would drive the motor-generator unit at an excessive speed were the intervening gears not provided with an overrunning clutch. The starting pedal should be released immediately, however, so as to reduce wear, etc. on this clutch. After the starting pedal is released, the gears are out of mesh and the generator is driven by the engine. If the engine speed is too low, the generator merely "motors," taking a small current to overcome its losses. It cannot drive the engine because of its overrunning clutch. When the engine speed is sufficiently high, the generator generates, etc., in the manner described in Par. 226.

228. The Cut-out Relay.—In the Delco system as used on some cars, and in several other starting and lighting systems as well, the generator is connected to the engine without the use of

an overrunning clutch and cannot "motor" at low engine speeds without taking excessive current in attempting to drive the engine. (Some models allow the generator to operate in this fashion but limit the armature current to 15 amp.) The generator must therefore be disconnected from the battery when the generator attempts to take energy from the battery. Also, when the generator voltage exceeds the electromotive force of the battery it must be connected to the battery. Moreover, if the generator is not so connected, it will build up to high voltage and overheat, due to large field and core losses.

Therefore, the generator and battery are usually connected by a cut-out relay (Fig. 267). The relay consists of a soft-iron core C and an armature A pivoted at P. There are two windings

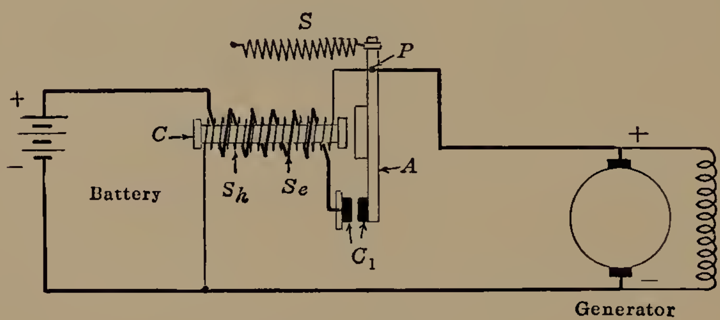


FIG. 267.—Cut-out relay.

on the core, a shunt-winding Sh, consisting of a considerable number of turns of fine wire, and a series-winding Se, consisting of a comparatively few turns of heavy wire. There are two contacts C_1 one on the armature A connected to the positive terminal of the generator, and a fixed contact connected to the positive terminal of the battery through the series-winding. The contacts C_1 are held open by means of the spring S. The shunt-winding is directly across the generator terminals. When the generator builds up to a sufficiently high voltage the current in the shunt winding causes the core C to attract the armature A, closing the contacts C_1. The battery is then connected across the generator terminals through the series winding. The series-winding is connected in such a manner that when current flows from the generator positive terminal to the battery positive terminal, it *assists* the shunt-winding and increases the pressure

between the contacts at C_1. When the generator electromotive force falls sufficiently, current flows from the battery positive terminal to the generator positive terminal and supplies energy to the generator. The current in the series-winding is reversed, and the series-winding *opposes* the shunt-winding. When the net flux in the core reaches a sufficiently low value, the relay opens, disconnecting battery and generator. In the usual automobile system, the relay closes with 7 to 8 volts across the generator and opens with from zero to 3 amp. reverse-current.

Cut-out relays are also used extensively in train-lighting systems employing axle drive, and in farm-lighting systems, etc. employing batteries, where automatic connection and disconnection are imperative.

APPENDIX A

Relations of Units

Length

1 inch = 2.54 cm.
1 foot = 30.48 cm.
1 mile = 1.609 km.

Area

1 circular mil = 0.7854 sq. mil.
1 circular mil = 0.000507 sq. mm.
1 square inch = 6.452 sq. cm.
1 square meter = 10.76 sq. ft.

Volume

1 cubic inch = 16.39 cu. cm.
1 liter = 1,000 cu. cm.
= 1.057 qt.
= 0.2642 gal.
1 gallon = 231 cu. in.

Weight

1 gram = 981 dynes
1 ounce (av.) = 28.35 grams
1 kilogram = 2.205 lb.
1 ton = 2,000 lb.
1 long ton = 2,240 lb.
1 metric ton = 1,000 kg.
= 2,205 lb.

Pressure

1 atmosphere = 14.70 lb. on square inch
= 29.92 in. of mercury at 32° F
= 760.0 mm. of mercury at 32° F.
= 33.94 ft. of water at 60° F.
1 lb. on sq. in. = 702.9 kg. on square meter

Work

1 joule (watt-second) = 10,000,000 ergs
1 gram degree Centigrade (gram calorie) = 4.184 joules
1 pound degree Fahrenheit (B.t.u.) = 252.2 gram degree Centigrade (gram-calorie)
= 778.3 ft.-lb.
1 kilogram-meter = 9.81 joules
= 7.233 ft.-lb.
1 foot-pound = 1.356 joules
1 horse-power-second = 178.3 gram degree Centigrade (gram-calorie)
= 0.7068 lb. degree Fahrenheit (B.t.u.)
= 550 ft.-lb.

Power

1 horsepower = 550 ft.-lb. per second
= 33,000 ft.-lb. per minute
= 0.7074 B.t.u. per second
= 746 watts

1 kilowatt = 1,000 watts
= 1,000 joules per second
= 1.341 h.p.
= 737.6 ft.-lb. per second
= 0.239 kg.-cal. per second

APPENDIX B

Circles and Spheres

Circle

$$A = \pi R^2 = \frac{\pi}{4} D^2 = 0.7854\ D^2$$

Sphere

$$A = 4\pi R^2 = 12.57 R^2 = \pi D^2$$

$$V = \frac{4}{3}\pi R^3 = \frac{1}{6}\pi D^3$$

$$= 4.189\ R^3 = 0.5236\ D^3$$

$\pi = 3.1416$
A = area (surface of sphere)
V = volume
R = radius
D = diameter

APPENDIX C

Specific Gravities

Aluminum	2.67	Mercury	13.60
Copper (drawn)	8.89	Nickel	8.60–8.90
Gold	19.26	Platinum	21.37
Iron, bar	7.48	Silver	10.55
Iron, wrought	7.80 to 7.90	Tin (cast)	7.29
Steel	7.60 to 7.80	Zinc (cast)	7.04–7.16
Lead	11.00		

1 *cu. ft. of water weighs* 62.4 *lb.*

APPENDIX D

Conversion of Thermometer Scales

$$\text{Degrees Fahrenheit} = C \times 1.8 + 32$$

$$\text{Degrees Centigrade} = \frac{F - 32}{1.8} = \frac{5}{9}(F - 32)$$

C = degrees Centigrade, F = degrees Fahrenheit

APPENDIX E

Heat Values of Fuels[1]

B.t.u.'s per pound

Bituminous coal...	10,000 to 15,000	Dry pine.........	9,153
Anthracite coal....	12,500 to 14,000	Gasoline..........	19,000 to 21,000
Connellsville coal..	15,000	Kerosene.........	20,093
Connellsville coke..	12,600	Fuel oils..........	18,500 to 19,000
Dry oak..........	8,316		

B.t.u.'s per cubic foot

Water gas................................	295
Coal gas................................	500 to 625
Natural gas.............................	1,000 to 1,400

APPENDIX F

Greek Alphabet

		Quantity	Mu	M	
Alpha	A	α			μ Permeability and "micro"
Beta	B	β			
Gamma	Γ		Nu	N	ν
		γ Conductivity (mho per centimeter — cube)	Xi	Ξ	ξ
			Omicron	O	o
			Pi	Π	π (3.1416)
Delta	Δ	δ	Rho	P	
Epsilon	E				ρ Resistivity
		ϵ Napierian log. base (2.718)	Sigma	Σ	Summation
					σ Magnetic or electrostatic surface density of charge
Zeta	Z	ζ			
Eta	H				
		η Efficiency			ς
Theta	Θ	θ, ϑ Phase displacement and power-factor angle.	Tau	T	τ
			Upsilon	Υ	υ
			Phi	Φ	φ, ϕ magnetic flux
Iota	I	ι	Chi	X	χ
Kappa	K	κ Dielectric constant	Psi	Ψ	ψ Electrostatic flux
Lambda	Λ		Omega	Ω	ohms
		λ Wave length		℧	mhos
					ω angular velocity

[1] "Mechanical Engineers' Handbook," McGraw-Hill Book Co. Inc.

QUESTIONS ON CHAPTER I

1. What substances are used as magnetic materials and why? Compare natural magnets with artificial magnets. What properties has each? In what way do the magnetic properties of soft iron (or steel) differ from those of hardened steel?

2. Describe a magnetic field. Does magnetism actually exist as lines? What is meant by the poles of a magnet? What relation exists between the poles of a magnet and the direction which the magnet assumes, when freely suspended in space?

3. What effects are noted when a bar magnet is broken and how may these effects be explained by Weber's theory?

4. Give the laws which govern the direction and the magnitude of the force existing between magnetic poles in space. Define a unit pole. What is pole strength and how is it measured?

5. What is the fundamental measure of field intensity? What is its relation to flux density in air? In magnetic materials?

6. Describe the compass needle. How may it be used to determine the polarity of magnets? In plotting magnetic fields?

7. How are magnetic figures produced and what do they signify?

8. What is meant by magnetic induction and what relation exists between the induced and the inducing poles? How may magnetic induction explain the magnetic attraction of iron by magnets?

9. What important law governs the geometrical shape which a magnetic field tends to assume?

10. State the advantages of the horseshoe magnet over the bar magnet. What advantages are gained in making magnets laminated? Give some practical uses of permanent magnets.

11. Describe a magnetic screen, giving its principle of operation.

12. What methods are used to magnetize permanent magnets?

13. What is the nature of the earth's magnetism? Why does the compass needle point true north at only a comparatively few places on the earth's surface? What is meant by the *dip* of the needle?

PROBLEMS ON CHAPTER I

NOTE: Problems marked with asterisk * are more difficult than the average and are intended for special assignment.

1. Two north poles, one having a pole strength of 18 unit poles and the other a pole strength of 24 unit poles, are placed 8 cm. apart in air. Considering these poles as concentrated at points, determine the force in dynes acting between the poles. Is this force attraction or repulsion?

2. A north pole having a strength of 40 unit poles and a south pole having a strength of 60 unit poles are placed 3 in. apart in air. Assuming

that these poles are concentrated at points, determine the force in grams acting between the two. Is this force attraction or repulsion?

3 A magnetic pole when placed 12 cm. (in air) from a north pole having a pole strength of 150 units is repelled with a force of 0.120 gram. Assuming that both poles are concentrated at points, determine the pole strength of the unknown pole. Is this unknown pole a north pole or a south pole?

4. The pole-faces of an electromagnet are 8 in. square and a flux of 700,000 maxwells exists between these pole-faces. Assuming that the magnetic field is uniformly distributed, determine: (*a*) the field intensity; (*b*) the force in grams exerted on a north pole of 120 units strength placed in this field; (*c*) in what direction does this north pole tend to move?

5. A bar magnet 10 cm. long and having poles of 80 units at its ends is placed in a uniform field in which the flux density is 8,000 lines per square inch, the axis of the magnet being perpendicular to the direction of the magnetic field. (*a*) What force in grams is acting on each pole? (*b*) In what direction does the magnet tend to move? (*c*) What is the turning moment of the magnet in gram-centimeters? (See page 6.)

6. A magnetized bar magnet is brought close to the poles of a horseshoe magnet, as shown in Fig. 6 (*A*). Sketch the resulting magnet field.

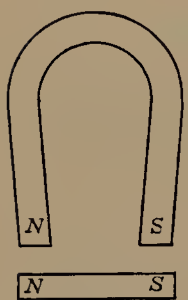


Fig. 6 (*A*).

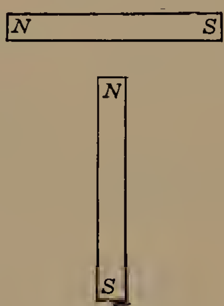


Fig. 9 (*A*).

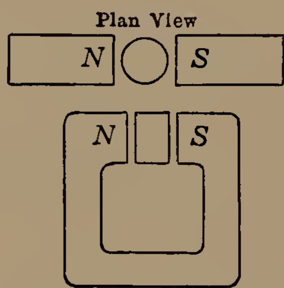


Fig. 10 (*A*).

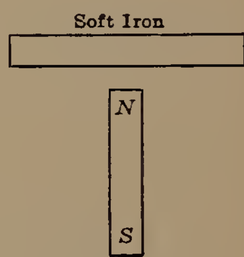


Fig. 11 (*A*).

7. Repeat problem 6, reversing the bar magnet.

8. Repeat problem 6, substituting an unmagnetized soft-iron bar for the bar magnet.

9. Sketch the magnetic field produced by the two bar magnets at right angles to each other, as shown in Fig. 9 (*A*).

10. Figure 10 (*A*) shows an elevation and a plan of a horseshoe magnet such as is used in some types of galvanometers (see Fig. 109, page 116), the plan view being drawn to an enlarged scale. Between the poles of this magnet is a cylindrical soft-iron core. Sketch the magnetic field in the plan view and indicate the induced poles on the core.

11. The north end of a permanent bar magnet, (Fig. 11 (*A*)) is brought near the middle of a soft-iron bar. Indicate the direction of the lines of induction through the bar and also the polarity of the induced poles.

QUESTIONS ON CHAPTER II

1. How may it be shown that a magnetic field exists about a conductor carrying current? What is the geometrical shape of the field if the conductor is cylindrical and there are no other magnetic fields in the vicinity? In what two ways may it be shown that such a field is cylindrical?

2. What definite relation exists between the direction of the current and the direction of the magnetic field which surrounds the conductor? Give two simple rules by which this relation may be remembered.

3. The current in a horizontal conductor flows from right to left. In what direction will the north end of a compass needle point if held over the wire? Beneath the wire?

4. What is the character of the force existing between two parallel conductors carrying current in the same direction? In opposite directions? Show that the direction of the force may be explained by the simple laws which govern the geometry of the magnetic field.

5. Sketch the direction of the magnetic field produced by a simple turn carrying current. Show that a solenoid may be produced by the combination of several such turns connected in series. Give three methods by which the polarity of the ends of such a solenoid can be determined, provided the direction of current in the solenoid be known.

6. Explain, by the simple laws of electromagnetic induction, the reason that a solenoid tends to draw an iron plunger within itself.

7. State the advantages of iron-clad solenoids. What characteristic does the "stop" give to the solenoid? Give one or two commercial applications of the solenoid.

8. Show that the operation of the telegraph relay is an illustration of the law of the magnetic circuit given in Par. 16, page 12.

9. Make a diagram of connections for a typical electric bell, pointing out its principle of operation.

10. By a simple sketch, indicate the magnetic circuits of a lifting-magnet. What are some of its commercial uses and what are its economic advantages? What is a magnetic separator?

11. Sketch the magnetic circuit and the flux path for one type of bipolar generator.

12. Sketch the magnetic circuits and the flux paths of a multipolar generator.

13. What is magnetic leakage? Does it represent energy loss? Why should it ordinarily be made a minimum? What general law should be followed in determining the position of generator field-coils?

PROBLEMS ON CHAPTER II

12. A portion of a direct-current feeder entering the duct of an underground cable system is shown in Fig. 12 (*A*). When a compass is held above the feeder, the needle deflects as shown. In what direction does the current in the feeder flow, in or out of the duct?

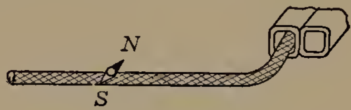


Fig. 12 (*A*).

Fig. 13 (*A*).

13. Figure 13 (*A*) shows two positive feeders of a trolley system running on a pole line and carrying current in the same direction. If the trolley wire drops upon the track, causing an enormous current to flow in the feeders for an instant, in what direction will these conductors tend to move and what is the direction of the force acting upon the insulators?

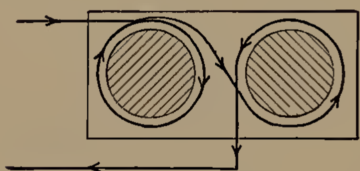


Fig. 14 (*A*).

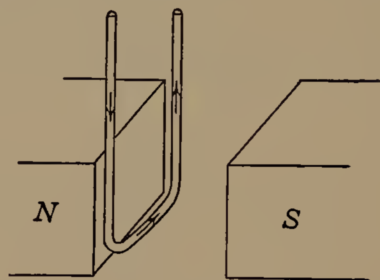


Fig. 15 (*A*).

14. Figure 14 (*A*) shows the end view of the two poles of an electromagnet and the direction of the current in the exciting coils. Indicate the polarity of each of the two poles.

15. Figure 15 (*A*) shows a loop of wire in a magnetic field, and the direction of the current in the loop. Does the loop of wire under these conditions strengthen or weaken the existing magnetic field?

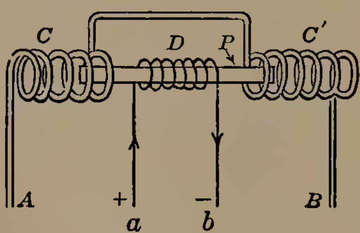


Fig. 16 (*A*).

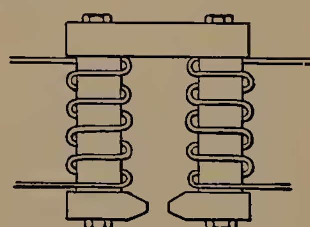


Fig. 17 (*A*).

16. In Fig. 16 (*A*) is shown the principle upon which one type of electric hammer operates. Two coils *C* and *C'* are connected in series and are in the positions shown. *P*, a soft-iron plunger running in guides, actuates the hammering device. A coil *D*, encircling the plunger *P*, is excited continuously with direct current. If the terminals *a* and *b* of the coil *D* are of the

polarity shown, indicate the polarity of the ends of the plunger P. If terminal A is $+$ and terminal B is $-$, in what direction will the plunger P tend to move? If the polarity of terminals A and B is reversed, in what direction will the plunger tend to move?

17. Figure 17 (A) shows two coils on a simple horseshoe magnet. Connect these coils so that they aid each other. Sketch the magnetic field between the poles.

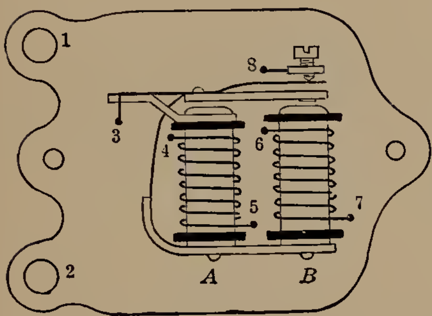


FIG. 18 (A).

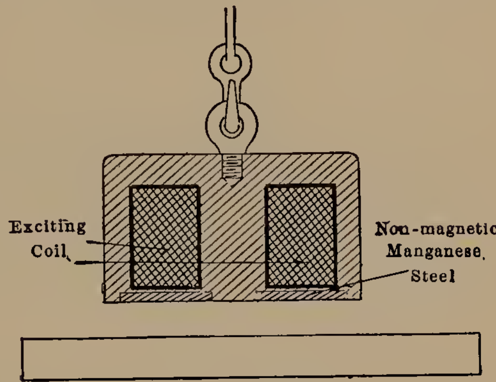


FIG. 19 (A).

18. Figure 18 (A) gives the diagram of a small buzzer. Connect the terminals 1–8 inclusive, so that the buzzer will be operative. Assuming that the terminal 1 is positive, indicate the polarities of the ends of the solenoids A and B.

19. Figure 19 (A) shows a simplified cross-section of a typical lifting-magnet and directly beneath it is a steel bar which it is about to pick up.

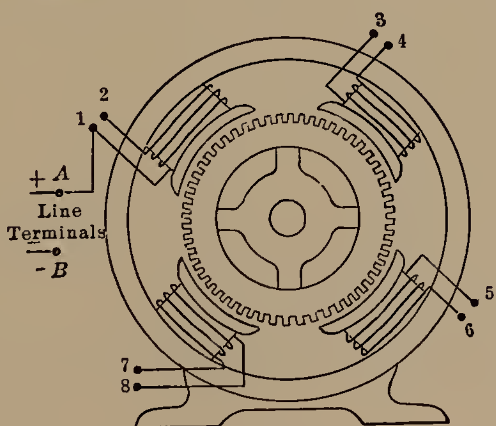


FIG. 20 (A).

The current flows through the exciting coils in a counter-clockwise direction when viewed from above. (a) What is the polarity of the center core? (b) Of the outer rim? (c) What polarity is induced at the center of the steel bar? (d) At its ends?

20. Figure 20 (A) shows the cross-section of the magnetic circuit of a four-pole dynamo, with the field windings shown diagrammatically. The positive line terminal A is connected to field terminal 1. Complete the field connections. Indicate the polarity of each pole. Indicate the yoke, the field core, the pole-shoes, the armature iron, and the armature spider. Sketch the various paths taken by the magnetic flux.

QUESTIONS ON CHAPTER III

1. When resistance is introduced into a circuit, what two effects are noted? Give a mechanical analogy and a hydraulic analogy which illustrate the effects that occur when current flows through resistance.

2. Into what two general divisions may substances, used for electrical purposes, be classified? Give examples of good conductors. Of good insulators.

3. What is the unit of resistance and how is it specifically defined? What is a megohm? A microhm?

4. In what way does the flow of electricity through conductors resemble the flow of water through pipes? How does an increase of cross-section affect the quantity of water flowing through a pipe if the pressure between its ends is maintained constant?

5. How does an increase of cross-section affect the quantity of electricity flowing through a conductor if the voltage pressure across its ends is maintained constant?

6. Upon what three factors does the resistance of a homogeneous conductor of uniform cross-section depend? What is meant by resistivity or specific resistance?

7. Upon what three factors does the conductance of a homogeneous substance of uniform cross-section depend? What is conductivity? What relation exists between conductance and resistance? Between conductivity and resistivity?

8. To what standard is the conductivity of commercial copper referred? What is meant by "per cent. conductivity?"

9. How is the equivalent resistance of a number of resistances in series determined? How is the equivalent conductance of a number of conductances in parallel determined? What equation gives the equivalent resistance of a parallel grouping of resistances?

10. Define a mil; a square mil; a circular mil. Show that the circular milage of a cylindrical conductor is obtained by squaring its diameter, expressed in mils.

11. Define a circular-mil-foot and state its resistance at 20° C.

12. What effect does an increase of temperature have on the resistance of the unalloyed conductors? In what two ways may this relation be expressed quantitatively? What common substance has a negative temperature coefficient of resistance? What substances have temperature coefficients that are practically zero and for what are such substances used?

13. Upon what simple relation is the A.W.G. based? Give the relation between the cross-sections of conductors having successive gage numbers. Of conductors differing by two numbers. By three numbers.

14. What is the approximate resistance of 1,000 ft. of No. 10 copper wire? What is its cross-section in circular mils; its diameter in mils, and its weight?

15. State the method by which the properties of a copper conductor of any size may be determined.

16. Why is copper so widely used as a conductor? Under what conditions are silver, aluminum, iron, and steel used as conductors? What is copper-clad steel and what is its use?

PROBLEMS ON CHAPTER III

21. Copper has a resistivity of 1.724 microhm-cm. unit at 20° C. Find the resistance, in microhms at 20° C. of a copper bar 4.0 m. long and having a cross-section 1.0 × 8.0 cm.

22. What is the resistance of a copper bus-bar, 12 ft. long and 0.50 by 4.0 in. cross-section, if the resistivity of the copper is 0.6788 microhms per inch-cube?

23. Determine the resistance of a cylindrical annealed aluminum rod 12.0 ft. long and having a diameter of 0.75 in., if the resistivity of aluminum is 2.77 microhm-cm.

***24.** What will be the diameter of a copper rod having the same length and resistance as the rod in problem 23?

25. The resistance of a 15-ft. length of iron wire having a diameter of 0.125 in. is 0.0865 ohm. What is the resistance of a 15-ft. length of wire of this same material, the diameter of which is 0.188 in.?

***26.** It is desired to wind a 0.08-ohm resistor with No. 6 B.W.G. iron wire having a diameter of 203 mils (0.516 cm.). The resistivity of the iron is 12.5 microhm-cm. What length of wire (in meters) is necessary?

27. What is the conductance of 940 ft. of 000 soft-drawn solid copper wire whose diameter is 410 mils? The conductivity of the copper is 580,000 mhos per centimeter-cube.

28. An aluminum bus-bar 12 ft. long has a cross-section 6.0 by 0.50 in. If the conductivity of aluminum is 0.61 that of copper (see problem 27), what is the conductance of this bus-bar?

***29.** Copper weighs 0.32 lb. per cubic inch and has a specific gravity of 8.89. Aluminum has a specific gravity of 2.67. In problem 28, (*a*) what will be the cross-section of a copper bus-bar of the same total length and the same conductance as the aluminum bus-bar? (*b*) What is the weight of this bus-bar in pounds? (*c*) What is the ratio of the weight of the aluminum to the copper bus-bar?

30. A reel contains 1,250 ft. of No. 2 A.W.G. insulated, stranded copper wire. This wire has a cross-section of 66,400 cir. mils. Its resistance at 20° C. is found to be 0.202 ohm. If the resistivity of the copper standard is 10.37 ohms-cir.-mil-ft. at 20° C., what is the per cent. conductivity of this wire?

31. A series arc circuit consists of 90 170-watt lamps connected by 4 miles of No. 6 wire (A.W.G.). Each lamp has a hot resistance of 3.7 ohms. What is the resistance of the cntire circuit? No. 6 wire has a resistance of 2.13 ohms per mile.

32. The heating element of an electric range has a resistance of 4.4 ohms. It requires a 70-ft. run of two No. 8 A.W.G. solid copper wires (140 ft. of wire) to feed this range from the service entrance. If the No. 8 wire has a resistance of 0.641 ohm per 1,000 ft., what is the total resistance of this circuit?

33. The hot resistance of the field windings of a 220-volt generator is 44.7 ohms. It is desired to reduce the field current to a value which would require a total field-circuit resistance of 108.6 ohms. The present field-rheostat has a maximum resistance of only 33.5 ohms. How much additional resistance is necessary?

34. Three incandescent lamps having hot resistances of 107, 68, and 63 ohms are all connected in parallel. (*a*) What is the conductance of each lamp? (*b*) What is the equivalent conductance of the combination? (*c*) What is the equivalent resistance of the combination?

35. An electric flatiron having a resistance of 35 ohms is connected in parallel with a toaster which has a resistance of 26 ohms. (*a*) What is the conductance of each? (*b*) What is the equivalent parallel conductance of the two? (*c*) What is the equivalent parallel resistance of the two?

36. A resistance of 12 ohms, a resistance of 16 ohms, and a resistance of 30 ohms are all connected in parallel, as shown in Fig. 36 (*A*). (*a*) What is the conductance of each of the three branches of the circuit? (*b*) What is the total conductance? (*c*) What is the equivalent parallel resistance of the combination?

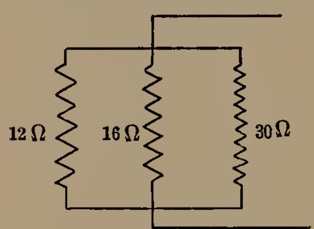


Fig. 36 (*A*).

37. A copper rod has a square cross-section, each side of which is 0.5 in. (*a*) How many square mils in the cross-section? (*b*) How many circular mils in the cross-section?

38. What is the cross-section in circular mils of a cylindrical rod 0.50 in. diameter? How many square mils in this cross-section?

39. A cable consists of 37 strands of No. 10 (A.W.G.) wires, each of which has a diameter of 102 mils. What is the cross-section of the cable in circular mils?

40. The wire, with which a relay magnet is wound, is examined and its diameter found to be 20.1 mils. (*a*) What is the cross-section of this wire in circular mils? (*b*) What is its A.W.G. number?

41. What is the diameter, in mils, of a cylindrical wire whose cross-section is 6,530 cir. mils?

42. A cable having a total cross-section of 790,000 cir. mils is made up of 61 strands. (*a*) What is the cross-section of each strand in circular mils? (*b*) What is the diameter of each strand in mils? (*c*) What is the gage number of the strands?

43. It is desired to make up a 1,700,000 C.M. cable with 127 strands. (*a*) What is the circular milage of each strand? (*b*) What is the diameter of each strand? (*c*) Give the gage number of the wire to be used in making up the cable.

44. What is the resistance of 1,800 ft. of 1,000,000-C.M. cable if the resistance of a circular-mil-foot is taken as 10.5 ohms?

45. A 2-mile length of copper conductor has a diameter of 125 mils. If the resistance of a circular-mil-foot is taken as 10.5 ohms, find; (*a*) the circular mils in this conductor; (*b*) its total resistance.

46. It is desired to wind an electric furnace tube with No. 10 A.W.G. Nichrome resistor material. A 110-ft. length is required. If Nichrome has a resistivity of 600 ohms-cir.-mil-ft. (about 60 times that of copper), what is the total resistance of the furnace heating element?

47. It is necessary that a resistor unit have a resistance of 16 ohms. Ideal wire, having a resistivity of 30 ohms-cir.-mil-ft. (about 30 times that of copper), is to be used. If No. 14 wire, having a diameter of 64.0 mils, is to be used, what length is required?

48. A 2.0-mile length of 0000 hard-drawn trolley wire has a resistance of 0.490 ohm at freezing temperature, 0° C. What is its resistance at the summer temperature of 50° C.?

49. The resistance of a No. 0, transmission line conductor is 10.4 ohms at 25° C. (*a*) What is its resistance at 0° C.? (*b*) What is its resistance at 20° C.?

50. The resistance of the shunt-field winding of a direct-current generator is measured and found to be 40.7 ohms after it has been standing for a considerable time in a room whose temperature is 22° C. What is the hot resistance of this same winding after it has been in operation and the winding has reached an average temperature of 60° C.?

51. The resistance between two marked segments of an armature is measured and found to be 0.264 after the machine has been standing for a considerable time in a room whose temperature is 24° C. After operating for 3 hr., the armature is stopped and the resistance between these same two segments is again measured and found to be 0.296 ohm. What is the temperature rise of the winding?

52. The resistance of the shunt-winding of a generator, when measured after the machine had stood for a considerable time in a room having a temperature of 21° C., was found to be 36.8 ohms. After the machine had been in operation 4 hr., the resistance of this same shunt-winding was found to be 41.0 ohms. What is the temperature rise in the winding over this period?

53. Without consulting the wire tables, determine the circular milage, the diameter, the weight, and the resistance of

(*a*) 1,800 ft. of No. 6 A.W.G. copper wire.
(*b*) 800 ft. of No. 22 A.W.G. copper wire.
(*c*) 400 ft. of 000 A.W.G. copper wire.

QUESTIONS ON CHAPTER IV

1. How is the ampere defined in absolute units? How is it further defined in order that it may be readily determined experimentally?

2. What is the unit of electrical quantity and how is it defined? What is the unit of electromotive force and how is it specifically defined?

3. In what way does the flow of electricity through a wire resemble the flow of water through a pipe? What is meant by pressure-drop in each case? Show that a difference of electrical pressure is necessary in order that the current may return from an electrical load to the negative terminal of the generator.

4. Show that voltage or electrical pressure may exist without flow of current. How should an ammeter be connected in a circuit? A voltmeter?

5. What is Ohm's law? Express this law algebraically. Transform this algebraic expression into two others and explain their meaning.

6. How are resistances in series combined to find the equivalent resistance of the combination?

7. How are resistances in parallel combined to find the equivalent resistance of the combination? What is the ratio of the currents flowing in two parallel resistances?

8. What is the unit of electrical power and how is it defined? Give three equations for determining electrical power. Under what conditions is it convenient to use each?

9. In what manner does electrical energy differ from electrical power? What is the unit of electrical energy?

10. How is heat related to mechanical and electrical energy? State the process by which chemical energy in coal appears as electrical energy and ultimately as mechanical energy. What is the order of efficiency of each conversion process?

11. How is a British thermal unit (B.t.u.) defined? A gram-calorie? What is the value of the gram-calorie in joules?

12. Discuss the method of determining the voltage at a load, provided the current and feeder resistance are known. How may the efficiency of transmission be expressed?

PROBLEMS ON CHAPTER IV

54. A 1,000-watt Mazda C lamp has a hot resistance of 0.9 ohm. What current does it take from a 30-volt circuit?

55. An electric flatiron which has a resistance of 35 ohms is connected across 115-volt d.-c. mains. What current and what power does it take?

56. An electric railway car-heater has a resistance of 160 ohms. What current does it take when the voltage at the car is 575 volts? What power does this heater develop?

57. The resistance of an electric hot plate is 10.4 ohms. What current and what power does it take from 115-volt mains?

58. A 6.6-amp., series Mazda lamp has a hot resistance of 11.65 ohms. What is the voltage across its terminals when it is in operation?

59. An electric toaster takes 3.5 amp. at 110 volts. What is its resistance and what power does it take?

60. The two exciting coils of an electromagnet, (Fig. 60 (*A*)) have resistances of 7.4 and 8.1 ohms respectively. When connected in series across 115-volt mains: (*a*) What current flows? (*b*) What is the voltage across each coil? (*c*) How much power is dissipated in each coil?

61. A certain 80-ohm relay requires 150 milliamperes for its operation. What voltage is necessary?

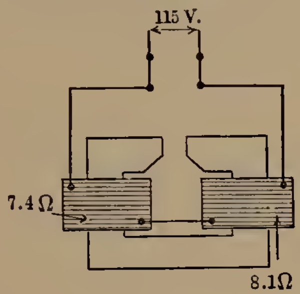


FIG. 60 (*A*).

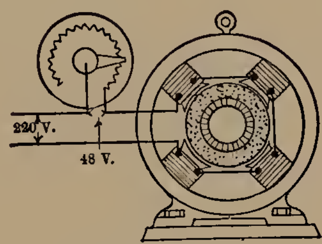


FIG. 63 (*A*).

62. If the relay of problem 61 were at the end of a signal line having a resistance of 12 ohms per wire, what should be the voltage of the battery at the sending end of the line?

63. The field circuit of a shunt motor, (Fig. 63 (*A*)) takes 6.0 amp. from the 220-volt mains. The voltage-drop across the rheostat is 48 volts. (*a*) What is the resistance of the field circuit? (*b*) What is the resistance of that portion of the rheostat which is in use? (*c*) What is the resistance of the field winding? (*d*) How much power is consumed in the rheostat and in the winding?

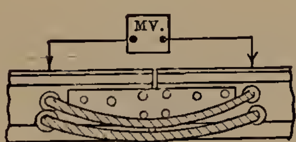


FIG. 64 (*A*).

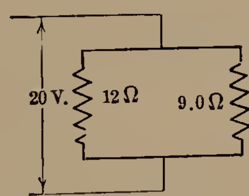


FIG. 66 (*A*).

64. A rail bonding is tested for its resistance by passing a current through it and measuring the voltage-drop, (Fig. 64 (*A*)). When the current is 142 amp., the voltmeter reads 17 millivolts. What is the resistance of the bonding?

***65.** A certain 3-mile, two-conductor feeder consists of a 2-mile length of 250,000 C.M. cable and a 1-mile length of 0000 (211,000 C.M.) cable in series. If the resistance of a circular-mil-foot of the copper is 10.5 ohms: (*a*) What is the resistance per conductor of the feeder? (*b*) If the current is 120 amp., what is the *total* voltage-drop due to each length of feeder and what is the total voltage-drop in the feeder?

66. Two resistances of 9.0 and 12.0 ohms respectively, (Fig. 66 (*A*)) are connected in parallel across a 20-volt source. (*a*) What current does each

take? (*b*) What single resistance would replace the two so far as the total current is concerned?

67. In order that the current in a magnet coil may be at its proper value of 6.5 amps., it is found necessary to connect in series with it a combination of two resistance units, one of 18 ohms and the other of 11 ohms in parallel (Fig. 67 (*A*)). What should be the resistance of a single resistance tube which could be used to replace these two?

68. If the magnet coil in problem 67 has a resistance of 10.5 ohms: (*a*) What is the line voltage? (*b*) How much power is taken by each of the resistance units and by the coil itself?

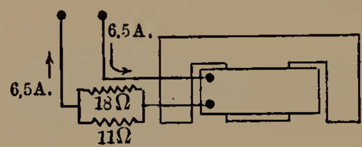


FIG. 67 (*A*).

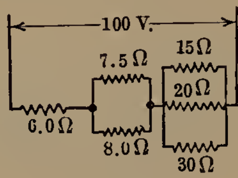


FIG. 69 (*A*).

69. Resistance units of 6.0, 7.5, 8.0, 15, 20, and 30 ohms respectively are arranged in a series-parallel grouping as shown in Fig. 69 (*A*), and connected across a 100-volt supply. Find: (*a*) the total current; (*b*) the current in each resistance; (*c*) the power taken by each resistance.

70. A trolley wire is paralleled for 2 miles by a 400,000 C.M. cable. The trolley wire has a resistance of 0.270 ohm per mile and the cable a resistance of 0.142 ohm per mile. When the combined load on the trolley system is 180 amp., how much current flows in each conductor?

71. The series-field winding of a compound generator has a resistance of 0.0840 ohm. When the total current delivered by the generator is 60 amp., it is desired that only 38 amp. flow in the series-field winding. What should be the resistance of the series-field shunt or diverter? (See page 254.)

***72.** A lighting system in a certain building consists of 18 100-watt lamps, each lamp having a hot resistance of 120 ohms. The lamps are all connected in multiple. This load is supplied from 115-volt bus-bars located 140 ft. (42.7 m.) from the load center over No. 10 (A.W.G.) wire having a cross-section of 10,000 C.M. (see Fig. 72 (*A*)). Assuming a resistance of 10.4 ohms per circular-mil-foot, what current and what power is delivered to the load center? What is the voltage at the load center?

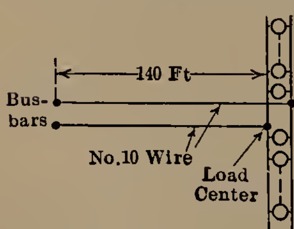


FIG. 72 (*A*).

73. The series field of a compound generator has a resistance of 0.0246 ohm. How much power is lost in this series field when the current in the field is 110 amp.?

74. A small electric resistance furnace requires 1,900 watts for its operation. (*a*) If it is to be supplied from 115-volt mains, what should be its resistance? (*b*) In order to reduce the temperature of the electric furnace, to a lower value the watts must be reduced to 1,200. What series resistance is necessary and how many watts are dissipated in this series resistance?

75. A shunt motor takes 40 amp. at 115 volts and has an efficiency of 0.83. What is its output in watts? In horsepower?

76. A shunt generator which delivers 217 amp. at 230 volts and has an efficiency of 0.912 is driven by a synchronous motor. What is the horsepower output of the synchronous motor?

77. An incandescent lighting installation consists of 20 lamps which take 0.93 amp. each at 112 volts and 15 lamps which take 0.57 amp. each at 112 volts. On the average, these lights are all in use for 4½ hr. a day. At 9 c. per kilowatt-hour and on a 30-day basis, what is the monthly lighting bill?

***78.** The oven of an electric range requiring 24 amp. at 110 volts is in operation, on the average, for 1¾ hr. a day, 30 days in the month. Two hot plates requiring 11 amp. each at 110 volts are in operation on the average for 3 hr. a day, 30 days in the month. At an energy rate of 10 c. per kilowatt-hour for the first 10 kw-hr. and 2 c. per kilowatt-hour thereafter, what is the monthly cost of operation?

***79.** A motor which delivers an average of 4.7 h.p., 8 hr. a day, 25 days per month, has an efficiency of 0.80. On a basis of 5 c. per kilowatt-hour for the first 200 kw-hr., and 3 c. per kilowatt-hour thereafter, what is the monthly energy bill?

80. One gallon of water equals 3.79 liters. At a cost of 2 c. per kilowatt-hour, how much does it cost to heat 10 gal. of water from 20° to 100° C.? Assume 85 per cent efficiency.

***81.** A 1-qt. (0.946 liter) electric percolator in use 30 days a month raises the temperature of the water from 20° to 100° C. and evaporates 10 per cent of the water once each day. It requires 539 gram-cal. to evaporate 1 gram of water at atmospheric pressure (latent heat of vaporization). At 10 cts. per kilowatt-hour, what is the monthly cost of operation of the percolator? Neglect losses.

82. A lamp load requiring 60 amp. is fed from the 117-volt bus-bars over a feeder having a resistance of 0.045 ohm per conductor (Fig. 82 (*A*)).

(*a*) What is the voltage at the lamp terminals?

(*b*) What is the efficiency of transmission? Show graphically the voltage variation along the feeder (see Fig. 69, page 67). Neglect the voltage-drop from the feeding center to the lamps.

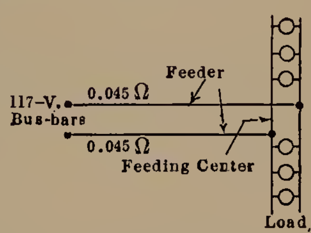


Fig. 82 (*A*).

***83.** A motor delivering 10 h.p. at an efficiency of 0.85 receives its power over a feeder from 230-volt bus-bars. The voltage at the motor terminals should not be less than 215 volts.

(*a*) What should be the resistance per conductor of the feeder?

(*b*) What is the total power loss in the feeder?

(*c*) What is the efficiency of transmission?

84. If the distance from the motor to the bus-bars in problem 83 is 450 ft. (106.5 m.), what size wire (A.W.G.) is required?

QUESTIONS ON CHAPTER V

1. Distinguish between the internal voltage or electromotive force and the terminal voltage of a battery when the battery is on open-circuit; when the battery delivers current.

2. Why does the terminal voltage of a battery drop when the battery delivers current? How may the voltage-drop due to the battery resistance be measured? How is the internal resistance determined?

3. What is the direction of current flow at the positive terminal of a battery when the battery delivers energy; when it receives energy?

4. Before a source of electrical energy, such as a generator, may deliver energy to a battery, how much voltage must first be supplied? What is meant when a battery is said to be "floating" on the line?

5. What effect is noted when the line voltage is increased to a value greater than the electromotive force of the battery? Show that the current which enters the positive terminal of a battery obeys Ohm's law.

6. When battery cells are connected in series, how may the total electromotive force of the combination be determined? How may the total resistance be determined? Discuss the effect of reversing a cell.

7. What is the equivalent electromotive force and resistance of a number of equal batteries when connected in parallel?

8. State Kirchhoff's first law. When is a current preceded by a positive sign? By a negative sign?

9. State Kirchhoff's second law. When is a product of current and resistance preceded by a positive sign? By a negative sign? Give a simple hydraulic analogy.

10. In applying Kirchhoff's second law to an electric circuit, what is the sign which should precede a rise in potential? A drop in potential?

11. Show that if a number of equations, involving Kirchhoff's laws, be applied to a network, at least one equation must involve the first law.

12. Show that a correct solution of a problem may be obtained, even when the assumed direction of the currents is incorrect.

13. Indicate the manner in which Kirchhoff's first law may be applied to the currents of an electric network so as to reduce the number of separate equations.

PROBLEMS ON CHAPTER V

85. A dry cell has an open-circuit electromotive force of 1.42 volts. When short-circuited through an ammeter of negligible resistance, it delivers a current of 12.0 amp. What is the internal resistance of the cell? Neglect any polarization.

86. What is the terminal voltage of the dry cell of problem 85, when it delivers 2.0 amp.?

87. A 6-volt starting battery, consisting of three lead cells in series, has a total electromotive force of 6.3 volts and an internal resistance of 0.023 ohm. What is its terminal voltage when it is doing starting duty, delivering a current of 110 amp.?

88. A storage-cell battery consists of 60 cells connected in series. On open-circuit a voltmeter across this battery indicates 130.0 volts. When the battery delivers 40 amp., the voltmeter indicates 124.6 volts. (*a*) What is the internal resistance of the battery? (*b*) Of each cell?

89. A generator whose electromotive force or open-circuit terminal voltage is 130.0 volts is connected to the battery of problem 88. The positive terminal of the generator is connected to the positive terminal of the battery and the negative terminal of the generator to the negative terminal of the battery. (*a*) What current flows? (*b*) The terminal voltage of the generator is then raised to 136.0 volts. What voltage is available for sending current in at the positive terminal of the battery? (*c*) Find the current delivered to the battery, using Ohm's law.

90. A 6-volt lighting and ignition battery consists of three cells in series, each of which has an electromotive force of 2.20 volts, and an internal resistance of 0.020 ohm. What voltage must be impressed across the battery terminals in order that the battery may be charged at a 15-amp. rate?

91. Six dry cells having electromotive forces of 1.40, 1.42, 1.38, 1.41, 1.36, and 1.37 volts and resistances of 0.10, 0.09, 0.12, 0.10, 0.12, and 0.12 ohm respectively, are all connected in series with positive to negative terminals. What current does this battery deliver to an external resistance of 12.0 ohms?

92. Repeat problem 91 with the second cell, having an electromotive force of 1.42 volts and a resistance of 0.09 ohm, reversed.

***93.** A storage battery consists of 140 cells, each of which has an electromotive force of 2.05 volts and a resistance of 0.0010 ohm.

(*a*) What current does this battery deliver when a rheostat consisting of eight resistances of 32 ohms each, connected in parallel, is put across the battery terminals?

(*b*) What power is developed within the battery?

(*c*) What is the terminal voltage of the battery?

(*d*) How much power does the battery deliver?

(*e*) How much power is lost within the battery?

94. Two dry cells, each having an electromotive force of 1.35 volts and an internal resistance of 0.10 ohm, are connected in parallel. What current will this battery supply to a miniature incandescent lamp whose hot resistance is 1.0 ohm?

95. A storage battery installation consists of two batteries *A* and *B*, each consisting of 55 cells. Each battery has an electromotive force of 112 volts and an internal resistance of 0.080 ohm. The two batteries are connected in parallel with terminals of like polarity connected together. (*a*) What current does the combination deliver to an external load whose resistance is 1.5 ohm? (*b*) What is the terminal voltage of the battery?

96. A battery used for marine-engine ignition consists of two groups of dry cells connected in parallel. Each group consists of six cells. The cells are all equal, each having an electromotive force of 1.32 volts and an internal resistance of 0.11 ohm.

(*a*) What is the equivalent electromotive force and internal resistance of the combination?

(b) What steady current does it deliver when the ignition circuit, which has a resistance of 9.2 ohms, is connected across its terminals?

(c) What is its terminal voltage under these conditions?

97. Four electrical conductors A, B, C, and D meet at a common junction O. In A the current is 12.0 amp., flowing toward the junction; in B the current is 7.0 amp., flowing away from the junction. The current in C is two-thirds that in D and both have the same direction with respect to the junction. What are the values and the directions of the currents in C and D?

***98.** Two batteries, (Fig. 98 (A)) one having an electromotive force of 12 volts and a resistance of 0.50 ohm, and the other an electromotive force of 5.0 volts and a resistance of 0.30 ohm, are connected together at their negative terminals by a 1.0-ohm resistance and at their positive terminals by a 4.0-ohm resistance. Assume that the current I flows in a counter-clockwise direction and that point A is at zero potential. Write an equation, involving Kirchhoff's second law, commencing at point A. Solve for the current I. What is the potential at points B, C, and D? Make a graph of the potential variation in the circuit (see Fig. 77, page 79).

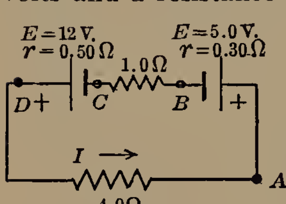


FIG. 98 (A).

***99.** A battery having an electromotive force of 14.0 volts and an internal resistance of 1.0 ohm supplies current to a circuit consisting of 12 ohms connected in series with 8.0 ohms and 4.0 ohms in parallel, the 12-ohm resistance being connected to the positive terminal of the battery (Fig. 99 (A)). Assuming that the currents I_1, I_2, and I_3 flow in the directions indicated in the figure, commence at point A and write two equations involving Kirchhoff's second law. Write a third equation involving the currents at junction B. Solve for I_1, I_2, and I_3. What is the terminal voltage of the battery?

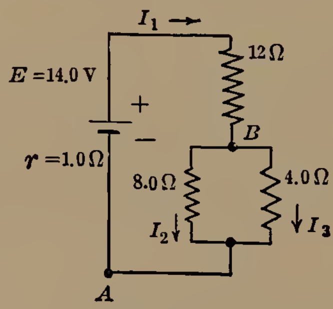


FIG. 99 (A).

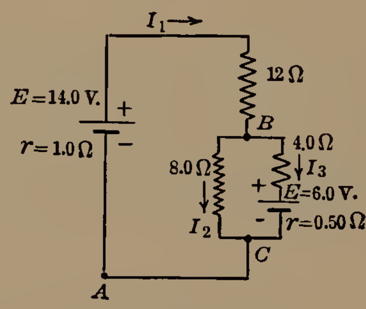


FIG. 100 (A).

***100.** Figure 100 (A) shows the network of Fig. 99 (A), except that a battery having an electromotive force of 6.0 volts and an internal resistance of 0.50 ohm is connected in the 4-ohm branch, acting in opposition to the assumed direction of I_3. Find the three currents I_1, I_2, I_3, and the voltage between points B and C.

101. Two batteries, Fig. 101 (*A*), having electromotive forces of 8.0 and 12.0 volts and resistances of 2.0 and 3.0 ohms, respectively, are connected in multiple with a 6-ohm resistance across their common terminals. Find the current and the power delivered by each battery, and the power consumed in the 6-ohm resistance.

***102.** A 5-mile trolley system is shown in Fig. 102 (*A*) The 5-mile trolley is 0000 hard-drawn copper having a resistance of 0.270 ohm per mile. This trolley is fed at a point 2 miles from the station by a 400,000-C.M. feeder having a resistance of 0.142 ohm per mile. The combined resistance of rail and ground return is 0.05 ohm per mile. The bus-bar voltage at the station is maintained constant at 600 volts. A car 2 miles from the station is taking 70 amp. Find the voltage at the car and the power taken by the car.

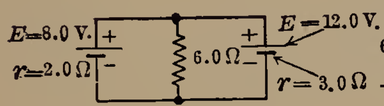


Fig. 101 (*A*).

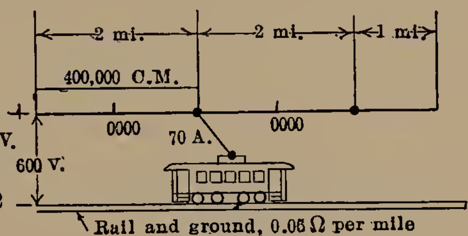


Fig. 102 (*A*).

***103.** Find the voltage at the car and at the end of the line when the car, problem 102, has moved a mile further from the power station and still takes 70 amp.

***104.** Find the voltage at the car, the power taken by the car, and the efficiency of transmission, problem 102, when the car takes 60 amp. at the end of the line.

***105.** In the railway system, problem 102, the car takes 60 amp. at the end of the line and a second car simultaneously takes 70 amp. when it is one mile from the station. By mean of Kirchhoff's laws, find: (*a*) The voltage at each car. (*b*) The power taken by each car.

Note: Owing to the fact that railway loads are widely fluctuating and are continually changing their position in the system, it is rarely possible, in practice, to formulate definite problems. However, the foregoing problems illustrate the influence of such important factors as the amount of copper which is necessary for maintaining a reasonably constant voltage at the car, the effect of the resistance of the return circuit, etc. on the car voltage. Although railway electrical engineers may not formulate definite equations in designing the electrical distribution system of railways, as is done in the foregoing problems, the general principles illustrated in these problems, together with economic considerations, govern the size of feeders, the nature of the rail bonding, etc.

QUESTIONS ON CHAPTER VI

1. What effect is noted if two copper strips be immersed in dilute sulphuric acid and a low-reading voltmeter be connected to the two strips? What effect is noted if one of the copper strips be replaced by a zinc strip?

2. What two conditions must be fulfilled in order that an electromotive force may exist in an electrolytic cell? When the external circuit is considered, what is the polarity of the copper with respect to the zinc? Why? When the cell itself is considered, what is the polarity of the copper with respect to the zinc? Why? What is meant by one metal being electro-positive to another?

3. Define *electrode, anode, cathode, electrolyte.* In what way is the electrical energy, delivered by an electric cell, supplied? Define a galvanic cell.

4. Define a primary cell. A secondary cell. State four qualities which are desirable, for commercial primary cells.

5. Upon what factors does the internal resistance of a cell depend? How may the resistance be decreased? How is the electromotive force affected by the size of the cell?

6. What is meant by *polarization?* In what two ways does polarization reduce the cell capacity? How may the effects of polarization be reduced?

7. Of what materials do the cathode and the anode of the Daniell cell consist? What electrolytes are used and how are the electrolytes kept separated?

8. In what ways does the gravity cell resemble the Daniell cell? In what particular does it differ?

9. Of what material does the cathode of the Edison-Lalande cell consist? The anode? What is used for electrolyte? Why is mineral oil necessary? What is the approximate electromotive force of the cell?

10. Of what material does the cathode of a Le Clanché cell consist? The anode? What electrolyte is used? What is the electromotive force of the cell? For what purposes is it used?

11. How is the standard ampere reproduced experimentally? The standard ohm? How is the International Volt defined? Of what materials is the cathode of a Weston cell made? The anode? What electrolyte is used? Why should the ordinary voltmeter *not* be used to measure the electromotive force of such a cell?

12. Of what materials is a dry cell made? What is the usual value of its electromotive force? What is the approximate terminal voltage when it delivers current? State one common cause of dry cells becoming exhausted. Give a few of the ordinary uses of dry cells.

13. Show that a storage cell on discharge operates on the same principle, electrolytically, as a primary cell.

14. What effect is noted on each of two lead strips immersed in dilute sulphuric acid, when a current is sent from one to the other through the acid? What is the electromotive force between the strips after current has flowed for some time?

15. What changes occur in the strips after they have delivered current until the cell is exhausted?

16. How does the specific gravity change during charge? During discharge? Why?

17. How is the active material formed on the Planté type of plate? In what manner is a large surface exposed to the action of the acid in the Manchester plate?

18. Describe the construction of a Faure or pasted type of plate. How is the positive plate formed? The negative? What are the advantages of the pasted plate? Disadvantages? Where is this type of plate commonly used?

19. Under what conditions are Planté plates used in stationary batteries? Pasted plates?

20. What materials are used for tanks? Why is considerable space between the plates and bottom of tanks desirable? How are lead joints and seams in lead-lined tanks made tight?

21. What type of separator has proved most satisfactory? Why? What precautions are necessary in handling wood separators?

22. What procedure should be taken when making dilute acid from concentrated acid?

23. How is specific gravity measured? How may specific gravity be used to determine the condition of charge of a cell?

24. Why is it usually necessary to add only water to make up the loss of electrolyte? What is the general effect of the specific gravity on the freezing temperature of the electrolyte?

25. State briefly the method of installing a stationary battery.

26. What are the primary requirements of vehicle and automobile batteries? Why is the specific gravity carried to high values in this type of cell? What type of plates is used?

27. What is meant by the "normal rating" of a battery? What is meant by the 8-hr. rate? The 3-hr. rate? Why do the ampere-hours delivered apparently decrease as the rate of discharge increases?

28. Describe the method commonly employed for charging batteries of moderate capacity.

29. Give one other method used to charge batteries, stating where it is used. How may the correct polarity be determined?

30. In what two ways does the voltage show the condition of charge? Why should not excessive gassing be permitted to occur? State the dangers involved in bringing a flame near a charging battery.

31. In what way does the electrolyte of the Edison storage cell differ from that of the lead-lead-acid cell? What material is used for the positive electrode or cathode? The anode? How is the conductivity of the cathode increased?

32. In what respect do the chemical reactions in this type of cell differ materially from those of the lead-lead-acid cell?

33. If doubt exists, what method is used to insure an Edison cell being completely charged? Why must care be taken in the matter of replenishing the electrolyte?

34. What are the advantages of the Edison cell? Name some of its applications.

35. State the fundamental principles of electroplating. What one characteristic is essential in the electrolyte? What material is commonly used for the anode? What is the character of the power supply required by electroplating processes?

PROBLEMS ON CHAPTER VI

106. A Le Clanché cell has an internal electromotive force of 1.42 volts. When a resistance is connected across its terminals, the cell delivers a current of 1.2 amp. and its terminal voltage immediately drops to 1.15 volt. Even when the current is maintained constant at 1.2 amp. the terminal voltage continues to fall until it reaches an almost constant value of 0.98 volt. (*a*) What is the initial internal resistance of the cell? (*b*) What is the *apparent* internal resistance of the cell after its terminal voltage has become sensibly constant? (*c*) To what is the difference in (*a*) and (*b*) due?

107. A dry cell has an electromotive force of 1.38 volts and an internal resistance of 0.10 ohm. After delivering a current of 1.4 amp. for some time, its terminal voltage becomes 1.04 volts. (*a*) What is the apparent cell resistance with this current flowing? (*b*) Why does this value of resistance differ from the 0.1 ohm?

108. A battery consists of 12 gravity cells connected in series. Each cell has an electromotive force of 1.10 volts, and an internal resistance of 0.09 ohm. What current will this battery deliver over a telegraph system which has a resistance of 49.8 ohms?

109. An unsaturated Weston cell has an electromotive force of 1.0179 volts and an internal resistance of 220 ohms. What will a 1.5-scale direct-current voltmeter having a resistance of 164 ohms read, when connected across its terminals? Why is it impracticable to measure the electromotive force of such a cell with an ordinary voltmeter?

***110.** Figure 110 (*A*) shows a 2-volt storage cell which delivers current through the adjustable resistance R and the 15-ohm resistance wire ac, connected in series. The negative terminal of a Weston cell whose electromotive force is 1.0181 volts is connected to the negative terminal a of the storage cell. The resistance R is adjusted until the storage-battery current is 0.10 amp. A slider b is movable along the resistance wire ac. At what point (ohms from a) on the resistance wire ac will the voltage from a to b be 1.0181 volts? What will a galvanometer read if connected between points bd, under these conditions? (This gives a means of determining the electromotive force of a Weston cell without its delivering current and is the underlying principle of the potentiometer. See page 141.)

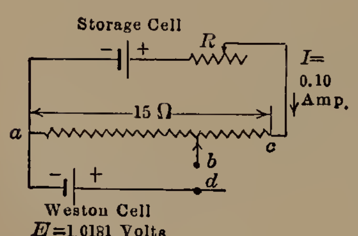


Fig. 110 (*A*).

***111.** A dry cell is substituted for the Weston cell in problem 110 and a galvanometer is connected between points bd. When the slider b is 14.48 ohms from a, the galvanometer reads zero. What is the electromotive

force of the dry cell? What current does it deliver under these conditions? Is this method of determining its electromotive force more or less accurate than using a voltmeter? Explain.

***112.** The voltage across the terminals of a small storage cell, when charged at the 2.0-amp. rate, is 2.48 volts. When the circuit is opened the terminal voltage of the cell is 2.16 volts. (*a*) What is the apparent internal resistance of the cell? (*b*) Approximately how many joules are converted into chemical energy every minute during charge? (*c*) How many joules are converted into heat during this same period?

***113.** The apparent internal resistance of the cell of problem 112, when on discharge, is 0.15 ohm. (*a*) What is its terminal voltage when discharging at the 2.0-amp. rate? (*b*) How many joules per minute does it develop? (*c*) How many joules per minute is lost within the cell? (*d*) What is the efficiency of the foregoing cell (ratio of energy delivered by cell to energy delivered to cell, problem 112)?

***114.** Plot two curves with specific gravity as abscissas and parts by volume and by weight of water as ordinates, using the data of Par. 95, page 102. From these curves determine the number of liters and the number of kilograms of water which it is necessary to add to 4.0 liters of sulphuric acid, sp. gr. 1.84, to give a solution having a specific gravity of (*a*) 1.22, (*b*) 1.25. How should the acid and water be mixed?

***115.** A battery rated at 160 amp-hr. is charged at the 8-hr. rate. Its specific gravity at the beginning of charge was 1.160. Assuming that the specific gravity increases according to the curve, (Fig. 99, page 104) how many ampere-hours have been delivered to the battery when a hydrometer indicates 1.190? How many additional ampere-hours will be necessary to charge the battery completely, assuming that the specific gravity will then be 1.210?

116. The hydrometer, problem 115, reads 1.180 after the battery has been discharging for some time. How many ampere-hours has it probably discharged? What type of battery would this probably be?

117. When fully charged a battery having Planté plates will discharge 60 amp. for 8 hr. before it is completely discharged. What is the discharge rate in amperes if, after being fully charged, it is completely discharged in 5 hr.? (See page 106, Par. 99.)

118. It is desired to charge a 3-cell, 6-volt battery from 115-volt d.-c. mains. When charged at the 12-amp. rate the terminal voltage is 2.5 volts per cell. (*a*) What series resistance (ohms and carrying capacity) is necessary? (*b*) What per cent of the power delivered by the line is used to charge the battery?

119. In problem 118 if it requires 8 hr. to charge the battery and energy costs 6.0 c. per kilowatt-hour, how much does it cost to charge the battery?

120. Repeat problem 118 with two batteries connected in series, each similar to the single battery.

121. Repeat problem 119 with three batteries, connected in series, each similar to the single battery.

122. A 10-volt, separately excited, motor-driven generator is used to charge a 4-cell, 640 amp-hr. storage battery. The battery requires an 8-hr.

charge at an average voltage of 7.4 volts and 90 amp. (*a*) If the generator has an efficiency of 0.80, what horsepower does the motor deliver to the generator? (*b*) If the motor has an efficiency of 0.82, and operates from 220-volt mains, what current does it take? (*c*) At 5.0 c. per kilowatt-hour, what is the cost of charging this battery?

123. A 320 amp-hr. storage battery, consisting of 55 cells, floats across 110-volt bus-bars. The average terminal voltage of each cell is 2.42 volts when the battery is being charged at the 8-hr. rate from another generating source. (*a*) How much power is required to charge the battery? (*b*) How many kilowatt-hours are required to charge the battery to its full ampere-hour rating? (*c*) With energy costing 2 c. per kilowatt-hour, what does it cost to charge the battery?

124. Each cell of the battery, problem 123, has an open-circuit electromotive force of 2.00 volts and an internal resistance of 0.0105 ohm. When the battery is floating across the 110-volt bus-bars, to what value must the bus-bar voltage drop in order that the battery may deliver 25 amp.

125. A 16-cell, 200 amp-hr. storage battery is being charged at the 8-hr. rate. The initial value of the terminal voltage of each cell is 2.18 volts and rises according to the curve, (Fig. 103, page 108). What power is the battery taking at the end of 5 hr. of charge? At what value of the terminal voltage should the rate of charge be reduced and why?

126. If the charging of the storage battery, problem 125, is completed in 7 hr., how much energy has it received? (Note: Average the ordinates between 0–1, 1–2, etc., (Fig. 103, page 108) to obtain the average voltage.)

127. An Edison storage battery consisting of five series-connected cells is charged at a constant rate of 18 amp. for 7 hr. The initial value of the voltage of each cell is 1.55 volts and the voltage varies with charge according to the upper curve (Fig. 106, page 111). What is the total energy required to charge the battery? (Note: Average the ordinates, Fig. 106, to obtain the average voltage during charge.)

128. An electrolytic process requires 100 amp. at 24 volts. The generator supplying this energy has an efficiency of 0.80 and is driven by a 220-volt motor whose efficiency is 0.82. (*a*) How many watts does the motor deliver to the generator? (*b*) What is the horsepower output of the motor? (*c*) What is the watt input to the motor? (*d*) What current does the motor take? (*e*) At 3 c. per kilowatt-hour, what is the cost per week of operating the electrolytic bath on the basis of 45 hr. a week?

QUESTIONS ON CHAPTER VII

1. What position does a coil, carrying a current, seek, when placed in a magnetic field? Show: (*a*) that the poles produced by the coil are attracted or repelled by the poles producing the magnetic field; (*b*) that the coil seeks such a position that the magnetic flux of the system is a maximum.

2. How is this principle utilized in the D'Arsonval galvanometer? Describe the construction of the moving coil. How is it suspended? How is

the current led to the coil? What is the purpose of the soft-iron core? How is the moving system damped? What methods are used for reading the deflection?

3. What is the purpose of galvanometer shunts? Describe one type, giving the ratios of the shunt resistances to the galvanometer resistance for various shunting ratios. Sketch the connections of the Ayrton shunt. What are the advantages of this type of shunt over the first type?

4. Show that the movement of the Weston direct-current instrument is an evolution of the D'Arsonval galvanometer. Describe the changes which were found necessary to make the instrument portable.

5. How may the Weston ammeter be made to measure currents of any desired magnitude when its own moving system is designed only for very small currents? Show that the ammeter may be considered as a voltmeter which reads merely the voltage-drop across a fixed resistance.

6. Show that an ammeter may have an indefinite number of ranges. Why do ammeter shunts have four posts, two for the current and two for the connection to the instrument? Discuss the sources of error and the precautions which must be taken when using an external-shunt ammeter. In what way does the internal-shunt ammeter differ from the external-shunt type?

7. Discuss the similarity between the voltmeter and the ammeter. Sketch the internal connections of a voltmeter having two voltage ranges. Why does a voltmeter have high resistance?

8. How may the range of a voltmeter be increased? What is a multiplier?

9. Describe briefly the principle upon which hot-wire instruments operate. State their uses and their disadvantages.

10. Describe the manner in which the voltmeter-ammeter method may be used for measuring low resistances. Why is a rheostat, connected in series with the circuit, often necessary?

11. Why does it become necessary to use special contacts for the voltmeter connection when accurate measurements of very low resistances are desired?

12. Sketch the two connections which are commonly employed when a voltmeter alone is used for measuring resistance. What are the approximate magnitudes of resistances which can be measured by this method? Why is a voltmeter having an unusually high resistance often desirable?

13. Make a simple diagram of the Wheatstone bridge, giving the principles upon which it operates. What are the ratio arms? What is the rheostat arm? How may the condition of balance be determined?

14. Describe briefly a systematic procedure for obtaining a balance. Describe the decade bridge.

15. Make a simple sketch of the slide-wire bridge, showing its similarity to the Wheatstone bridge; compare the two types of bridge with regard to accuracy and simplicity.

16. Show the application of the slide-wire bridge principle to the location of a ground in a cable. Why are the positions of the battery and galvanometer in the Murray-loop test reversed from their positions shown in Fig. 127, page 135?

17. Show that a sensitive galvanometer, connected in series with very high resistance, such as the resistance of insulating materials, may be used as an ammeter to measure the current leaking through the insulation. How is the galvanometer calibrated in terms of resistance?

18. Why does the galvanometer deflection, when measuring the leakage current, vary with time? What is standard practice as regards the time of electrification?

19. Make a sketch of a simple potentiometer in which a 2-volt battery supplies current to a resistance-wire while a standard cell, in series with a galvanometer, is so connected to the resistance-wire that the galvanometer may be made to read zero. Also indicate the method by which this resistance-wire, carrying current, may be used to measure an unknown electromotive force.

20. Make a simple sketch of a Leeds & Northrup student's potentiometer. How is the potentiometer current adjusted to its proper value by means of the standard cell? Show the method used to measure an unknown electromotive force. Name the accessories that are necessary with this type of potentiometer.

21. Sketch the connections of a volt-box and show its use in the measurements of electromotive forces which are in excess of the range of the potentiometer. What is a drop-wire?

22. Show how current may be measured accurately with a potentiometer, which is fundamentally a device for measuring electromotive force. What are standard resistances and in what values of resistance are they usually made?

23. Sketch the internal connections of a wattmeter. Upon what principle does it operate?

24. In what manner does the watthour meter differ essentially from the other types of instruments which have been described? How is the armature connected to the circuit? The field-coils? Why is an auxiliary coil necessary and how is it connected? Why are retarding magnets necessary?

25. What relation exists between the revolutions of the disc and the watthours registered? How is the meter calibrated? What adjustments are made if the meter is too fast near full load? Too slow?

26. What adjustments are made if the meter is too fast at light load? Too slow?

27. Sketch the connections of a three-wire meter. Where is it used?

PROBLEMS ON CHAPTER VII

129. A D'Arsonval galvanometer whose resistance is 482 ohms deflects 20 cm. with a current of 3.90 microamperes. What potential difference across its terminals will produce full-scale deflection of 25 cm.?

130. The galvanometer of problem 129 is shunted with a 60-ohm resistance. (*a*) What is its deflection when the line current is 22.0 microamperes? (*b*) What is the voltage across its terminals?

131. Calculate the values of three resistances to shunt the galvanometer of problem 129 so that its current will be one-tenth, one one-hundredth, and one one-thousandth the line current, respectively (see Fig. 111, page 118).

***132.** Figure 132 (*A*) shows a galvanometer whose resistance is 800 ohms, used in connection with an Ayrton shunt whose total resistance *AB* is 10,000 ohms. The resistance *Aa* is 1.0 ohm. When the line contact *C* is at *a*, find: (*a*) the galvanometer current in microamperes when the line current *I* equals 5,000 microamperes. (*b*) Find the current which would flow through the galvanometer if the line contact *c* were at *B*. (*c*) Compare the ratio of resistances *Aa* and *AB* with the respective galvanometer currents. (Hint: Use equation 24, page 59.)

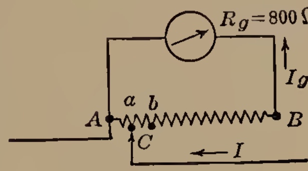


Fig. 132 (*A*).

***133.** Repeat problem 132 for a line current of 4,000 microamperes with the line contact *c* connected to *b*. The resistance *Ab* is 10.0 ohms.

134. A direct-current ammeter without a shunt gives full-scale deflection when the current in its moving system is 0.025 amp. The ammeter resistance is 1.5 ohms. What voltage, in millivolts, across its terminals will cause full-scale deflection?

***135.** It is desired that the ammeter of problem 134 be made to have a full scale of 10 amp. (*a*) What should be the resistance of its shunt? (*b*) What must be the resistance of a shunt in problem 134 to give a full-scale deflection of 500 amp.? (*c*) How much power is lost in the shunt with full-scale deflection in both (*a*) and (*b*)? (Note: The current in the instrument is negligible as compared with the current in the shunt.)

136. An ammeter has a resistance of 2.20 ohms and its shunt a resistance of 0.0040 ohm (Fig. 136 (*A*)). (*a*) When the external current is 100.0 amp., how much current flows in the ammeter and how much in its shunt? (*b*) What is the ratio of ammeter to shunt current?

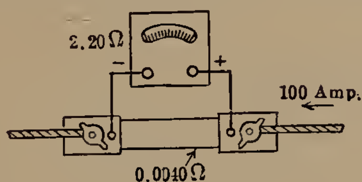


Fig. 136 (*A*).

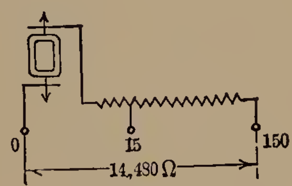


Fig. 137 (*A*).

137. A two-scale voltmeter has a resistance of 14,480 ohms between the terminals marked 0 and 150 volts, (Fig. 137 (*A*)). (*a*) What is the resistance between the 0 terminal and the 15-volt terminal? (*b*) What voltmeter current gives full-scale deflection in each case?

138. What should be the resistance of a multiplier to give the voltmeter of problem 137 a full-scale deflection of 600 volts?

139. Two 300-scale voltmeters *a* and *b* have resistances of 31,500 and 34,200 ohms. What will each read when the two are connected in series across a 500-volt circuit?

140. A 300-scale voltmeter, having a resistance of 31,500 ohms, in series with a 50,000-ohm resistance, is connected between a trolley and ground. What is the trolley voltage when the voltmeter reads 215 volts?

141. A low resistance is measured by the voltmeter-ammeter method, (Fig. 120, page 127). The current of 32.0 amp. is taken from 110-volt mains through a rheostat. The voltmeter connected directly across the resistance indicates 3.42 volts. (*a*) What is the value of the unknown resistance? (*b*) What is the resistance of the rheostat? (*c*) How much power is being dissipated in the resistance and in the rheostat?

*__142.__ The resistance of a copper rod 0.250 in. (0.635 cm.) diameter and 46 in. (116.8 cm.) long is measured by the voltmeter-ammeter method, shown in Fig. 121, page 128. The voltmeter contact points are 40 in. (101.6 cm.) apart. When the ammeter reads 75.4 amp. the voltmeter reads 42.6 millivolts; the temperature is 20° C. (*a*) What is the resistance of a 1-ft. (30.5 cm.) length of the rod? (*b*) What is the resistance of a circular-mil-foot of the rod? (*c*) If the resistance of standard copper is 10.37 ohms per circular-mil-foot at 20° C., what is the per cent. conductivity of this copper?

*__143.__ An aluminum rod 0.200 in. (0.508 cm.) diameter and 50 in. (127.0 cm.) long is connected in series with a 0.001-ohm standard resistance (Fig. 143 (*A*)). When current flows in the system a millivoltmeter connected across the standard resistance reads 30.2 millivolts (see dotted lines). The millivoltmeter is then connected across the contact points on the aluminum rod 42 in. (106.7 cm.) apart and reads 45.1 millivolts. (*a*) What is the resistance per foot of the aluminum? (*b*) What is the resistance per circular-mil-foot of the aluminum? (Note: In this method of resistance measurement, it is not necessary that the instrument read millivolts. If its scale readings are proportional to the voltage across the instrument terminals, the ratio of the two readings may be used.)

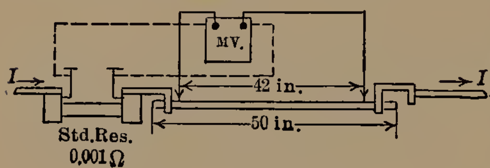


FIG. 143 (*A*).

144. A 300-scale, 37,000-ohm voltmeter when connected across d.-c. mains (Fig. 144 (*A*)), switch S being closed, reads 224.0 volts. When switch S is opened, connecting the resistance x in series with the voltmeter, the voltmeter reads 87.0 volts. (*a*) What current flows through the voltmeter and x when S is open? (*b*) What is the voltage across x? (*c*) What is the value of x in ohms?

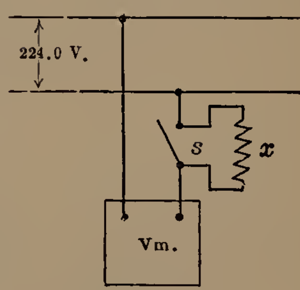


FIG. 144 (*A*).

*__145.__ A voltmeter V_1 (Fig. 145 (*A*)), when connected across d.-c. mains reads 600 volts. A voltmeter V_2, whose resistance is 100,000 ohms, is connected between the positive line and the core of a cable. The sheath of the cable is connected to the negative line. V_2 reads 12.0 volts. (*a*) What current flows through V_2? (*b*) What is the voltage across the cable? (*c*) What is the insulation resistance of

the cable? (*d*) If the cable is 1,840 ft. long, what is the insulation resistance in megohms, of a mile length?

146. A 150-scale, 100,000-ohm voltmeter is used to measure the insulation resistance of the armature of a dynamo. The voltmeter is connected first between a positive main and neutral, which is grounded, and reads 122 volts. The negative terminal of the voltmeter is then connected to the commutator and reads 4.0 volts (see Fig. 146 (*A*)). The frame of the machine is grounded. What is the resistance to ground of the armature?

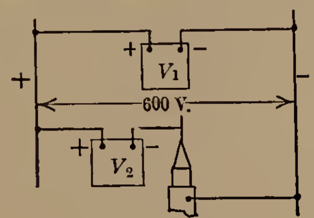


Fig. 145 (*A*).

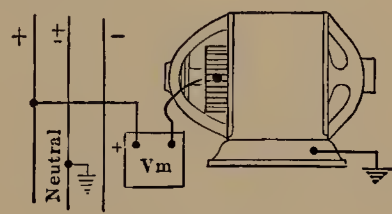


Fig. 146 (*A*).

147. It is desired to measure an unknown resistance X between 3 and 4 ohms to four significant figures, by means of a Wheatstone bridge (see Fig. 123, page 131). What should be the ratio of M to N? If the rheostat arm P reads 3,492 ohms when a balance is obtained, what is the value of the unknown resistance?

148. In the slide-wire bridge, (Fig. 127, page 135), R is a known resistance of 10 ohms. A balance is obtained when the slider reads 12.2 cm. What is the value of the unknown resistance X?

149. In problem 148, an unknown resistance is substituted for R and a balance is obtained when the slider reads 63.4 cm. What is the value of this unknown resistance?

150. Two single-conductor cables, each 2,000 ft. (610 m.) long and containing No. 14 A.W.G. copper conductors, are used for signal purposes. It is desired to locate a fault which has developed in one cable, by the Murray-loop method. The two cables are looped at the far end, being connected as shown in Fig. 128, page 136. A balance is obtained when the slider reads 68.8 cm. on the 100-cm. slide wire. How far from the home end is the fault?

***151.** A cable is tested for its insulation resistance, the connections shown in Fig. 129, page 138 being used. The Ayrton shunt has a resistance of 10,000 ohms, and the galvanometer a resistance of 1,200 ohms. When the cable is short-circuited, and the Ayrton shunt is set at 0.0001, the galvanometer deflects 24.0 cm. The electromotive force of the battery is 300 volts. (*a*) What is the resistance of the circuit? (*b*) What current flows in the circuit? (*c*) The short-circuit is removed from the cable. The galvanometer deflects 16.4 cm. when the shunt is set at 1.0, after the cable has been electrified 1 min. What current now flows in the circuit? (*d*) What is the insulation resistance of the cable in megohms?

***152.** The insulation of another cable is tested by the same method as that given in problem 151. When the cable is short-circuited, the deflection

is 24.2 cm. with the shunt set at 0.0001. After the short circuit is removed, the deflection after 1 min. is 19.4 cm. with the shunt set at 0.1. (*a*) What is the insulation resistance of this cable in megohms? (*b*) If the cable is 2,400 ft. long, what is the insulation resistance in megohms, of a mile length?

***153.** A 2-volt storage cell *A* (Fig. 153 (*A*)) delivers current to a 150-ohm resistance-wire *ca* through the rheostat *R*. The rheostat *R* is adjusted until the current in the circuit is 0.01 amp. The negative terminal of a battery *B* is connected to *a* the negative side of *A*. The positive terminal of *B* is connected through a galvanometer and key to a movable contact *b* on the wire *ca*. If the electromotive force of *B* is 1.0186 volts, what is the resistance *a* to *b* when the galvanometer does not deflect on depressing the key in the galvanometer circuit?

***154.** The battery *B* in problem 153 is removed and a dry cell whose electromotive force is unknown is substituted. A balance is obtained when the resistance *ab* is 142.4 ohms. What is the electromotive force of the dry cell? What current does it deliver under the condition of balance?

155. In a certain potentiometer, the resistance between each 0.1-volt contact is 10.0 ohms. What is the working current of the potentiometer?

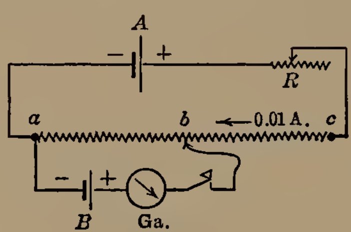


FIG. 153 (*A*).

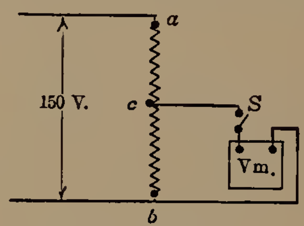


FIG. 156 (*A*).

156. A drop-wire *ab* (Fig. 156 (*A*)) having a resistance of 1,000 ohms, is connected across 150 volts. At its midpoint *c* a tap is brought out, to which a voltmeter *V*, having a resistance of 2,000 ohms, is connected through switch *S*. (*a*) What is the voltage from *c* to *b* when *S* is open? (*b*) What does the voltmeter read when switch *S* is closed?

157. It is desired to calibrate a 75-scale ammeter by means of a potentiometer, the connections being as shown in Figs. 132 and 134, pages 144 and 147. What value of standard resistance should be used?

158. A 10-amp., 115 volt watthour meter is tested for its accuracy by the method described in Par. 126, page 152. The watthour meter constant is 0.4. The rotating element makes 40 revolutions in 53.4 sec. The average power over this time interval is 1,122 watts. (*a*) What is its percentage of correct registration at this load? (*b*) What adjustment is necessary to bring the registration nearer to its correct value?

159. The meter of problem 158 is tested at light load. The rotating element makes two revolutions in 65.2 sec. and the average power over this interval is 46.6 watts. (*a*) What is the percentage of correct registration? (*b*) What adjustment is necessary to bring the meter nearer to correct registration?

QUESTIONS ON CHAPTER VIII

1. In what respect does the magnetic circuit resemble the electric circuit? What three factors, which do not exist to any appreciable extent in the electric circuit, make it difficult to determine magnetic relations accurately?

2. Define ampere-turns; magnetomotive force; gilbert; reluctance; oersted; permeance; permeability; flux; maxwell; gauss.

3. Define unit reluctance. Upon what three factors does reluctance depend? Write the equation which gives the reluctances of any magnetic conductor having a uniform cross-section. Write the general equation which gives the reluctance of any magnetic circuit consisting of several parts in series, each having uniform cross-section. Repeat for permeances in parallel.

4. Sketch the general shape of a curve plotted with flux density as ordinates and magnetomotive force per centimeter as abscissas. Why does the curve ultimately become concave downwards?

5. How is the permeability determined from the foregoing curve? Sketch a typical permeability curve.

6. Write the general equation for determining the flux in a single portion of a magnetic circuit. Write the general equation for the flux in the magnetic circuit when the circuit consists of several parts in series, each of uniform cross-section.

7. Sketch typical magnetization curves for silicon steel, cast steel, cast iron, and open-hearth sheet steel. Compare their magnetic properties and state their respective fields of use.

8. Discuss carefully the effect which is noted when a magnetic material forming a closed magnetic circuit is subjected to magnetomotive force, after which this magnetomotive force is reduced to zero. How may the magnetic flux within the specimen be brought to zero? What effect results from carrying the magnetomotive force to a negative value equal to the maximum positive value which was used? Describe the effects which result when the magnetomotive force is carried from its maximum negative value to its maximum positive value.

9. What is meant by "remanence?" "Coercive force?" Why does hysteresis represent energy loss?

10. Define linkages. Show that linkages are associated with every electric current. Sketch some typical examples. In what manner does "inductance" differ from "linkages?" Define inductance.

11. What effect is noted if a galvanometer is connected across the terminals of an insulated coil, and the north pole of a bar magnet is thrust into the coil? What effect is noted when the magnet is stationary? When it is withdrawn?

12. State the relation which exists between the direction of the induced electromotive force and the change of the magnetic linkages.

13. Upon what three factors does the induced electromotive force depend? Why is the equation preceded by a negative sign?

14. State Lenz's law. Upon what fundamental law is Lenz's law based?

15. When a direct current attempts to build up in a circuit having self-inductance, what effect is produced by the flux which this current produces? Show that the resulting reactions tend to *prevent* the increase of current.

16. Make a sketch illustrating the rise of current with time.

17. Theoretically, how much time is required for the current to reach its Ohm's law value? Practically, what is the order of magnitude of the time required for the current to reach its Ohm's law value?

18. In what manner does the current die out in an inductive circuit carrying current when the circuit is short-circuited?

19. Upon what three factors does the electromotive force of self-induction depend?

20. Show that an expenditure of energy is not necessary to maintain a steady magnetic field. Write the equation giving the energy of the magnetic field.

21. How may the energy of the magnetic field manifest itself? Show how the energy of the magnetic field is utilized for gas ignition.

22. Show that a change of current in one circuit may induce an electromotive force in another circuit entirely insulated from the first one. What relation does the induced electromotive force in the second circuit bear to the change of current in the first circuit?

23. Define "coefficient of coupling." How may this coefficient be increased? Define "mutual inductance."

24. Show how mutual inductance is utilized in the operation of the induction coil. Why is a condenser used?

25. Make a wiring diagram of a typical battery-ignition system. State the functions of the cam, the interrupter, the condenser, the ignition coil, and the distributor. Compare this system of ignition with induction-coil ignition.

PROBLEMS ON CHAPTER VIII

160. The field-winding of a certain four-pole generator consists of 280 shunt turns and 8 series turns per pole. When the shunt-field current is 3.2 amp. and, with no current in the series field, what is the magnetomotive force per pole in ampere-turns? When the shunt-field current is 1.84 amp.?

161. If the series turns, problem 160, are so connected as to aid the shunt turns and the current in the series field is 40 amp., what is the magnetomotive force per pole in ampere-turns for each of the two values of shunt-field current?

162. Repeat problem 161, with the series field connected incorrectly, that is, with its ampere-turns opposing the shunt-field ampere-turns.

***163.** A steel rod 1 cm. diameter, fits closely into two holes in a cast-iron yoke Y (Fig. 163 (A)). The yoke is so massive that its reluctance is negligible as compared with that of the rod. An exciting coil C having 200 turns surrounds the steel rod. With the steel rod in place, the flux in the rod is measured and found to be 3,800 maxwells when the current in the coil is 0.30 amp. The rod is then removed and the flux through the center of the

coil is found to be 2.98 maxwells. (*a*) What is the flux density B in the iron rod in gausses? (*b*) What is the permeability of the steel at this value of flux density? (*c*) What is the total magnetomotive force in gilberts, acting on the rod? (*d*) What is the magnetomotive force H per centimeter length of the rod if the rod is 20 cm. in length between the inside faces of the yoke? (*e*) Divide the flux density B in (*a*) by the intensity of the magnetic field H in (*d*) and compare with (*b*).

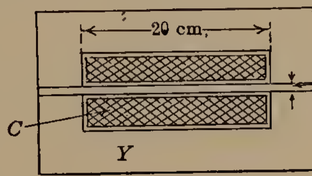


FIG. 163 (*A*).

*164. When the value of the current in the exciting coil is increased four times its value in problem 163, to 1.20 amp., the flux in the iron increases from 3,800 to 10,800 maxwells. Repeat (*a*) to (*e*) inclusive, problem 163. Explain the change in permeability.

*165. In problem 163 (*a*) Calculate the reluctance of the air space occupied by the steel rod. (*b*) Calculate the reluctance of the steel rod. (*c*) Find the ratio of (*a*) to (*b*).

*166. In problem 165, calculate the permeance in (*a*) and (*b*) and find the ratio of the permeance of the iron to that of the air space.

167. A portion of a magnetic circuit consists of two cylinders of iron, joined end to end (Fig. 167 (*A*)). One cylinder has a length of 12 cm., a diameter of 2.0 cm., and a permeability of 200; and the other has a length of 20 cm., a diameter of 2.5 cm., and a permeability of 800. Assuming that the flux is distributed uniformly over each cross-section for its entire length, find the reluctance of each cylinder and the combined reluctance of the two. Express results in the proper units.

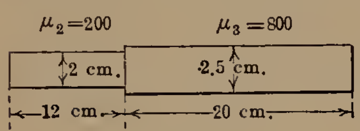


FIG. 167 (*A*).

168. The magnetic circuit of problem 167 is clamped between the pole-faces of an electromagnet between which a magnetomotive force of 800 amp-turns exists. Assuming the permeabilities to remain at the values given in problem 167, find the total magnetomotive force in gilberts, and the total flux density in each cylinder.

169. A transformer core made of silicon steel has a mean length of 200 cm. and the maximum flux density in this core is 16,000 lines per square centimeter. (*a*) From Fig. 146, page 162, determine the permeability of the steel at this flux density. (*b*) How many ampere-turns per centimeter length of iron are necessary?

170. The cast steel yoke of a dynamo has a cross-section of 150 sq. cm., a mean length of 30 cm., and the flux is 2×10^6 maxwells. (*a*) What is the flux density? (*b*) From Fig. 144, page 160, determine the permeability of the steel at this flux density. (*c*) What is the reluctance of the yoke? (*d*) How many ampere-turns are required to send the flux through the yoke?

*171. An anchor-ring solenoid consists of a steel ring having a radial saw-cut of 0.7 mm. length. The cross-sectional area of the ring has a diameter of 1 cm. and the mean diameter across the ring is 8.0 cm. This solenoid

is wound with 150 turns of wire. When the current in the winding is 1.5 amp., the flux in the ring is 8,000 maxwells. (*a*) Find the c.g.s. flux linkages. (*b*) What are the c.g.s. flux linkages per ampere? (*c*) What is the self-inductance in henrys of the solenoid at this value of flux density?

***172.** Repeat problem 171 with a flux density in the solenoid of 9,000 gausses, and a current of 1.25 amp.

***173.** The current in problem 171 is reversed in 0.04 sec., which causes the flux to build up to 8,000 maxwells in the reverse direction. (*a*) What is the total change of flux in maxwells? (*b*) What is the average rate of change of flux in maxwells per second? (*c*) Compute the average induced electromotive force, using equation (64), page 167. (*d*) What is the average rate of change of current? (*e*) Compute the induced electromotive force, using equation (65), page 172.

***174.** What is the energy in joules stored in the magetic field of problem 171?

175. The field circuit of a two-pole shunt generator has a resistance of 150 ohms. When the terminal voltage is 120 volts, the flux is 1,500,000 maxwells. (The shunt-field circuit is connected directly across the machine terminals.) (*a*) If there are 2,000 turns per pole, what is the self-inductance of the field? (*b*) What is the energy stored in the magnetic field?

***176.** In Fig. 176 (*A*) are shown two coils *A* and *B* lying in parallel planes and having a common axis. Coil *A* has 400 turns and coil *B* has 250 turns. When 1.0 amp. flows in coil *A* with *B* open-circuited, the effective flux linking *A* is 2,000 maxwells, and of this flux 600 maxwells link *B*. (*a*) What is the self-inductance of coil *A*? (*b*) What is the coefficient of coupling of the two circuits?

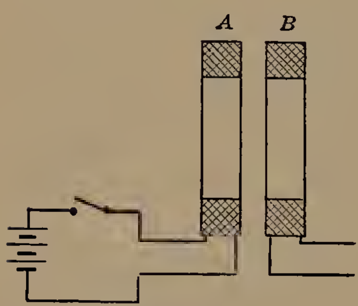


Fig. 176 (*A*).

***177.** The current in coil *A* is interrupted in 0.02 sec. Using equation (64), page 167, calculate the average induced electromotive force in coils *A* and *B*, respectively.

***178.** At what average rate does the current change, problem 177? Knowing the induced electromotive force in *B*, find the mutual inductance, using equation (67), page 176.

179. The primary of an ignition coil has 100 turns and, when the current is 0.6 amp., the flux in the core is 3,000 maxwells. The primary circuit is interrupted in 0.01 sec. It is necessary to induce 1,000 volts in the secondary in order that the spark may jump the gap. How many secondary turns are necessary?

***180.** In problem 179, what is the mutual inductance between the primary and the secondary? Use equation (67), page 176.

QUESTIONS ON CHAPTER IX

1. Compare *static* electricity with *dynamic* electricity.

2. Describe a simple method of producing electricity by friction, giving some of the materials which may be used. How may the presence of such electricity be detected?

3. If an electrified rod be held near a suspended pith ball, describe the phenomena which occur. Account for these phenomena. What is meant by an "induced charge?"

4. What is an electrophorus? What occurs when the metal plate is placed on the charged resinous material? Why is the top of the plate grounded before raising? How is the energy, involved in producing the positive charge, supplied?

5. What is the function of an electroscope? How is it constructed? What occurs when the brass knob is touched with a positive charge? What occurs, with a totally discharged electroscope, when a positive charge is brought into the vicinity but is not given to the knob?

6. How may a charge of opposite sign to the inducing charge be given to an electroscope without actual contact with the inducing charge? How may the sign of a charge be determined with the electroscope?

7. What is meant by "bound charge?" "Free charge?"

8. Describe the construction of a typical influence machine. How are the charges made to build up on the combined carriers and inductors? How is the charge removed from the carriers? What is the object of the Leyden jars?

9. What is meant by an electrostatic field? How may such a field be represented? Give five properties of electrostatic lines.

10. Explain the phenomena of the pith ball being attracted, repelled, etc. (Fig. 166, page 188) by means of the laws of induction and the laws of the electrostatic field.

11. Define a unit electrostatic charge. Compare with the unit magnetic pole.

12. State Coulomb's law of force between charged bodies.

13. Show that, so far as outside electrostatic effects are concerned, the charge on a sphere acts as if it were concentrated at the center of the sphere.

14. What effect is produced when two insulated ellipsoids are connected to the two terminals of a static machine?

15. How do the charges distribute themselves over the ellipsoids when the ellipsoids are brought so that two ends are adjacent but not in contact?

16. If the ellipsoids are well insulated, what effect is noted when the wires, connecting the ellipsoids to the induction machine, are disconnected?

17. One end of a positively charged insulated ellipsoid is brought near one end of an insulated ellipsoid having no initial electrical charge. What two effects are noted?

18. What effect is noted when the further end of the second ellipsoid is grounded? What is a "free charge" and a "bound charge?"

19. State the law governing the force existing between electric charges of unlike sign; of like sign.

20. State the laws of electrostatic induction.

21. Distinguish carefully between dielectric and insulator. What properties should a good insulator have? A good dielectric?

22. Compare electrostatic lines with magnetic lines of induction and with electric current, with particular reference to their effects on their respective mediums as their densities increase.

23. What is meant by "dielectric strength?" "Voltage gradient?" Give a simple example of air ruptured by excessive voltage gradient.

24. Define a condenser. What occurs when the two terminals of a well-insulated condenser are connected across a source of direct-current voltage? What occurs when the source of voltage is disconnected? What occurs when the charged condenser is short-circuited? Give a hydraulic analogy of these effects.

25. What relation exists between the charge on the condenser, and the voltage across the condenser plates? Define capacitance. What is the unit of capacitance in the practical system? Why is capacitance usually expressed in microfarads?

26. Show that, if the air in an air condenser be replaced by other dielectrics, the capacitance increases. What is meant by "specific inductive capacity" or "dielectric constant?" Of what order of magnitude are the dielectric constants of the ordinary dielectrics?

27. Derive the equation which gives the equivalent capacitance of a number of capacitances in parallel.

28. Repeat question 27 for a number of capacitances in series. What two fundamental relations are used in deriving this last equation?

29. State three equations which give the energy stored in a condenser. How does the stored energy in a given condenser vary with the voltage?

30. Make a diagram of connections used for measuring the capacitance of a condenser by the ballistic method. How is the ballistic throw of the galvanometer determined? What does a steady deflection of the galvanometer indicate? How is the constant of the apparatus determined?

31. Make a diagram of connections for measuring the capacitance of a condenser by the bridge method, employing alternating current.

32. Write the equation which gives the relation existing among the ratio arms and capacitances in the other two arms.

33 Describe briefly a method for locating an "open" in a cable by capacitance measurements. Make a diagram of connections. Derive the equation which gives the distance to the fault.

PROBLEMS ON CHAPTER IX

181. A sphere in air having a radius of 1 cm. is charged with 100 electrostatic units (statcoulombs) of electricity. (*a*) What is the force in dynes on a unit charge just at the surface of the sphere? (*b*) At a distance of 10 cm. from the center of the sphere?

182. Two spheres in air, 25 cm. apart between centers and each of a radius of 2 cm., are connected across the terminals of an influence machine.

The voltage is such that each acquires a charge of 120 statcoulombs. What force exists between the spheres? Is this force attraction or repulsion?

183. A unit positive charge is placed midway between the spheres, problem 182. (*a*) What force in dynes does the positively-charged sphere exert on this unit charge? (*b*) In what direction does it tend to make the unit charge move? (*c*) What force does the negatively-charged sphere exert on this unit charge? (*d*) In what direction does it tend to make the unit charge move? (*e*) What is the total force acting on the unit charge?

184. (*a*) Find the force in grams acting between two positive charges in air, one of 200 units and the other of 150 units, the centers of the charges being 40 cm. apart. (*b*) In what direction do the two charges tend to move?

185. A sample of 5-mil varnished cambric is tested for dielectric strength between flat, parallel electrodes. It punctures at 14,000 volts. At what voltage gradient, in volts per mil, does it puncture?

186. A sample of bond paper 4 mils thick punctures at 1,000 volts when tested between flat, parallel electrodes. What is the voltage gradient in volts per mil at break-down? In volts per millimeter?

187. The capacitance of a condenser is 0.000042 farad. (*a*) What is the charge in the condenser when the voltage across its terminals is 250 volts? (*b*) In what units is this charge given?

188. (*a*) What charge in microcoulombs exists on a 60-μf. condenser when it is connected across 600 volts? (*b*) How long would it be necessary for a steady current of 0.05 amp. to flow in order to charge this condenser with the microcoulombs given in (*a*)?

189. A certain condenser has a charge of 420 microcoulombs when its voltage is 120 volts. (*a*) What is its capacitance? (*b*) How much energy is stored in this condenser? (*c*) What is the energy when the voltage is doubled? (*d*) What is the ratio of energy in (*c*) to that in (*b*)?

190. A condenser has a capacitance of 220 μf. (*a*) What voltage is necessary to charge this condenser with 4,800 microcoulombs? (*b*) How much energy is stored in this condenser? (*c*) What is the energy when the charge is 9,600 microcoulombs?

191. A steady current of 0.005 amp. flows to charge a 60-μf. condenser. (*a*) What is the voltage across the condenser at the end of 6 sec.? (*b*) What energy is stored in the condenser?

192. An air condenser, having a capacitance of 0.0043 μf., is connected across 600 volts. (*a*) What is the charge on the condenser? (*b*) What energy is stored in this condenser? (*c*) This condenser is immersed in oil having a dielectric constant of 2.44. The voltage is maintained at 600 volts. What is the charge which now exists on this condenser? (*d*) What is the stored energy?

193. Two air condensers, having capacitances of 0.0072 μf. and 0.0036 μf., are connected in parallel. (*a*) What is the resulting capacitance of the combination? (*b*) If a charge of 100 microcoulombs is given to the combination, what is the charge on each condenser?

194. The 0.0036-μf. condenser, problem 193, is immersed in hot paraffin, which is then allowed to cool. The resulting dielectric constant is 2.1. Repeat (*a*) and (*b*).

195. Three condensers connected in parallel have charges of 2,200, 2,800, and 3,600 microcoulombs respectively when the voltage across the combination is 200 volts. (*a*) What is the resultant capacitance of the combination? (*b*) How much energy is stored in the combination?

196. Two condensers in which the leakage is negligible have capacitances of 16 and 20 μf. and are connected in series across 400 volts. (*a*) What is the resultant capacitance of the combination? (*b*) What is the charge on each condenser? (*c*) What is the voltage across each condenser?

197. The capacitance of two series-connected condensers is measured and found to be 6.67 μf. One of the condensers has a capacitance of 12 μf. (*a*) What is the capacitance of the other condenser? (*b*) If 100 volts be impressed across the combination, what is the voltage across each condenser?

198. Three condensers, having capacitances of 40, 60, and 80 μf. respectively, are connected in series across 600 volts. (*a*) What is the resultant capacitance of the combination? (*b*) At the instant of closing the switch, what is the voltage across each condenser?

199. Find the resultant capacitance of three series-connected condensers having respective capacitances of 2.8, 6.0, and 8 μf.

200. The voltage across the 6-μf. condenser in problem 199, is 42 volts. (*a*) Find the voltage across the combination. (*b*) Find the energy stored in each condenser.

201. The capacitance of an unknown condenser is measured by the method given on page 202, Par. 161. When a standard condenser, having a capacitance of 1.0 μf., is in circuit, the maximum throw of the galvanometer on charge is 10.8 cm. With the unknown condenser, the maximum throw of the galvanometer is 7.2 cm. (*a*) What is the galvanometer constant (Eq. 80, page 203)? (*b*) What is the unknown capacitance?

202. The capacitance of an unknown condenser C_x is measured by the method given on page 203, the connections shown in Fig. 178, page 203, being used. The standard condenser C_2 has a capacitance of 0.2 μf. When the bridge is in balance, *i.e.*, no perceptible sound is heard in the telephone receiver, R_1 is found to be 443 ohms, and R_2 1,000 ohms. What is the value of C_x?

203. If $C_x = 2.2$ μf., $C_2 = 0.8$ μf., and $R_2 = 1{,}000$ ohms (Fig. 178, page 203) to what value should R_1 be adjusted in order to obtain a balance?

***204.** A cable is tested for an "open" by the method given on page 204, Par. 162, the connections given in Fig. 179 being used. The perfect and the faulty cable each has a length of 1,200 ft. The Ayrton shunt is set at 1.0, when the capacitance measurement of the length x of the faulty cable is made and the ballistic throw of the galvanometer is 12.4 cm. When the capacitance measurement of the perfect cable plus the looped length $l-x$, is made, the Ayrton shunt is set at 0.1 and the ballistic throw of the galvanometer is 4.4 cm. Find the distance x.

QUESTIONS ON CHAPTER X

1. Define a generator. Upon what principle does a generator operate?

2. In what manner are the flux linkages in a generator coil varied?

3. Show that the electromotive force in a single coil may also be considered as being due to the active conductors of the coil *cutting* flux. When considering the electromotive force induced in a generator armature, why is this last point of view the more simple? Show the application of this principle to the generation of electromotive force in single armature coils.

4. State Fleming's right-hand rule.

5. What is the fundamental equation which gives the induced electromotive force in a single conductor cutting a magnetic field? What is the order of magnitude of this electromotive force in the ordinary generator?

6. Sketch the values of induced electromotive force for various positions of a flat coil rotating at a uniform rate in a uniform magnetic field. For what positions of the coil is the induced electromotive force zero? A maximum? Why does the electromotive force vary for different positions of the coil? Why does it reverse its sign?

7. What is the character of the electromotive force generated in such a coil? How may current be conducted to the external circuit? Describe slip-rings. What is mean by the "neutral plane"?

8. Why is the current taken from slip-rings not suited for direct-current purposes? How may a unidirectional current be obtained from a coil which generates alternating current? Describe a commutator and its operation.

9. Show how the current to the external circuit may be made practically steady.

10. Describe the construction of a gramme-ring winding. What are its advantages and its disadvantages?

11. What are the advantages of the drum-winding over the gramme-ring winding? Distinguish between end connections and active conductors. What is meant by "pitch," by "fractional pitch"? In what form and in what manner are armature coils placed in the slots?

12. How are the coils connected to commutator segments? In what manner is it desirable to number the coil-sides when designing a winding?

13. Define "front pitch," "back pitch," and "average pitch." What two limitations are placed on the pitch? How is the approximate average pitch determined?

15. Define a progressive winding and a retrogressive winding. What determines each?

15. Why is it often necessary to place several coil-sides in a single slot? What is meant by a "multiple coil?" What additional restriction is placed on the pitch used with this type of winding?

16. How may the number of paths through an armature be determined? What is the effect on the current rating of increasing the number of paths through the armature? The voltage rating? The kilowatt rating? How

may the number of paths through a simplex lap-winding be readily determined?

17. In what essential manner does the wave-winding differ from the lap-winding? What is meant by "front pitch," "back pitch," and "average pitch?" In a wave winding what relation must exist among certain winding elements after each passage around the armature?

18. Why is a wave-winding necessarily much more restricted in the choice of pitch than a lap-winding? When is a wave-winding retrogressive? Progressive? Why is the use of a "dummy coil" sometimes necessary with wave-windings?

19. Why may two brushes only be necessary with wave-windings? Why is a greater number of brushes than two often used?

20. How many paths through the armature does a simplex wave-winding give?

21. State two distinct advantages of wave-windings. Where are wave-windings used? Lap-windings?

22. What is the character of the active iron which is used for armatures? How are armature stampings made? What is the armature spider? What are ventilating ducts and how are they formed? What is the general form of slot insulation and how are the coils held in position?

23. State the two functions of the yoke or frame of the dynamo. Of what materials and how is the frame made?

24. Of what materials are the field-cores made? Describe two types of construction.

25. Describe the construction of the commutator. How are the segments insulated from one another? How are they held in place?

26. What type of insulation is used for the wire of the field coils? How is the series winding often placed on the field structure?

27. Of what materials are brushes made? Where are carbon and where are copper brushes used? Approximately what brush pressure should be used?

PROBLEMS ON CHAPTER X

205. A conductor 25 cm. long moves downwards at a uniform velocity of 4,000 cm. per second in the magnetic field of Fig. 205 (*A*). The field may

Fig. 205 (*A*).

Fig. 206 (*A*).

be considered as having a uniform density of 5,000 gausses. The conductor is perpendicular to the magnetic field and it moves in a direction perpendicular to itself and to the magnetic field. (*a*) What is the direction of the induced electromotive force in this conductor? (*b*) What is the value, in volts, of this induced electromotive force?

206. A single-turn coil, (Fig. 206 (*A*)) rotates at a uniform velocity of 1,000 r.p.m. in a magnetic field having a uniform density of 4,000 gausses.

The coil has an axial length of 30 cm. and a breadth of 25 cm. (*a*) What electromotive force is being induced in the coil when its plane is perpendicular to the magnetic field? (*b*) When the plane of the coil is parallel to the magnetic field?

***207.** Figure 207 (*A*) shows a north pole of a four-pole generator from which flux enters the armature. The average flux density *under the pole* is 45,000 lines per square inch (6,980 gausses). The combined span of the four poles is 0.7 of the entire armature periphery. The axial length of the active armature copper is 12 in. (30.5 cm.) and the peripheral velocity of the armature is 120 ft. (36.6 m.) per second. (*a*) What is the induced electromotive force per conductor when the conductor is directly under the pole? (*b*) What is the average induced electromotive force per revolution per conductor such as *a*? (*c*) How many series-connected conductors must there be between brushes, in order that the machine may generate 155 volts between brushes?

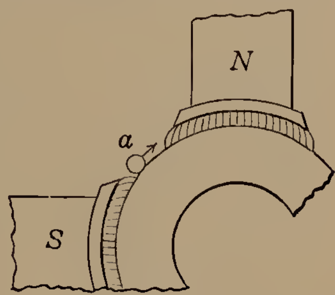


FIG. 207 (*A*).

***208.** The armature in problem 207 has a diameter of 24 in. (0.610 m.). What is its speed in r.p.m.?

209. On the armature of a certain four-pole generator there are 136 active conductors. The induced electromotive force per conductor is 7.4 volts. The armature winding is a simplex lap-winding, having four parallel paths between brushes. (*a*) What is the induced electromotive force between brushes? (*b*) If the current-carrying capacity per armature path is 40 amp., what is the current per terminal of the machine? (*c*) What is the kilowatt rating of the machine?

210. Repeat problem 209 for a simplex wave-winding in which there are but two parallel paths between terminals. (The number of active conductors must be reduced to 134 by means of a dummy coil in order that the winding may close (see page 222)).

211. There are 54 slots on the armature of a certain four-pole generator, and the winding is to be of the simplex lap-type having two coil-sides per slot. (*a*) How many coil-sides or winding elements will there be? (*b*) What should be the average pitch? (*c*) Give a possible value for the back pitch and another for the front pitch.

212. Sketch a portion of the winding, problem 211, using the values of back and front pitch selected (see Fig. 194, page 216).

213. (*a*) Using a front pitch and back pitch of 27, problem 212 (the average pitch is 27), on what element will the winding terminate after one passage around the armature, if a wave-winding is attempted? (*b*) Is it possible to design a wave-winding for this armature, if every winding element is included? (*c*) Give two values of the number of slots, in the immediate neighborhood of 54, which would make the wave-winding possible. (*d*) Which value gives a progressive winding and which gives a retrogressive winding?

214. A certain armature has 63 slots. A simplex lap-winding for a four-pole generator is to be designed with two elements per slot. (*a*) Give a possible value of front pitch and a possible value of back pitch. (*b*) Using these values, sketch a portion of the winding. (*c*) How many parallel paths are there between machine terminals?

215. (*a*) If the average pitch is 32, problem 214, is a wave-winding possible with this armature, if the two elements in every slot are to be included? (*b*) Sketch a portion of the winding, using a back pitch of 33 and a front pitch of 31 (see Fig. 200, page 223). (*c*) How many parallel paths between machine terminals are there?

216. An armature for a four-pole generator has 48 slots. A simplex lap-winding having four coil-sides per slot is to be used. (*a*) Select a possible value of back pitch and of front pitch. (*b*) Sketch the entire winding. (*c*) How many parallel paths between machine terminals are there in this winding?

217. Repeat problem 216 for a six-pole machine.

218. An armature for a six-pole machine has 98 slots and is to be wound with a simplex wave-winding having two coil-sides per slot. (*a*) What is the average pitch? (*b*) Select the nearest integer (whole number) to the average pitch and, using this value for front and back pitch, determine whether or not the winding is possible. (*c*) Sketch a portion of the winding.

QUESTIONS ON CHAPTER XI

1. From what fundamental relation is equation (84), page 231, giving the induced electromotive force in an armature, derived? What factors does this equation involve? How does the induced electromotive force vary with the flux per pole? The speed? The number of armature conductors? The paths through the armature?

2. In a given machine, what relation exists among the induced electromotive force, the flux, and the speed? With constant speed, to what factor is the induced electromotive force proportional?

3. Sketch a curve giving the relation between the flux per pole and the field ampere-turns. Discuss the shape of this curve and compare it with the saturation curves for iron and steel (see Fig. 146, page 162).

4. Show that a curve, obtained at constant speed, and plotted with field current as abscissas and induced electromotive force as ordinates, is similar to the curve having ampere-turns as abscissas and values of flux as ordinates.

5. Sketch a saturation curve taken with increasing and then decreasing values of field current. Why do the two curves not coincide?

6. Make a diagram of connections such as would be used for determining a saturation curve. Why should the machine be separately excited?

7. Make three sketches showing the connections for three types of generators in common use.

8. Describe in some detail the process by which a shunt generator "builds up" its voltage. What prevents the generator building up indefinitely?

9. Give four reasons why a generator may not build up. How may the probable reason for the machine not building up be determined in each case? What is the remedy in each case?

10. Sketch the armature and the field-cores of a bipolar generator. Show the field-coils diagrammatically and show the resulting flux distribution with no armature current.

11. Assume a direction of rotation for the armature and indicate the direction of the current in the various armature conductors. Show the general direction in the armature of the flux produced by these conductors acting alone.

12. Show the path of the resulting flux when the field is excited and when the armature delivers current. What relation does the direction of flux bear to the direction of rotation?

13. Why is it usually necessary to advance the brushes in the direction of rotation with increase of load? Show that, when the brushes are so advanced, certain conductors on the armature *demagnetize* the field and others *cross-magnetize* the field.

14. Of what two parts does commutation consist? What determines the time in which the reversal of current must take place? Why does the reversal of current in the individual coil often give rise to difficulties?

15. Why does it improve commutation to move the brushes ahead of the neutral plane? Why are carbon brushes preferable to metal brushes, in spite of their much higher resistance?

16. Why may the sparking under the brushes be severe, even though the voltages induced in the coils undergoing commutation are comparatively low in value? What is the order of magnitude of these voltages?

17. What is meant by "high mica?" What produces high mica? Describe two methods of reducing or eliminating high mica.

18. Why is it highly desirable that commutators be maintained in excellent condition at all times? How should carbon be removed from the surface of the commutator? Compare the uses of emery paper and sand paper on commutators. Describe the method of fitting the brushes to the commutator.

19. Why does sparking occur at the brushes of a generator under load conditions, if the brushes are allowed to remain in the neutral plane? State a method of neutralizing the flux which exists in the neutral zone because of armature reaction. Why should this flux be not only neutralized but reversed?

20. In a generator, what is the polarity of the interpole following a north main pole in the direction of rotation? Why?

21. How are the interpole windings connected in the circuits of the machine? Give reasons.

22. How are interpoles adjusted to the proper strength?

23. Why does the terminal voltage of a separately-excited generator operated with constant field current and at constant speed drop as the load increases? Give two reasons. Does the *induced* electromotive force remain constant with increase of load?

24. What relation exists between the induced electromotive force and the terminal voltage?

25. Sketch the connections which would be used in determining the characteristic of a shunt generator. What two factors are held constant during the test?

26. State the three factors which cause the terminal voltage of a shunt generator to drop as the load increases. What important factor, not present with the separately-excited generator, contributes to the drop in terminal voltage?

27. What is meant by a shunt generator "breaking down?" Why can the test for determining the operating characteristic often be carried to short-circuit? Sketch the entire characteristic from open-circuit to short-circuit and thence back to open-circuit. Give reasons for the shape of the characteristic.

28. What is a compound generator? Describe the series-winding and show how it is connected. Sketch a "long shunt" connection and a "short shunt" connection. Compare the effects of the two connections on the generator operation.

29. Through what agency does the series-winding affect the characteristic of the generator?

30. Sketch the characteristics of and define an overcompounded generator, a flat-compounded generator, and an undercompounded generator. Where is each used?

31. Sketch the connections of the series generator and give an example of its use.

32. What is a Tirrill regulator? What principle underlies its operation?

PROBLEMS ON CHAPTER XI

***219.** The armature of a 20-kw., four-pole, 1,100-r.p.m. generator has 64 slots and there are 12 series-connected conductors per slot. The armature is simplex lap-wound, giving four parallel paths. The pole-faces are 10 in. (25.4 cm.) square and the average flux density under the pole faces is 40,000 lines per square inch (6,200 gausses). (*a*) What is the flux per pole in maxwells? (*b*) Find the induced electromotive force when the generator runs at rated speed.

***220.** There are 47 slots on the armature of a certain four-pole generator. The flux in maxwells entering the armature from each north pole is 736,000 lines. The armature is to have a simplex wave-winding giving two parallel paths through the armature. The generator terminal voltage on open-circuit is to be 250 volts when the machine is operating at its rated speed of 1,350 r.p.m. How many conductors per slot should be used?

221. Find the induced electromotive force between brushes in a 32-pole, 120-r.p.m. generator when the flux per pole is 6,700,000 maxwells. The armature is simplex lap-wound and there are 14 slots per pole and 8 series-connected conductors per slot.

*222. A certain four-pole generator, when running at 800 r.p.m., generates 250 volts between brushes. There are 53 slots on the armature, eight series-connected conductors per slot, and the armature has a simplex wave-winding. What is the flux per pole when the machine is operating under these conditions?

*223. Find the constant K in equation (86), page 231, for the generator of problem 222.

224. The following data were taken during the experimental determination of a saturation curve:

Field current.....	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	3.75
Induced volts....	4.0	28.0	58.0	86.0	110.0	128.0	140.0	147.0	148.0

The speed was maintained constant at 1,000 r.p.m. Plot the saturation curve. Indicate the point at which saturation begins. What field current does the generator require to produce a no-load voltage of 120 volts? If the machine is self-excited, what field resistance gives 120 volts at the machine terminals at no load?

225. Replot the curve of problem 224 for a speed of 900 r.p.m. What field current is now required to produce a no-load voltage of 120 volts? If the machine is self-excited, what is the field resistance?

226. In problem 224, what field current flows as a result of the residual magnetism? What voltage is induced due to this field current, provided the initial field current has such a direction as to increase the residual magnetism? What field current flows as a result of this new induced voltage? (This is the process of building up.)

227. After the field current had reached the value of 3.75 amp., problem 224 it was then *decreased*, the speed being maintained constant at 1,000 r.p.m. The following data were obtained:

Field current......	3.5	3.0	2.5	2.0	1.5	1.0	0.5	0
Induced volts.....	148.0	143.0	132.0	116.0	94.0	66.0	37.0	10

With these data, plot a curve in conjunction with that of problem 224, indicating by arrows the direction of the cycle. Why do the two curves not coincide?

228. Plot the saturation curves given by the data of problems 224 and 227, for a speed of 1,200 r.p.m., indicating the direction of the cycle.

*229. There are 260 conductors on the surface of a two-pole armature and each conductor carries 18 amp. (*a*) When the brushes are in the geometrical neutral, how many cross-magnetizing and how many demagnetizing *ampere-conductors* are there? (*b*) How many cross-magnetizing and demagnetizing *ampere-turns* are there?

*230. Repeat problem 229 for the conditions existing when the brushes are advanced 15 space-degrees.

231. The no-load voltage of a separately-excited, 30-kw., 220-volt generator is 232 volts, and the terminal voltage when it is delivering its rated current is 220 volts. The speed is constant. The armature resistance is 0.060 ohm. What is the induced voltage when the machine is delivering

its rated load? Account for the difference between this induced voltage and the no-load voltage.

232. A 5-kw., 115-volt, shunt generator has an armature resistance of 0.12 ohm and a field resistance including rheostat of 60 ohms. (*a*) When the generator is delivering its rated current at rated voltage, what is the armature current? (*b*) What is the induced electromotive force in the armature?

233. How much power is lost in the armature and in the field circuit in problem 232? What is the total generated power?

234. In problem 232, the machine has a no-load voltage of 125 volts, the speed remaining unchanged. Account for the difference between this no-load voltage and the full-load induced voltage.

235. A 50-kw., 230-volt, shunt generator has a no-load voltage of 242 volts. The voltage-drop at rated load, due to armature reaction, is 3.0 volts and that due to the decreased field current is 4.0 volts. The armature resistance is 0.032 ohm, and the field-circuit resistance is 48 ohms. What is the terminal voltage at rated load? The speed at rated load is the same as that at no load.

236. (*a*) In the machine of problem 235 how much power is lost in the field at no load? (*b*) At rated load? (*c*) How much power is lost in the armature at rated load? (*d*) What is the total power generated?

237. A 230-volt, 50-kw., 900-r.p.m., flat-compounded generator has a shunt-field resistance, including rheostat, of 61 ohms, a series-field resistance of 0.010 ohm, and an armature resistance of 0.036 ohm. The no-load and the rated-load voltages are 230 volts at a constant speed of 900 r.p.m. The machine is connected long shunt. At rated load: (*a*) What power is lost in the shunt-field circuit? (*b*) What power is lost in the series field? (*c*) What power is lost in the armature? (*d*) What power is delivered to the load? (*e*) What is the total power generated?

238. Repeat problem 237 for short shunt. Neglect the slight increase in induced voltage due to the small increase in the voltage across the shunt field. This is, in part, compensated by the decreased series-field current.

239. A 25-kw., 230-volt, 800-r.p.m. compound generator has a no-load voltage of 220 volts and a rated-load voltage of 230 volts. The armature has a resistance of 0.080 ohm, the series field a resistance of 0.022 ohm, and the shunt-field circuit a resistance of 70 ohms. The machine is connected long shunt. (*a*) What is the shunt-field loss at no load? (*b*) At rated load? (*c*) What is the armature loss at rated load? (*d*) The series-field loss at rated load? (*e*) What power is delivered to the load? (*f*) What is the total power generated?

***240.** The generator of problem 239 is made flat-compounded by shunting the series field with a diverter whose resistance is 0.030 ohm. Both the no-load and the rated-load voltages are 220 volts. At rated load: (*a*) What is the armature current? (*b*) What is the armature loss? (*c*) What is the current in the series field? (*d*) What is the series-field loss? (*e*) What is the diverter loss? (*f*) What power is delivered to the load? (*g*) What is the total power generated?

*241. A 50-kw., 230-volt, 800-r.p.m., compound generator has interpoles. The armature resistance is 0.032 ohm, the series-field resistance is 0.012 ohm, the interpole-field resistance is 0.009 ohm, its diverter has a resistance of 0.020 ohm, and the shunt-field resistance is 52 ohms. The machine is connected long shunt. The no-load voltage is 225 volts and the rated-load voltage is 230 volts. Find at rated load: (*a*) The armature current; (*b*) the armature loss; (*c*) the series-field loss; (*d*) the interpole loss; (*e*) the diverter loss; (*f*) the shunt-field loss; (*g*) the total generated power.

The generator connections are shown in Fig. 241 (*A*).

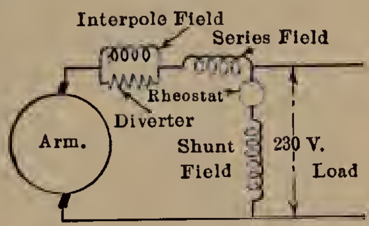


Fig. 241 (*A*).

QUESTIONS ON CHAPTER XII

1. Define a motor and compare the definition with that of a generator.

2. Upon what fundamental phenomenon does motor operation depend? Show by a sketch the field resulting from the placing of a single conductor carrying current in a uniform magnetic field, the conductor being perpendicular to the direction of the field.

3. In what manner may the force acting on the conductor be explained by the appearance of the resultant magnetic field?

4. To what three factors is the force acting on a conductor, placed in a magnetic field, proportional? What geometrical relation exists among the conductor, the direction of the field, and the direction of the force?

5. Give Fleming's left-hand rule. Describe another simple method by which the relations given by both Fleming's left-hand rule and Fleming's right-hand rule may be determined.

6. Define torque. What two quantities determine the magnitude of torque? In what units is torque expressed?

7. Show that, if a coil carries current and lies in a magnetic field with its plane parallel to the field, torque is developed. Illustrate by a sketch.

8. Show with sketches two or three positions of such a coil, including the position of maximum torque and the position of zero torque. How may a coil which begins to rotate be made to continue rotating when the coil reaches the position of zero torque?

9. Show with sketches the development of torque in a two-pole and in a multipolar machine, having numerous conductors on the armature surface.

10. In any given machine, to what two factors is torque proportional?

11. Why is the value of current in a motor armature, when in operation, not equal to the line voltage divided by the armature resistance?

12. Show that a motor armature, when in operation, must have an electromotive force induced within itself. By means of Fleming's left-hand and right-hand rules, or by other methods, show the relation of the induced electromotive force to the line voltage and to the armature current.

13. What effect does the counter electromotive force have on the value of current flowing into the armature? What is the *net* voltage acting in the armature circuit? How is the current determined?

14. Why must the line voltage exceed the counter electromotive force if a machine is to operate as a motor? What relation exists between the two voltages?

15. Describe a simple experiment which demonstrates clearly the existence of counter electromotive force.

16. In any given motor, what two quantities determine the speed? Write the equation for speed. How does the speed vary with the counter electromotive force? With the flux?

17. Sketch a bipolar motor showing the direction of the magnetic field due to the poles, the direction of current in the individual armature conductors with the brushes in the geometrical neutral, and the direction of rotation.

18. In question 17 show the direction of the armature magnetomotive force, and the direction of the resulting field. In what direction must the brushes be moved as the load is applied, in order that excessive sparking may not occur?

19. Show that in a motor a north-commutating pole must follow a north main pole taken in the direction of rotation.

20. What immediate reaction results from the application of load to a shunt motor? Show that the reactions which follow cause the motor to develop sufficient torque to carry the increase of load.

21. In a shunt motor, how does the internal torque vary with the current? Give reasons.

22. Why does the speed of a shunt motor ordinarily drop slightly with increase of load? What is the effect of armature reaction on the speed of the motor?

23. State some common industrial applications of shunt motors. Discuss the starting torque of such motors.

24. In a series motor, how does the torque vary with current if saturation and armature reaction are neglected? Why? What is the effect of saturation on the torque-current characteristic?

25. Sketch a typical current-speed characteristic of a series motor. Give two reasons for the decrease in speed with increase of load. What are the relative magnitudes of these two causes for decrease in speed?

26. Why does a series motor tend to race at light loads? Discuss the efficiency curve of a series motor.

27. Sketch the speed-current curve and the torque-current curve of a cumulative-compound motor. Discuss these curves. Name some industrial applications of this type of motor.

28. Sketch the speed-current curve and the torque-current curve of a differential-compound motor. Discuss these curves. Why is this type of motor little used?

29. Why must resistance be used when starting a motor? Show with a simple sketch the connections which would be used for starting a shunt motor.

30. Sketch the wiring diagram of a three-point starter, giving the reasons for each connection made. What is the object of the hold-up magnet?

31. Sketch the wiring diagram of a four-point starter. Compare its connections and its uses with those of the three-point starter.

32. Sketch the wiring diagram of one type of series-motor starter. What other types are used?

33. Distinguish between a controller and a starter.

34. Describe types of resistance units used in both starters and controllers.

35. With a given motor, what two factors only can be changed in order to change the speed?

36. What factor is changed in the armature-resistance method of speed control? Give two important objections to this method.

37. What is "multivoltage" speed control and what prevents wide use of this method?

38. With the ordinary motor, why cannot wide ranges of speed control be obtained by variation of field current? What ranges are obtainable with commutating poles?

39. Describe the method commonly used for controlling street-railway motors. How are two efficient operating speeds obtained?

40. What is meant by "multiple unit control?" Where is it used and what are its advantages?

41. Derive the equation giving the horsepower absorbed by a prony brake of the arm type.

42. Describe a simple rope-brake. Derive the equation which gives the power that it absorbs.

43. Why is it possible to measure speed quickly and accurately by means of a combination of direct-current magneto and voltmeter? How is the combination calibrated?

PROBLEMS ON CHAPTER XII

242. A conductor 30 cm. long lies in a uniform magnetic field having an intensity of 2,000 gausses, the direction of the conductor being perpendicular to the direction of the field. Compute the force in grams acting on the conductor when the current is 25 amp.

243. A rectangular coil of wire 30 cm. long and 20 cm. wide and having 15 turns lies in a magnetic field having a uniform intensity of 1,800 gausses. The plane of the coil is parallel to the direction of the field and the length of the coil is perpendicular to the direction of the field (see Fig. 235 (*a*), page 262). (*a*) Find the force acting on one of the 30-cm. coil-sides when the current in each turn of the coil is 20 amp. (*b*) What is the turning moment of the coil in gram-centimeters?

244. A motor armature 9 in. (22.9 cm.) in diameter and having an axial length of 10 in. (25.4 cm.) has 336 surface conductors. The motor has four poles. Seven-tenths of these conductors are directly under the pole-faces,

where the flux density averages 45,200 lines per square inch (7,000 gausses). The current per conductor is 20 amp. (*a*) What is the force in grams acting on any conductor which happens to be directly under a pole? (*b*) What is the total force acting tangentially along the periphery of the armature? Give results in kilograms and in pounds.

245. What torque in pounds-feet does the armature in problem 244 develop? In kilogram-meters?

246. A 30-in. (76.2-cm.) pulley is driven by a belt. The tension in the tight side of the belt is 194 lb. (88.0 kg.) and that in the loose side is 62 lb. (28.1 kg.). (*a*) What torque is acting on the pulley? (*b*) How many foot-pounds of work are performed in one revolution of the pulley? (*c*) If the pulley speed is 320 r.p.m., how many foot-pounds of work per minute are performed? (*d*) What horsepower is delivered to the pulley?

247. A certain gear having 35 teeth and a pitch diameter (diameter of the circle of contact) of 12 in. (30.5 cm.) is driven by a smaller gear having 15 teeth. The force acting at the points of contact of the teeth is 420 lb. (190 kg.). (*a*) What torque is applied to the driven gear? (*b*) What torque is developed by the smaller gear? (*c*) If the speed of the smaller gear is 640 r.p.m., what work is transmitted in foot-pounds per minute, neglecting friction? (*d*) What horsepower is transmitted?

248. A shunt motor operates with 230 volts at its terminals. The line current is 60 amp., the field resistance (including rheostat) is 96 ohms, and the armature resistance is 0.14 ohm. (*a*) Determine the armature current. (*b*) What counter electromotive force is this motor developing? (*c*) How much power is lost in the field circuit? (*d*) How much power is lost in heating the armature?

249. When the motor of problem 248 develops a counter electromotive force of 225 volts: (*a*) What is the *net* electromotive force acting through the armature circuit? (*b*) What is the value of the armature current? (*c*) What is the value of the line current?

250. The armature of the motor of problem 248 is brought to rest and by means of a series resistance the *line* current is made equal to 60 amp. (*a*) What is the value of the field current? (*b*) What is the value of the armature current? (*c*) Compare these results with those of problem 248 and explain the differences.

251. A 50-h.p., 230-volt, shunt motor requires line current of 180 amp. at rated load. The shunt-field current is 4.1 amp., and the armature resistance is 0.054 ohm. (*a*) What back electromotive force does the motor develop? (*b*) How much power is lost in heating the armature?

252. The motor of problem 251 runs at 760 r.p.m. at no load when the line current is 9.0 amp. Neglecting armature reaction, what is its speed at rated load?

***253.** If the armature reaction, problem 252, causes the value of the useful flux at rated load to be 2.5 per cent less than the value existing at no load, what is the rated-load speed?

***254.** The field current of the motor of problems 251 and 252 is reduced from 4.1 to 3.2 amp., when the motor is running without load. Owing to

saturation, the flux is reduced in the ratio of 4.1 to 3.5. The armature current does not change by any substantial amount. What is the no-load speed of the motor with this new value of field current?

255. A 5-h.p., 220-volt, 1,050-r.p.m., shunt motor has an armature resistance of 0.68 ohm and a field resistance of 207 ohms. At no load the motor takes 2.5 amp. from the line and runs at 1,060 r.p.m. Its rated-load current is 20.4 amp. Neglecting armature reaction, find the speed at rated load of this motor.

256. When a 10-h.p., 220-volt, series motor takes 24.0 amp. from the 220-volt line its speed is 950 r.p.m. and it develops 32.5 lb.-ft. (4.50 kg-m.) torque. It has an armature resistance of 0.19 ohm and a series-field resistance of 0.11 ohm. (*a*) Neglecting armature reaction and assuming that the motor operates on the straight portion of the saturation curve, what is its speed when it takes 41.0 amp.? (*b*) What internal torque does it develop when taking 41.0 amp.?

257. The motor armature of problem 256 cannot safely operate at a speed in excess of 1,700 r.p.m. What is the minimum safe current?

258. A 220-volt, 5-h.p., shunt motor and a 220-volt, 5-hp., series motor have rated-load speeds of 600 r.p.m. and both armatures require 22 amp. at rated load and develop 43.7 lb.-ft. (6.04 kg-m.) torque. Neglecting all losses: (*a*) What are their speeds when their armature currents are 11 amp.? (*b*) What are their internal torques when their armature currents are 11 amp.?

259. A 220-volt, 20-h.p., shunt motor runs at 840 r.p.m. when it takes its rated current of 76 amp. The field current is 2.1 amp. and the armature resistance is 0.10 ohm. The speed is reduced by inserting 0.8 ohm in the armature circuit. The motor current does not change. (*a*) What is the speed under these conditions? (*b*) What is the ratio of the torques with and without the additional resistance? (*c*) What percentage of the armature input is lost in the added resistance?

260. A prony brake similar to that shown in Fig. 252, page 284, is used to test a 115-volt, 5-h.p., shunt motor. The perpendicular distance from the center of the shaft to the center line of the balance is 2 ft. (0.610 m.). The dead weight or tare of the arm is +2.4 lb. (1.09 kg.). When the balance reads 11.1 lb. (5.03 kg.) the speed is 1,230 r.p.m., the ammeter giving the current input to the motor reads 32.4 amp., and the line voltmeter reads 116.0 volts. At this load: (*a*) What is the external torque of the motor? (*b*) What is the horsepower output of the motor? (*c*) What is the watt output of the motor? (*d*) What is the watt input to the motor? (*e*) What is the efficiency of the motor?

261. When the motor of problem 260 delivers its rated load of 5-h.p. the balance reads 13.3 lb. (6.03 kg.); the speed is 1,210 r.p.m.; the ammeter reads 39.9 amp. and the voltmeter reads 116.3 volts. Repeat (*a*) to (*e*) inclusive, problem 260.

262. A rope-brake similar to that shown in Fig. 254, page 287, is used to determine the output of a 115-volt, 2-h.p., shunt motor. The brake drum is 10 in. (0.254 m.) in diameter, the speed is 1,560 r.p.m., one balance reads

16.2 lb. (7.35 kg.) and the other reads 4.1 lb. (1.86 kg.). The ammeter which measures the motor current reads 13.7 amp. and the line voltmeter reads 114.5 volts. (*a*) What torque does the motor develop? (*b*) What horsepower does it develop? (*c*) What is its efficiency at this load?

263. When the motor, problem 262, delivers its rated load of 2 h.p. the speed is 1,530 r.p.m., one balance reads 22.7 lb. (10.3 kg.), and the other reads 6.2 lb. (2.81 kg.). The ammeter reads 18.1 amp. and the voltmeter reads 114.5 volts. Find (*a*), (*b*), and (*c*), problem 260, at this load.

QUESTIONS ON CHAPTER XIII

1. What two disadvantageous effects do energy losses within dynamos have on their operation? To what is the armature resistance loss due and how is its value determined? What precautions should be taken when measuring the armature resistance?

2. Of what does the loss in the shunt-field circuit consist? How is it determined?

3. What fundamental laws account for eddy-current losses in the armature iron? How are these losses reduced to a small value?

4. Why does hysteresis loss occur in the armature iron?

5. Of what two factors only are the iron losses and the friction losses functions? What are these losses called?

6. Give three equations which may be used for calculating efficiency. Under what conditions is each used?

7. State the approximate rated-load efficiencies of the following: 1-h.p. motor; 5-h.p. motor; 10-h.p. motor; 20-h.p. motor; a 500-kw. dynamo.

8. What difficulty is met when it is attempted to measure the efficiency of a large motor by measuring simultaneously the output and input? How do errors in such measurements affect the results obtained?

9. What difficulty is encountered when an attempt is made to measure the input to a generator?

10. How is a machine connected when it is desired to determine its stray power? Make any distinctions, if such exist, between the methods used for determining the stray power of a motor and those used for determining the stray power of a generator.

11. What effect do errors in the measurement of stray power have on the calculated efficiency?

12. What one factor determines in a large measure the rating of electrical apparatus? Name another factor which may limit the output of direct-current machinery.

13. Give the approximate temperature to which cotton and fibrous insulations may be subjected without rapid deterioration.

14. How may the approximate temperature of windings be determined by thermometer? What is meant by "hot-spot" temperature? How is its approximate value determined?

15. Describe the method, involving change of resistance, which is used for determining "hot-spot" temperature. What precautions are taken when making such a measurement with an armature? With a shunt field?

16. Give four reasons why it is desirable to operate several generating units in parallel rather than to attempt to use one or possibly two units.

17. Show that, if two shunt generators operate in parallel, the generator with the more drooping characteristic takes a lesser share of the load.

18. What relation should exist among several shunt generators for the most satisfactory parallel operation? State the procedure to be followed in putting an idle generator in service; in removing a generator from service. How may the load between generators be adjusted?

19. Show that two overcompounded generators, operating in parallel, are in an unstable condition of operation. How may this condition of instability be eliminated?

20. Make a wiring diagram of two compound generators in parallel. Where must the ammeters be connected?

21. Why is overload protection in an electric circuit necessary? Compare circuit breakers and fuses from the point of view of: first cost; space required; convenience; operating cost; speed of operation.

22. Upon what part of the switchboard should circuit breakers be mounted and why?

23. What operating difficulties are encountered in the operation of automobile electric power plants which are not present in central stations?

24. Describe the principle underlying "third brush" regulation. How may the battery charging rate be regulated?

25. Describe the vibrating type of circuit breaker.

26. What circuits are energized in the Delco system when the ignition switch is closed? Why is it desirable that the generator operate as a motor? Why is the generator disconnected when the cranking operation begins?

27. Why are two overrunning clutches used with this system? What happens when the generator speed becomes so low that its electromotive force is less than that of the battery?

28. Describe the cut-out relay. Where is it used?

PROBLEMS ON CHAPTER XIII

264. A 10-kw., 220-volt, shunt generator has the following losses at rated load: Armature resistance loss, 540 watts; field loss (including rheostat), 480 watts; stray power, 630 watts. What is the efficiency of the generator at rated load?

265. The generator of problem 264 operates as a motor. The armature current, the field current, and the stray power remain practically the same. What is its efficiency as a motor?

266. A 30-kw., 230-volt, 900-r.p.m., shunt generator has an armature resistance of 0.06 ohm, a field resistance of 80 ohms, and the stray-power loss at rated voltage and current is 1,180 watts (see problem 268). (*a*) What is its rated-load current? (*b*) What is the rated-load armature current? (*c*) What is the rated-load armature loss? (*d*) What is the rated-

load field loss? (*e*) What are the total losses at rated load? (*f*) What is its efficiency at rated load and voltage?

267. The generator voltage, problem 266, is adjusted to 230 volts at half load. The field resistance is increased to 85 ohms and the stray power is now 1,150 watts. Repeat (*a*) to (*f*).

268. The generator of problem 266 is connected as shown in Fig. 258, page 293, in order to measure its stray power. The machine is operated as a motor without load. The field current is adjusted to $^{230}/_{80} = 2.88$ amp. by means of the field rheostat and the speed is brought to 900 r.p.m. with the armature rheostat. The voltmeter across the armature reads 236 volts and the armature current is 5.00 amp. What is the stray-power loss?

269. A 10-h.p., 115-volt, 1,500-r.p.m., shunt motor, when operating near its rated load and at rated speed, takes 79.0 amp. at 115 volts from the line. Its armature resistance is 0.076 ohm and the field current is 3.2 amp. The motor is then run light, connections similar to those shown in Fig. 258, page 293, being used. The field current is maintained at 3.2 amp. and the speed is brought to 1,500 r.p.m. by means of the armature rheostat. The voltmeter across the armature then reads 109 volts and the armature ammeter reads 6.3 amp. Under the above conditions: (*a*) What is the armature current when this motor is operating near it rated load? (*b*) What is the field loss? (*c*) What is the stray power? (*d*) What is the motor output in watts?

270. In problem 269: (*a*) What is the motor efficiency? (*b*) What is the motor output in horsepower? (*c*) What is the torque at the motor shaft?

271. A 5-kw., 115-volt, 1,200-r.p.m., shunt generator has an armature resistance of 0.12 ohm and a field resistance, including the rheostat, of 60 ohms. The machine is run light as a motor in order to determine its stray power, the connections shown in Fig. 258, page 293, being used. When the field current is adjusted to 1.92 amp. and the speed to 1,200 r.p.m. by means of the armature rheostat, the armature current is 2.8 amp. and the voltmeter across the armature terminals reads 120 volts. Find: (*a*) the rated line current of the generator; (*b*) the rated armature current; (*c*) the armature loss at rated load; (*d*) the field loss at rated load; (*e*) the stray power, (*f*) the total loss; (*g*) the rated-load efficiency.

272. The armature copper and the field copper of a 220-volt, 10-h.p. motor are measured for their resistances after the motor has been standing idle for several hours in a room whose temperature is 20° C. The resistance of the armature between two marked segments is 0.28 ohm and that of the field excluding the rheostat is 148 ohms. The machine is then loaded with its rated load. After running 3 hr., the armature resistance is measured and found to be 0.307 ohm, and the field resistance 160.5 ohms. (*a*) What is the apparent temperature increase of the armature during this time? (*b*) Of the field? (*c*) Using the correction factor given by the A. I. E. E. Standardization Rules (see page 296), what are the "hot-spot" temperatures in the two cases?

273. After the machine of problem 272 operates 6 hr. at rated load, it is stopped and the armature resistance is again measured between the two

marked segments and found to be 0.325 ohm. The field resistance at this time has increased to 169.5 ohms. Find (*a*), (*b*), and (*c*), problem 272.

274. A 100-h.p., 600-volt, series (railway) motor is put under test for temperature rise. The initial series-field resistance is first measured after the machine has been standing for some time in a room whose temperature is 20° C., and is found to be 0.108 ohm. The armature copper resistance between marked segments under these conditions is found to be 0.152 ohm. The motor is then operated at 75-h.p. continuously for 10 hr. At the end of this period the series-field resistance is again measured and found to be 0.118 ohm and the armature resistance between the same marked segments 0.168 ohm. (*a*) Find the corrected "hot-spot" temperature of the field; (*b*) of the armature. (*c*) Are these operating temperatures safe for cotton insulation? (Note: Railway motors are rated on intermittent duty and usually are unable to operate continuously at their rated horsepower without overheating.)

***275.** Two 550-volt, 50-kw., shunt generators *A* and *B* are operated in parallel. The characteristic of *A* is such that when its no-load voltage is 550 volts the rated-load voltage is 500 volts. The characteristic of *B* is such that when its no-load voltage is 550 volts its rated-load voltage is 520 volts. For all practical purposes the characteristics may be considered as being straight lines. Both machines are connected in parallel at 550 volts with no load on the system. When *B* is delivering its rated current of 91 amp.: (*a*) What current is *A* delivering? (*b*) What is the voltage of the system? (*c*) What power is *A* delivering?

***276.** Find the current delivered by *B*, in problem 275 when *A* delivers 50 amp. Find the total power of the system.

277. Two shunt generators are operating in parallel. Their common terminal voltage is 115 volts. Generator *A* has an armature resistance of 0.08 ohm and its induced electromotive force is 119 volts. Generator *B* has an armature resistance of 0.12 ohm and its induced electromotive force is 120 volts. (*a*) What current does generator *A* deliver? (*b*) What current does generator *B* deliver? (*c*) What is the total power delivered to the bus-bars by the two generators?

INDEX